proaches R^2, as can be done with a causal interpretation based on a position representation of the wave function.[3]

If one starts with the causal interpretation of the quantum theory obtained in the position representation, with the quantum potential, $U=-\hbar^2\nabla^2R/2mR$, one can naturally make an arbitrary canonical transformation on the *particle* variables. Because it does not alter the quantum potential, however, such a transformation does not lead to an alternative causal interpretation of the quantum theory, but only to a redescription of the same causal interpretation in new mathematical terms, which are, however, usually a great deal more complicated than were the original terms. A new causal interpretation could come from such a procedure only if the canonical transformation on the particle variables were simultaneously accompanied by a corresponding linear transformation on the wave function. But such a linear transformation does not seem even in the simplest cases to lead to an acceptable causal interpretation. It would appear, therefore, that a causal interpretation of the quantum theory can be obtained only if we use the space-time representation of the wave function as a basis. This result is perhaps not too surprising, if one considers the fact that in all fields other than the quantum theory, space and time have thus far stood out as the natural frame for the description of the progress of physical phenomena.

The author would also like to express his disagreement with Epstein's (and Halpern's[4]) statement that the relation, $p=\partial S/\partial q$, constitutes a "quantum condition." Actually, it is a *consistent subsidiary condition* on the equations of motion, which if adopted at any time, say $t=0$, permits one to explain causally and continuously such processes as transitions between stationary states and interference in scattering problems.[2] On the other hand, Bohr's original quantum conditions were restrictions *contradicting* the equations of motion (which predicted continuous radiation in the case of the hydrogen atom, for example), and which could not explain either the process of transition or the appearance of interference in scattering problems. It would seem preferable to use different words to describe concepts which are so different.

[1] S. Epstein, preceding letter [Phys. Rev. **89**, 319 (1952).
[2] D. Bohm, Phys. Rev. **85**, 166 (1952).
[3] D. Bohm, Phys. Rev. (to be published).
[4] O. Halpern, Phys. Rev. **87**, 389 (1952).

The Thermal Neutron Fission and Capture Cross Section of U^{232}

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It has been shown[1] that Pa^{232} is a short-lived beta-emitter which decays to U^{232}, a long-lived alpha-emitter. U^{232} of good isotopic purity, therefore, can be prepared by the neutron irradiation of Pa^{231}, which has been shown to have a capture cross section for pile neutrons of 290 barns.[2]

Three samples of Pa^{231} were subjected to three widely different total neutron fluxes, and the resulting U^{232} was examined for fissionability. The Pa^{231}, irradiated as the oxide, was dissolved in a mixture of nitric and hydrofluoric acids, heated to near dryness to remove fluoride, and dissolved in 0.1*M* nitric acid. The uranium was extracted from this solution with diethyl ether after saturation with ammonium nitrate. The ether was washed with saturated ammonium nitrate and thin samples were prepared for alpha-counting and fission counting both by direct evaporation and by electroplating on platinum disks.

The weight of U^{232} on each plate was determined from the α-emission rate, assuming a half-life of 70 years.[3] The fissionability of each sample was then determined[4] in the thermal column of the Argonne heavy water pile. The rate of fission was compared with a sample of Pu^{239} of known weight, and the fission cross section of the U^{232} determined from the known fission cross section of Pu^{239} and the rate of fission in each sample of known weight. The data are summarized in Table I.

The indicated error in fission cross section values is limited to the probable errors in fission rate counting and alpha-emission rate counting. In addition, an uncertainty[3] of about 15 percent in the value for the half-life of U^{232} results in a corresponding uncertainty in the absolute value of the fission cross section. These measurements therefore yield a value of 83 ± 15 barns for the thermal neutron fission cross section of U^{232}.

The consistency of the results under conditions of large variation in total flux and amount of U^{232} formed seems to rule out any important effect from a fissionable contaminant present either as an impurity or formed during the irradiation. The fissionable isotope U^{233} would be formed by a second-order capture on either U^{232} or Pa^{232}, the latter giving Pa^{233}, a 27.4-day beta-emitter which decays to U^{233}. Sample 3 was examined for Pa^{233}, and the amount present indicated an amount of U^{233} sufficient to increase the apparent fission cross section of U^{232} by 0.50 barn. If one assumes that the remaining small increase in the observed fission cross-section value for sample 3 over those for samples 1 and 2 is real and further assumes that the increased value is due only to U^{233} formed by $U^{232}(n,\gamma)U^{233}$, then a rough estimate can be made of the pile neutron capture cross section of U^{232}. A value of about 200 barns is indicated, but since the observed differences are not outside the limits of experimental error, we conclude only that the capture cross section of U^{232} probably does not exceed 500 barns.

