Peckar's discussion of the validity of his method rested on the quadratic approximation to the unperturbed energy. Whenever Peckar's effective mass equation is valid, its solutions are entirely equivalent to the solutions of Slater's equation. However, Slater's solution has a wider application because of the use of the operator $E_0(-i\hbar\nabla)$ instead of its quadratic approximation.

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[2] S. Peckar, J. Phys. (U.S.S.R.) **10**, 431 (1946).
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Nuclear Spin of $_{95}Am^{241}$

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THE spectrum of americium-241 has been photographed with a 30-foot spectrograph and found to contain many lines with wide hyperfine structure in the form of flag patterns, all apparently with six components. In lines that are well resolved there is a low degradation in spacing and intensity, and since the J values are expected to be high, the number of components is presumably spin-limited, with $I=5/2$. The existence of an appreciable quadrupole moment is indicated by a noticeable departure from the interval rule for some lines.

The Ground State of N^{14}

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TO explain the large ft value, 10^9, for the C^{14} β-decay, it has been proposed[1] that the ground state of N^{14} is an almost pure $^3D_1$ state, the C^{14} ground state being a $^1S_0$ state as in other even-even nuclei. The β-decay then requires $\Delta L=2$, and is therefore second-forbidden, giving an ft value of the experimental magnitude instead of the value $\sim 10^3$ expected in analogy with the $^1S_0\rightarrow{}^3S_1$ β-decay of He^6. The $^3D_1$ assignment is supported by the experimental value of the magnetic moment, 0.40 (in units $eh/2Mc$), since a pure $^3D_1$ state of maximum symmetry would have the value 0.31, while the magnetic moment of the $^3S_1$ state would be 0.88.

A serious difficulty is that the ft value of 10^9 demands that the $^3D_1$ state be extremely pure,[1,2] having no more than 1 part in 10^5 admixture of $^3S_1$ state, and conversely, that the ground state of C^{14} have an equally small admixture of $^5D_0$ state. This appears extremely unlikely. It is known that the ground state of the deuteron has $\sim$4 percent $^3D_1$ mixed with the $^3S_1$ ground state, due to the tensor force. Similarly the experimental magnetic moment of Li^6, 0.82, requires 10 percent $^3D_1$ admixture in the $^3S_1$ ground state. A mixture of just this magnitude is indicated by tensor force calculations.[3]

As an alternative explanation for the forbiddenness of the β-decay, we suggest that the ground state of N^{14} is predominantly a P state, either $^3P_1$ or $^1P_1$. If we further assume that the main spin-orbit force present is the tensor force, then it follows that in a second-order perturbation calculation, the tensor force will mix in some D and F states, but no $^3S_1$ state. A small amount of $^3S_1$ state will appear only in the next order of the perturbation calculation, due to the above-mentioned small admixture of D state. On this model, then, one would expect only 10^{-1}—10^{-2} percent admixture of $^3S_1$ state in the ground state. Furthermore, most of the $^3S_1$ admixture probably would come from configurations other than the lowest one, the $(1s)^4(2p)^{-2}$ configuration, and hence would not contribute to the β-decay. One would therefore expect an ft value of 10^7—10^9. The direct $^1S_0\rightarrow{}^{1,3}P_1$ transition would remain second-forbidden.

The suggestion that the ground state of N^{14} is predominantly a P state is, of course, beset with serious difficulties. A $^1P_1$ state does occur in the $(1s)^4(2p)^{-2}$ configuration, with a magnetic and moment value of 0.50 in fair agreement with the experimental data. However, this state belongs to the supermultiplet (111), and hence if the central forces are predominantly of the Majorana exchange type, this state should lie $\sim$5 Mev above the more symmetric S (and D) states.[4] To obtain a $^3P_1$ state of maximum symmetry one must go to a configuration that presumably lies much higher than the $(1s)^4(2p)^{-2}$ one, i.e., the $(1s)^3(2p)^{-2}(3d)$ or $(1s)^4(2p)^{-3}(3p)$ configurations. The kinetic energy of such configurations should be $\sim$10 Mev higher than that for the $(1s)^4(2p)^{-2}$ configuration. Also the magnetic moment of such a state would be 0.69, in poor agreement with the experimental value.

In any event, the experiment suggested by Messiah,[5] to observe the $N^{14}(n,d)C^{13*}$ angular distribution,[6] would determine the L value of the ground state of N^{14}. If the ground state is a P state, then one would observe an "$l=2$" angular distribution, just as if it were a D state (an "$l=1$" distribution being forbidden by parity considerations). However, the cross section for the reaction should be 10—100 times smaller for the P state (it would come only from the small amount of D state admixture) than for the pure $^3D_1$ state.

