The meson field (φ) for particles with integral spin is supposed to be determined by a wave equation of the type

$$(i\beta^{\lambda}\partial_{\lambda}-m)\varphi=\text{const}\times\Sigma_{i,j}(\bar{\Psi}_i\Psi_j+\bar{\Psi}_j\Psi_i), \quad (1)$$

where the Ψ_i are nucleon wave functions, and m is a multiple of thc π-meson mass. Bhabha and others[10–13] have shown that relativistie restrictions imposed on the matrices β^{λ} require that they should satisfy

$$[\beta^{\lambda}, [\beta_{\mu}, \beta_{\nu}]]=\delta_{\mu}{}^{\lambda}\beta_{\nu}-\delta_{\nu}{}^{\lambda}\beta_{\mu}. \quad (2)$$

The masses of particles derived from nucleon interactions would then be simple multiples of the π-meson mass. The lowest spin state of such particles would be spin 0. These particles of zero spin may decay by neutrino emission to particles of spin $\frac{1}{2}$, of which the μ-meson is the representative of lowest (nonvanishing) mass. Another mode of decay is possible in which one or more mesons of spin 0 are created, with a Q-value which is either very small, or a multiple of m_{π}.

The classification of mesons with mass less than the proton mass is therefore as follows:

	Charged	Uncharged	Spin $\frac{1}{2}$
1.	$\pi^{\pm}\rightarrow\mu^{\pm}+\nu$	$\pi^0\rightarrow 2\gamma$	$\mu^{\pm}\rightarrow e^{\pm}+2\nu$
2.	$\zeta^{\pm}\rightarrow\pi^{\pm}+\pi^0$	$\zeta^0\rightarrow\pi^{+}+\pi^{-}$	
3.	$\tau^{\pm}\rightarrow\pi^{+}+\pi^{-}+\pi^{\pm}(?)$	$\tau^0(V_2{}^0)\rightarrow\pi^{+}+\pi^{-}$ or $\zeta^{\pm}+\pi^{\mp}$ or $\pi^{+}+\pi^{-}+\pi^0$	
4.	$\psi^{\pm}\rightarrow\pi^{\pm}+\zeta^0(?)$		$\kappa^{\pm}\rightarrow\mu^{\pm}+\pi^{+}+\pi^{-}$ or $\mu^{\pm}+2\nu$
5.	$\chi^{\pm}\rightarrow\pi^{\pm}+\tau^0$	$\chi^0\rightarrow\pi^{+}+\pi^{-}$ or $\psi^{\pm}+\pi^{\mp}$	

Most of these reactions are given in the Report of the International Physics Conference, Copenhagen, June, 1952, except $V^{\pm}$, as it was called there, has been identified with the spin-$\frac{1}{2}$ $\kappa^{\pm}$, since its mass would otherwise not fall in with this scheme.

It remains to account for those particles whose masses are greater than the proton mass. It seems that there is a "mass barrier" at the proton mass which is necessary to ensure the stability of the proton, and prevents the decay of isobaric states to particles whose masses are *all* below the proton mass. We assume the nucleon may be regarded as consisting of two particles, one a massive core of spin zero and the other of spin one-half, bound together by the meson forces, which lead to the existence of one or more energy levels, excited states corresponding to the possibly observed isobaric states of the nucleon.[1,6] To account for the existence of charged as well as neutral mesons of zero spin, it is necessary to suppose that the core (denoted by F) may be positively charged or neutral. If the spin-$\frac{1}{2}$ particle is identified with either an electron, or a neutrino, a simple explanation of β decay can be given:

$$N=F^0+\nu\rightarrow F^0+\nu+e^{+}+e^{-}\rightarrow P+\nu+e^{-} \quad (P=F^0+e^{+})$$

or

$$N=F^{+}+e^{-}\rightarrow F^{+}+e^{-}+\nu+\nu\rightarrow P+\nu+e^{-} \quad (P=F^{+}+\nu).$$

In the real excited state recognized as a $V_1{}^0$ particle, a μ^{-} meson may replace the electron.

Thus, the structure of the nucleon is in some ways comparable with that of the hydrogen atom, but in this instance there are no definite selection rules and the quanta may correspond to mesons of various masses. In the interaction of two nucleons, either or both may transit to excited states, through the exchange of spin-$\frac{1}{2}$ particles. These excited states may be real or virtual. As an example of a process in which virtual intermediate states are important, we cite the interaction of a π^{-} with a proton:

$$\begin{aligned}P+\pi^{-}&=(F^{+}+\nu)+\pi^{-}\\&\rightarrow(F^0+\nu)\text{ excited}\\&\rightarrow(F^0+\nu)+(\pi^0\text{ or }\gamma)\\&\rightarrow N+(\pi^0\text{ or }\gamma).\end{aligned}$$

We do not, however, suppose that nuclear forces are due directly to meson coupling, but rather to exchange between the spin-$\frac{1}{2}$ particles, which are actually coupled to the core by the meson field.