Table I. Fission cross section of U^{232} resulting from irradiations of Pa^{231}.

Sample No.	Total *nvt*	Weight of U^{232} formed, μg	Fission cross section $\times10^{24}$, cm^2
1	3.41×10^{16}	2.3×10^{-3}	82.9 ± 1.5
2	8.87×10^{17}	0.32	83.1 ± 1.5
3	4.06×10^{19}	26.2	84.6 ± 1.5[a]

[a] Corrected for contribution of U^{233} known to be present from the decay of Pa^{233}.

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[1] J. W. Gofman and G. T. Seaborg, *The Transuranium Elements: Research Papers* (McGraw-Hill Book Company, New York, 1948), Paper No. 19.14, National Nuclear Energy Series, Plutonium Project Record, Vol. 14B, Div. IV.
[2] Elson, John, and Sellers, Argonne National Laboratory Report 4282 (1951), to be published.
[3] James, Florin, Hopkins, and Ghiorso, *The Transuranium Elements: Research Papers* (McGraw-Hill Book Company, New York, 1948), Paper No. 22.8, National Nuclear Energy Series, Plutonium Project Record, Vol. 14B, Div. IV.
[4] For a description of the apparatus, see reference 3, Paper 22.29.

A Criticism of a Recent Unified Field Theory

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I WOULD like to show by an example that Dr. Albert Einstein's recent unified field theory is apparently not in agreement with the Newtonian and Coulomb laws of force between charged masses.

Consider two charged and one uncharged mass, separated by distances large compared with their physical dimensions, all substantially at rest at time $t=0$. Under the influence of their mutual attractions and repulsions, the three masses will undergo accelerations closely predicted by Newton's and Coulomb's laws. Corresponding to this motion, there will be (let us assume) a solution in Einstein's theory for all the quantities in it, including the gravitational and electromagnetic potentials which he indicates by the symbols $g_{\underline{ij}}$ and $g_{\underset{\vee}{ij}}$, as functions of the four chosen coordinates of space-time:

$$g_{\underline{ij}}=f_{\underline{ij}}(x_1, x_2, x_3, x_4) \quad (i, j=1, 2, 3, 4),$$

$$g_{\underset{\vee}{ij}}=f_{\underset{\vee}{ij}}(x_1, x_2, x_3, x_4) \quad (i, j=1, 2, 3, 4).$$

It is a characteristic of the new field theory that if the above equations form a satisfactory solution, then another satisfactory solution is

$$g_{\underline{ij}}'=f_{ij}(kx_1, kx_2, kx_3, kx_4) \quad (k=\text{constant}),$$
$$g_{\underset{\vee}{ij}}'=f_{ij}(kx_1, kx_2, kx_3, kx_4) \quad (k=\text{constant}),$$

representing a different physical reality. In addition, if $g_{\underline{ij}}$ and $g_{\underset{\vee}{ij}}$ satisfy certain conditions which Einstein gives for free space at (x_1, x_2, x_3, x_4), then $g_{\underline{ij}}'$ and $g_{\underset{\vee}{ij}}'$ satisfy the conditions at (kx_1, kx_2, kx_3, kx_4).

Consequently, in the new solution, as in the original one, there are three, isolated, non-free-space, space-time regions, representing three material bodies in motion. Taking the three coordinates x_1, x_2, and x_3 as ordinary Cartesian spatial coordinates and x_4 as the time, it can be shown by taking successive space-time sections at constant time that the second set of three bodies are similar in shape to the original bodies, $1/k$ as large in linear dimensions, $1/k$ times as far apart, and have an acceleration k times as large as the corresponding bodies of the original example, at time $t=0$.

In Einstein's theory, the formula for charge density is

$$I_{123}=dg_{\underset{\vee}{12}}/dx_3+dg_{\underset{\vee}{23}}/dx_1+dg_{\underset{\vee}{31}}/dx_2,$$

and it can be calculated immediately from this equation that a charge on a body in the second example is $1/k^2$ as large as the charge on the corresponding body of the first example. One of the bodies remains uncharged; since its acceleration is k times as large as before, and the direction of acceleration is unchanged, the gravitational field of each of the charged masses at the uncharged mass must be k times as large as before. Since they are $1/k$ as far away, their masses must be $1/k$ as large as before to account for their gravitational effect. Since their accelerations have multiplied by k, the force on each of the charged masses is the same as on the corresponding body in the first example. These forces are (1) the Coulomb force of each charged mass on the other, (2) the gravitational force of each charged mass on the other, and (3) the gravitational force of the uncharged mass on the charged bodies. Since the charged bodies have $1/k$ times their original masses and are $1/k$ as far apart, the second force is the same as in the first example. Therefore, the sum of forces (1) and (3) must be the same as before. Since the mass and position of the uncharged body were arbitrary, both forces (1) and (3) must be individually the same as previously. But the charged masses bear charges $1/k^2$ as large as originally and are $1/k$ times as far apart, so that the Coulomb force (1) should be $1/k^2$ as large as originally, and the theory leads to a contradiction with Coulomb's law.