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[6] The mirror reaction $N^{14}(p,d)N^{13*}$ should serve equally well, since the 2.4-Mev level of N^{13} is the mirror analog of the 3.1-Mev level of C^{13}.

Validity of Born Approximation in Stripping*

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THE deuteron stripping process has been discussed in Born approximation in several recent publications,[1,2] and the result of this approximation has been shown[2] to be in very good agreement with the supposedly more accurate method of boundary conditions.[3] Born approximation assumes the incident wave function to be a satisfactory first approximation to the wave function of the system; hence it is generally considered unreliable for low energy reactions, where the perturbations are comparatively much stronger. But in the case of stripping, it will be shown that the result of the Born approximation happens to be obtainable by another route, which employs only the physically plausible assumptions of Butler's paper. It becomes clear why the Born approximation gives a satisfactory result.

The calculation will be presented as for an infinitely heavy target, and with Coulomb interaction with the deuteron omitted, although it is apparent how the latter should be inserted. By convention, the captured particle will be called the "neutron."

Let ξ denote the internal coordinates of the target nucleus. The complete Hamiltonian is

$$H=H_0(\xi)+V_{\xi N}+V_{\xi P}+V_{NP}+T_N+T_P. \quad (1)$$

Here the V_{ij} are the various potential energies, as indicated, and T_j are the kinetic energy operators for the deuteron particles.

The complete wave function is Ψ, and is expanded as follows:

$$\Psi=\varphi_i(\xi)\psi_D(N,P)+\Sigma_j\int d^3kA_j(\mathbf{k})\psi_j(\xi,N)e^{i(\mathbf{k}\cdot\mathbf{r}_P)}. \quad (2)$$

Here the φ_j are normalized energy eigenstates of the target nucleus, ψ_j are normalized energy eigenstates of the product nucleus, and ψ_D is the incident deuteron wave function.

Substitution into the Schrödinger equation permits solving for $A_j(\mathbf{k})$:

$$A_j(\mathbf{k})=\frac{1}{(2\pi)^3\{E-E_j-T(k)\}}\{\langle\psi_j e^{i(\mathbf{k}\cdot\mathbf{r}_P)}|V_{N\xi}+V_{P\xi}|\varphi_i\psi_D\rangle + \langle\psi_j e^{i(\mathbf{k}\cdot\mathbf{r}_P)}|V_{P\xi}+V_{NP}|\Sigma_l\int d^3k'\psi_l A_l(\mathbf{k}')e^{i(\mathbf{k}'\cdot\mathbf{r}_P)}\rangle\}, \quad (3)$$

where

$$T(k)\equiv\hbar^2k^2/2M. \quad (4)$$

An alternative form is

$$A_j(\mathbf{k})=\frac{1}{(2\pi)^3\{E-E_j-T(k)\}}\{\langle\psi_j e^{i(\mathbf{k}\cdot\mathbf{r}_P)}|V_{P\xi}+V_{NP}|\Psi\rangle + \langle\psi_j e^{i(\mathbf{k}\cdot\mathbf{r}_P)}|V_{N\xi}-V_{NP}|\varphi_i\psi_D\rangle\}. \quad (5)$$

Finally, Eq. (5) is substituted into Eq. (2), giving an integral equation for the problem:

$$\Psi=\varphi_i\psi_D+\Sigma_j\int d^3k\frac{\psi_j e^{i(\mathbf{k}\cdot\mathbf{r}_P)}}{(2\pi)^3\{E-E_j-T(k)\}} \times\{\langle\psi_j e^{i(\mathbf{k}\cdot\mathbf{r}_{P'})}|V_{P\xi}+V_{NP}|\Psi\rangle + \langle\psi_j e^{i(\mathbf{k}\cdot\mathbf{r}_{P'})}|V_{N\xi}-V_{NP}|\varphi_i\psi_D\rangle\}. \quad (6)$$

It is a very basic approximation in stripping that $V_{P\xi}$ is unimportant, so it will be ignored without any further justification. To achieve Born approximation, it is only necessary to replace Ψ by $\varphi_i\psi_D$. Then V_{NP} goes out, and Eq. (6) gives directly the starting point of Daitch and French. The connection with Butler's paper is obtained by observing that the final state wave function, ψ_j, localizes the neutron; hence V_{NP} has no matrix elements to protons which pass far from the nucleus, as happens in stripping. Thus Butler's assumptions prescribe that we should immediately strike V_{NP} from Eq. (6), and we return once again to the Born result.

