It is a characteristic of the new field theory that if the above equations form a satisfactory solution, then another satisfactory solution is

$$g_{\underline{ij}}'=f_{ij}(kx_1, kx_2, kx_3, kx_4) \quad (k=\text{constant}),$$
$$g_{\underset{\vee}{ij}}'=f_{ij}(kx_1, kx_2, kx_3, kx_4) \quad (k=\text{constant}),$$

representing a different physical reality. In addition, if $g_{\underline{ij}}$ and $g_{\underset{\vee}{ij}}$ satisfy certain conditions which Einstein gives for free space at (x_1, x_2, x_3, x_4), then $g_{\underline{ij}}'$ and $g_{\underset{\vee}{ij}}'$ satisfy the conditions at (kx_1, kx_2, kx_3, kx_4).

Consequently, in the new solution, as in the original one, there are three, isolated, non-free-space, space-time regions, representing three material bodies in motion. Taking the three coordinates x_1, x_2, and x_3 as ordinary Cartesian spatial coordinates and x_4 as the time, it can be shown by taking successive space-time sections at constant time that the second set of three bodies are similar in shape to the original bodies, $1/k$ as large in linear dimensions, $1/k$ times as far apart, and have an acceleration k times as large as the corresponding bodies of the original example, at time $t=0$.

In Einstein's theory, the formula for charge density is

$$I_{123}=dg_{\underset{\vee}{12}}/dx_3+dg_{\underset{\vee}{23}}/dx_1+dg_{\underset{\vee}{31}}/dx_2,$$

and it can be calculated immediately from this equation that a charge on a body in the second example is $1/k^2$ as large as the charge on the corresponding body of the first example. One of the bodies remains uncharged; since its acceleration is k times as large as before, and the direction of acceleration is unchanged, the gravitational field of each of the charged masses at the uncharged mass must be k times as large as before. Since they are $1/k$ as far away, their masses must be $1/k$ as large as before to account for their gravitational effect. Since their accelerations have multiplied by k, the force on each of the charged masses is the same as on the corresponding body in the first example. These forces are (1) the Coulomb force of each charged mass on the other, (2) the gravitational force of each charged mass on the other, and (3) the gravitational force of the uncharged mass on the charged bodies. Since the charged bodies have $1/k$ times their original masses and are $1/k$ as far apart, the second force is the same as in the first example. Therefore, the sum of forces (1) and (3) must be the same as before. Since the mass and position of the uncharged body were arbitrary, both forces (1) and (3) must be individually the same as previously. But the charged masses bear charges $1/k^2$ as large as originally and are $1/k$ times as far apart, so that the Coulomb force (1) should be $1/k^2$ as large as originally, and the theory leads to a contradiction with Coulomb's law.

A more general discussion and criticism of this theory will appear in a forthcoming article.

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A Comment on a Criticism of Unified Field Theory

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DR. C. P. Johnson's[1] argument touches upon a point of view of fundamental importance, which deserves a detailed discussion. In order to bring out the essential point, I will first bring up an analogy to the case at hand.

Question: Are the laws of the electromagnetic field invariant with respect to a change of sign of the electric charges, or, equivalently, of the electromagnetic field components? One is inclined to answer this question negatively, on the basis of our empirical knowledge. For, if we find a solution representing an atom with a positively charged nucleus and negatively charged surrounding particles, then there exists also a second solution for which the nucleus is negatively charged, and the particles around it positively charged, in contradiction to empirical results, according to which the nucleus is always positively charged; hence, one concludes that the equations do not possess the invariance property stated above.

This conclusion, however, is unwarranted. In fact, suppose the laws do possess this invariance property; it is possible that the predominance of nuclear charges of the one sign is due to the fact that configurations of opposed charges are unstable in their interactions. This would lead to a situation in which the one sign for the nuclear charge is predominant. A consideration of the mathematical possibilities shows that this alternative explanation (in which the laws possess the above invariance) appears more plausible.

I now turn my attention to the problem in which we are interested here.

In order for a system of field equations to be acceptable from a physical point of view, it has to account for the atomistic structure of physical reality. This comprises two general characteristic features:

(1) the quasi localization of mass (i.e., energy) and electrical charge;

(2) regions of space corresponding to a "particle" have discrete masses and charges. That is to say, if there exist elementary solutions of the equations which depend upon a continuous parameter, then the field equations must prevent the coexistence within one system of such elementary solutions pertaining to arbitrary values of their parameters. If a theory does not possess these two features, that is, if these features do not follow as conclusions from the theory, then the theory is inadmissable.

We now separate all conceivable systems of field equations into two classes, according to whether the individual equations are "homogeneous with respect to degree of differentiation" or not. By "homogeneous" we mean a type of equation such as is exemplified by the gravitational equations of empty space ($R_{ik}=0$). The R_{ik} consist of an aggregate of terms, which are either linear in the second derivatives of g_{ik} or else quadratic in the first derivatives of g_{ik}. We then say that R_{ik} is "homogeneous (of second order) in differentiation with respect to coordinates."

It seems to me that all relativistic systems of equations, which have a unitary structure, i.e., which are not composed of logically independent sets of terms, possess this property of homogeneity; this applies also to the system of equations which I call "generalized gravitational theory."

Now it seems that every such homogeneous system of equations must be incompatible with the requirement (2) given above. This is because any homogeneous system of equations possesses a family of solutions which depend, in a continuous way, on a parameter k. This is, in fact, the property which Johnson has used in his argument.

Let the field variables be denoted by g for short, and let $g(x)$ be a solution of the field equations; then also $g(kx)$ is a solution for any value of k. We refer to such a manifold of solutions as a family of "similar solutions." What is physically important here is the fact that both the mass and the charge of a "particle" vary continuously with k (all solutions being imbedded in the same Minkowski space). It would seem then that such a world, built out of solutions with continuously varying k values, violates the requirement (2).

However, the conclusion is based on the assumption that such solutions, with arbitrarily differing values of k, can coexist in the same world, without destroying each other through their interactions; whereas, it could be, for example, that the interaction terms would introduce inadmissable singularities into the field (this is what happens in the static case of two bodies in the theory of pure gravitation). If, however, the field equations exclude the possibility of coexistence of similar solutions in one and the same world, such an objection to the theory can no longer hold; Johnson's argument cannot be carried out then, for it too is based on the assumption of coexistence of similar solutions.

The above considerations show how careful one has to be in using general arguments to form a reliable judgment about the admissibility of a field theory from the empirical point of view.

[1] C. P. Johnson, Jr., preceding letter [Phys. Rev. **89**, 320 (1953)].