Substitution into the Schrödinger equation permits solving for $A_j(\mathbf{k})$:

$$A_j(\mathbf{k})=\frac{1}{(2\pi)^3\{E-E_j-T(k)\}}\{\langle\psi_j e^{i(\mathbf{k}\cdot\mathbf{r}_P)}|V_{N\xi}+V_{P\xi}|\varphi_i\psi_D\rangle+\langle\psi_j e^{i(\mathbf{k}\cdot\mathbf{r}_P)}|V_{P\xi}+V_{NP}|\Sigma_l\int d^3k'\psi_l A_l(\mathbf{k}')e^{i(\mathbf{k}'\cdot\mathbf{r}_P)}\rangle\}, \quad (3)$$

where

$$T(k)\equiv\hbar^2k^2/2M. \quad (4)$$

An alternative form is

$$A_j(\mathbf{k})=\frac{1}{(2\pi)^3\{E-E_j-T(k)\}}\{\langle\psi_j e^{i(\mathbf{k}\cdot\mathbf{r}_P)}|V_{P\xi}+V_{NP}|\Psi\rangle+\langle\psi_j e^{i(\mathbf{k}\cdot\mathbf{r}_P)}|V_{N\xi}-V_{NP}|\varphi_i\psi_D\rangle\}. \quad (5)$$

Finally, Eq. (5) is substituted into Eq. (2), giving an integral equation for the problem:

$$\Psi=\varphi_i\psi_D+\Sigma_j\int d^3k\frac{\psi_j e^{i(\mathbf{k}\cdot\mathbf{r}_P)}}{(2\pi)^3\{E-E_j-T(k)\}}\times\{\langle\psi_j e^{i(\mathbf{k}\cdot\mathbf{r}_P')}|V_{P\xi}+V_{NP}|\Psi\rangle+\langle\psi_j e^{i(\mathbf{k}\cdot\mathbf{r}_P')}|V_{N\xi}-V_{NP}|\varphi_i\psi_D\rangle\}. \quad (6)$$

It is a very basic approximation in stripping that $V_{P\xi}$ is unimportant, so it will be ignored without any further justification. To achieve Born approximation, it is only necessary to replace Ψ by $\varphi_i\psi_D$. Then V_{NP} goes out, and Eq. (6) gives directly the starting point of Daitch and French. The connection with Butler's paper is obtained by observing that the final state wave function, ψ_j, localizes the neutron; hence V_{NP} has no matrix elements to protons which pass far from the nucleus, as happens in stripping. Thus Butler's assumptions prescribe that we should immediately strike V_{NP} from Eq. (6), and we return once again to the Born result.

Equation (6) appears hopeful as a starting point for the investigation of corrections to the simple stripping calculation, and such an investigation is planned.

This paper is the outcome of a conversation with Professor J. B. French, and of many discussions with Dr. S. T. Butler.

* This work was performed while the author held a U. S. Atomic Energy Commission Postdoctoral Fellowship.

[1] Bhatia, Huang, Huby, and Newns, Phil. Mag. **43**, 485 (1952).
[2] P. B. Daitch and J. B. French, Phys. Rev. **87**, 900 (1952).
[3] S. T. Butler, Proc. Roy. Soc. (London) **208**, 559 (1951).

The Causal Interpretation of Quantum Mechanics

SAUL T. EPSTEIN
Boston University, Boston, Massachusetts
(Received October 16, 1952)

RECENTLY Bohm[1] has proposed a causal interpretation of quantum mechanics. It is based on the following observation: If one writes the Schrödinger function in *coordinate* space as $\psi=R_s\exp(iS_s/\hbar)$ (the subscript s denotes Schrödinger), then S_s satisfies an equation of the classical Hamilton-Jacobi type but containing, in addition to the ordinary potential, a "quantum-mechanical potential."

This suggests that one can consider ψ to describe the motion of a classical particle in the combined field of these potentials with, however, the "quantum condition" that one chooses only those solutions of the equations of motion such that the particle's momentum is given by

$$P=\partial S_s/\partial q. \quad (1)$$

Alternatively, one may say that one picks out only the trajectories described by the *particular* solution $S=S_s$ of the associated Hamilton-Jacobi equation.[1,2]

In this note we would like to point out another possibility of this type. We write the Schrödinger function in *momentum* space as $\phi=\rho_s\exp(i\sigma_s/\hbar)$ (ρ_s and σ_s real). One then finds that σ_s satisfies an equation of the classical Hamilton-Jacobi type but containing, in addition to the ordinary potential, a momentum-dependent "quantum-mechanical potential."[3]

This suggests that one can consider ϕ to describe the motion of a classical particle in the combined field of these potentials with, however, the "quantum condition" that one chooses only those solutions of the equations of motion such that

$$q=-\partial\sigma_s/\partial P. \quad (2)$$

Alternatively, one may say that one picks out only the trajectories described by the *particular* solution $\sigma=\sigma_s$ of the associated Hamilton-Jacobi equation.

In general, one must of course integrate the equations of motion to find P as a function of time and then use (2) to find q as a function of time. However, there is one case where this is unnecessary, and one sees that the present description gives different motions than Bohm's description. We consider a bound state of zero angular momentum. Then

$$S_s=\sigma_s=-Et.$$

Therefore, in Bohm's case $P=0$, $q=$constant, and the particle can be at rest anywhere; while in the present case $q=0$, the particle stays at the origin (and for the particular case of a harmonic oscillator, $P=$constant).

There are three further points we wish to mention:

(a) By starting from representations intermediate between the coordinate and momentum representations, it would seem that we could generate any number of such descriptions.

(b) It is tempting to speculate that this apparent multiplicity of causal descriptions is connected with the multiplicity of phase space-descriptions found by Moyal.[4]

(c) Finally we have *not* investigated whether any of these alternative descriptions can, when combined with the hypothesis of molecular chaos, yield the conventional probability interpretation of quantum-mechanics.

[1] D. Bohm, Phys. Rev. **85**, 166, 180 (1952).
[2] O. Halpern, Phys. Rev. **87**, 389 (1952); D. Bohm, Phys. Rev. **87**, 389 (1952).
[3] To get the classical Hamilton-Jacobi equations in momentum space one inserts $q=-\partial\sigma/\partial P$ in

$$H(P,q)+\partial\sigma/\partial t=0.$$

Since we have momentum-dependent potentials, P is the canonical momentum and not necessarily mq as in Bohm's description.
[4] J. E. Moyal, Proc. Cambridge Phil. Soc. **45**, 99 (1949).

Comments on a Letter Concerning the Causal Interpretation of the Quantum Theory

DAVID BOHM
Faculdade de Filosofia, Ciências e Letras, Universidade de São Paulo, São Paulo, Brazil
(Received November 17, 1952)

IN a recent letter,[1] Epstein has made the interesting suggestion that alternative causal interpretations[2] of the quantum theory may be possible, starting, for example, from a momentum representation of the wave function, or from other representations between that of position and momentum. In the present letter, the author would like to give the reasons why he thinks that this proposal cannot, in fact, be carried out.

To illustrate the difficulties involved, let us consider the problem of the hydrogen atom. In the momentum representation, the potential energy, e^2/r, takes the form of an integral operator which cannot be expressed as a convergent series of the operators, $x_i=i\hbar\partial/\partial p_i$. As a result, when the wave function is expressed as a product, $Re^{iS/\hbar}$, it does not seem to be possible to obtain the equivalent Hamilton-Jacobi equation for S, and the conservation equation for R^2. Moreover, without such a conservation equation, it is difficult to see how one could, with the aid of the hypothesis of molecular chaos, demonstrate that the probability density ap-