A more general discussion and criticism of this theory will appear in a forthcoming article.

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A Comment on a Criticism of Unified Field Theory

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DR. C. P. Johnson's[1] argument touches upon a point of view of fundamental importance, which deserves a detailed discussion. In order to bring out the essential point, I will first bring up an analogy to the case at hand.

Question: Are the laws of the electromagnetic field invariant with respect to a change of sign of the electric charges, or, equivalently, of the electromagnetic field components? One is inclined to answer this question negatively, on the basis of our empirical knowledge. For, if we find a solution representing an atom with a positively charged nucleus and negatively charged surrounding particles, then there exists also a second solution for which the nucleus is negatively charged, and the particles around it positively charged, in contradiction to empirical results, according to which the nucleus is always positively charged; hence, one concludes that the equations do not possess the invariance property stated above.

This conclusion, however, is unwarranted. In fact, suppose the laws do possess this invariance property; it is possible that the predominance of nuclear charges of the one sign is due to the fact that configurations of opposed charges are unstable in their interactions. This would lead to a situation in which the one sign for the nuclear charge is predominant. A consideration of the mathematical possibilities shows that this alternative explanation (in which the laws possess the above invariance) appears more plausible.

I now turn my attention to the problem in which we are interested here.

In order for a system of field equations to be acceptable from a physical point of view, it has to account for the atomistic structure of physical reality. This comprises two general characteristic features:

(1) the quasi localization of mass (i.e., energy) and electrical charge;

(2) regions of space corresponding to a "particle" have discrete masses and charges. That is to say, if there exist elementary solutions of the equations which depend upon a continuous parameter, then the field equations must prevent the coexistence within one system of such elementary solutions pertaining to arbitrary values of their parameters. If a theory does not possess these two features, that is, if these features do not follow as conclusions from the theory, then the theory is inadmissable.

We now separate all conceivable systems of field equations into two classes, according to whether the individual equations are "homogeneous with respect to degree of differentiation" or not. By "homogeneous" we mean a type of equation such as is exemplified by the gravitational equations of empty space ($R_{ik}=0$). The R_{ik} consist of an aggregate of terms, which are either linear in the second derivatives of g_{ik} or else quadratic in the first derivatives of g_{ik}. We then say that R_{ik} is "homogeneous (of second order) in differentiation with respect to coordinates."

It seems to me that all relativistic systems of equations, which have a unitary structure, i.e., which are not composed of logically independent sets of terms, possess this property of homogeneity; this applies also to the system of equations which I call "generalized gravitational theory."

Now it seems that every such homogeneous system of equations must be incompatible with the requirement (2) given above. This is because any homogeneous system of equations possesses a family of solutions which depend, in a continuous way, on a parameter k. This is, in fact, the property which Johnson has used in his argument.

Let the field variables be denoted by g for short, and let $g(x)$ be a solution of the field equations; then also $g(kx)$ is a solution for any value of k. We refer to such a manifold of solutions as a family of "similar solutions." What is physically important here is the fact that both the mass and the charge of a "particle" vary continuously with k (all solutions being imbedded in the same Minkowski space). It would seem then that such a world, built out of solutions with continuously varying k values, violates the requirement (2).

However, the conclusion is based on the assumption that such solutions, with arbitrarily differing values of k, can coexist in the same world, without destroying each other through their interactions; whereas, it could be, for example, that the interaction terms would introduce inadmissable singularities into the field (this is what happens in the static case of two bodies in the theory of pure gravitation). If, however, the field equations exclude the possibility of coexistence of similar solutions in one and the same world, such an objection to the theory can no longer hold; Johnson's argument cannot be carried out then, for it too is based on the assumption of coexistence of similar solutions.

The above considerations show how careful one has to be in using general arguments to form a reliable judgment about the admissibility of a field theory from the empirical point of view.

[1] C. P. Johnson, Jr., preceding letter [Phys. Rev. **89**, 320 (1953)].