Equation (6) appears hopeful as a starting point for the investigation of corrections to the simple stripping calculation, and such an investigation is planned.

This paper is the outcome of a conversation with Professor J. B. French, and of many discussions with Dr. S. T. Butler.

* This work was performed while the author held a U. S. Atomic Energy Commission Postdoctoral Fellowship.

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The Causal Interpretation of Quantum Mechanics

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RECENTLY Bohm[1] has proposed a causal interpretation of quantum mechanics. It is based on the following observation: If one writes the Schrödinger function in *coordinate* space as $\psi=R_s\exp(iS_s/\hbar)$ (the subscript s denotes Schrödinger), then S_s satisfies an equation of the classical Hamilton-Jacobi type but containing, in addition to the ordinary potential, a "quantum-mechanical potential."

This suggests that one can consider ψ to describe the motion of a classical particle in the combined field of these potentials with, however, the "quantum condition" that one chooses only those solutions of the equations of motion such that the particle's momentum is given by

$$P=\partial S_s/\partial q. \quad (1)$$

Alternatively, one may say that one picks out only the trajectories described by the *particular* solution $S=S_s$ of the associated Hamilton-Jacobi equation.[1,2]

In this note we would like to point out another possibility of this type. We write the Schrödinger function in *momentum* space as $\phi=\rho_s\exp(i\sigma_s/\hbar)$ (ρ_s and σ_s real). One then finds that σ_s satisfies an equation of the classical Hamilton-Jacobi type but containing, in addition to the ordinary potential, a momentum-dependent "quantum-mechanical potential."[3]

This suggests that one can consider ϕ to describe the motion of a classical particle in the combined field of these potentials with, however, the "quantum condition" that one chooses only those solutions of the equations of motion such that

$$q=-\partial\sigma_s/\partial P. \quad (2)$$

Alternatively, one may say that one picks out only the trajectories described by the *particular* solution $\sigma=\sigma_s$ of the associated Hamilton-Jacobi equation.

In general, one must of course integrate the equations of motion to find P as a function of time and then use (2) to find q as a function of time. However, there is one case where this is unnecessary, and one sees that the present description gives different motions than Bohm's description. We consider a bound state of zero angular momentum. Then

$$S_s=\sigma_s=-Et.$$

Therefore, in Bohm's case $P=0$, $q=$constant, and the particle can be at rest anywhere; while in the present case $q=0$, the particle stays at the origin (and for the particular case of a harmonic oscillator, $P=$constant).

There are three further points we wish to mention:

(a) By starting from representations intermediate between the coordinate and momentum representations, it would seem that we could generate any number of such descriptions.

(b) It is tempting to speculate that this apparent multiplicity of causal descriptions is connected with the multiplicity of phase space-descriptions found by Moyal.[4]

(c) Finally we have *not* investigated whether any of these alternative descriptions can, when combined with the hypothesis of molecular chaos, yield the conventional probability interpretation of quantum-mechanics.

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[2] O. Halpern, Phys. Rev. **87**, 389 (1952); D. Bohm, Phys. Rev. **87**, 389 (1952).
[3] To get the classical Hamilton-Jacobi equations in momentum space one inserts $q=-\partial\sigma/\partial P$ in

$$H(P,q)+\partial\sigma/\partial t=0.$$

Since we have momentum-dependent potentials, P is the canonical momentum and not necessarily mq as in Bohm's description.
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Comments on a Letter Concerning the Causal Interpretation of the Quantum Theory

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IN a recent letter,[1] Epstein has made the interesting suggestion that alternative causal interpretations[2] of the quantum theory may be possible, starting, for example, from a momentum representation of the wave function, or from other representations between that of position and momentum. In the present letter, the author would like to give the reasons why he thinks that this proposal cannot, in fact, be carried out.

To illustrate the difficulties involved, let us consider the problem of the hydrogen atom. In the momentum representation, the potential energy, e^2/r, takes the form of an integral operator which cannot be expressed as a convergent series of the operators, $x_i=i\hbar\partial/\partial p_i$. As a result, when the wave function is expressed as a product, $Re^{iS/\hbar}$, it does not seem to be possible to obtain the equivalent Hamilton-Jacobi equation for S, and the conservation equation for R^2. Moreover, without such a conservation equation, it is difficult to see how one could, with the aid of the hypothesis of molecular chaos, demonstrate that the probability density ap-