Most of the real excited states have short lives, decaying to the ground state with meson emission. In case both nucleons are excited, as is likely at high energies, two but not more than two mesons of the various species will be created in a nucleon-nucleon interaction. At low energies only π-mesons could be emitted; but at higher energies, there is an increasing probability for mesons of greater mass to result; and, as the experimental evidence suggests,[5,8] the proportion of heavier mesons may be quite high. This model of meson production is intermediate between the multiple[14–16] and plural[17] models which have hitherto been advanced. Obviously a succession of real transitions between excited states is possible. Possible observed reactions[1,6,7] are

$$V_1{}^0\rightarrow P+\pi^{-},$$
$$V_1{}^{\pm}\rightarrow V_1{}^0+\pi^{\pm}.$$

A further feature of this model is the resolution of the anomaly of the copious production of $v_1{}^0$ mesons in spite of their relatively long lifetimes. Since they mostly appear in real states as the result of nuclear collisions and not through the absorption of mesons, (though this is possible in principle), the coupling constant which characterizes the decay does not have to be as large as other theories would require. The interaction of the spin-zero mesons with the nucleon must clearly be quite strong, at least at low energies, on this model; however, as has been shown, there is no need to postulate any additional coupling to account for β-decay.

We wish to acknowledge discussions with Professor Fermi and Professor Wentzel on the above topics and also to thank Professor Wentzel for pointing out that a somewhat similar model for the nucleon had been discussed by him[18] in 1936.

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Interpretation of Isomeric Transitions of Electric Quadrupole Type

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IN the recent classification of nuclear isomers,[1] the transitions of electric quadrupole type are unique in possessing several examples of lifetimes appreciably shorter than predicted on the basis of the shell model. In some cases, the predicted lifetimes are more than a factor of a hundred too long. It appears evident that we have here the effect of some type of cooperative nuclear motion.[1]

A natural interpretation of these transitions is obtained in the model describing the nucleus in terms of the coupled single particle motion and nuclear surface oscillations.[2,3] According to such a model, the low-lying states of the nucleus arise either by excitation of the particle structure with an accompanying readjustment of the surface, or by an excitation of the surface without a change of the particle quantum numbers. In many cases, the first few excited states are of the former character and can therefore be classified by means of the shell model. The readjustment of the surface implies, however, that transition probabilities between two such states will in general be appreciably smaller than estimates based on shell model wave functions, as is in fact observed.[4]

In regions of high surface deformability, one obtains very low-lying states of collective excitation, which may be considered as rotational levels of the deformed nucleus. In the case of even-even nuclei, these states yield a spectrum with $I=0, 2, 4$, etc., even parity, and with energy values

$$E_I=(\hbar^2/2\mathcal{J})I(I+1), \tag{1}$$

neglecting the influence of rotation-vibration interaction. The effective moment of inertia $\mathcal{J}$ is related to the deformation of the nucleus and is given by

$$\mathcal{J}=3B\beta^2 \tag{2}$$

in terms of β the deformation parameter, and B, the mass parameter of the surface oscillators.[5] The wave functions for these states are obtained from the strong coupling solutions $\Omega=K=0$.

These states possess the spins, parities, low energies, and very short lifetimes of the observed levels. The transition from the first excited state to the ground state is of electric quadrupole type, with a probability per unit time given by

$$T=(\kappa^5/300\hbar)e^2Q_0^2, \tag{3}$$

where κ is the wave number of the emitted photon, and Q_0 the intrinsic quadrupole moment of the nucleus, measured with respect to the symmetry axis. For a uniformly charged incompressible nucleus, we have

$$Q_0=[3/(5\pi)^{\frac{1}{2}}]\cdot ZR_0^2\beta, \tag{4}$$

with Z and R_0, the nuclear charge and radius.

From the empirically measured lifetime and energy, one can determine the deformation of the nucleus, expressed by Q_0, listed in column 5 of Table I. The energies E and transition probabilities

TABLE I. Electric quadrupole transitions in even-even nuclei with measured half-lives.

Nuclide	E (kev)	$\log T$ (sec^{-1})	$S=T_{\text{obs}}/T_{sp}$	$Q_0(10^{-24}\,\text{cm}^2)$
Er^{166}	80	7.91	180	10
Yb^{170}	84	7.94	150	9
Hf^{176}	89	8.01	150	9
Os^{186}	134	8.64	55	6
Hg^{196}	411	$\sim$11.0	$\sim$50	$\sim$6
Pb^{204}	374	6.34	1.7×10^{-3}	

T in columns two and three are taken from reference 1, with the exception of Hf^{176} [F. K. McGowan, Phys. Rev. **87**, 542 (1952); a total conversion coefficient of 4 has been assumed], and Hg^{196} [R. L. Graham and R. E. Bell, Phys. Rev. **84**, 380 (1951)]. Column four gives the factor S by which the transition probability exceeds that calculated for a single proton transition of type $d_{\frac{5}{2}}\rightarrow s_{\frac{1}{2}}$. A comparison with a two-proton transition of type $(j^2)_{J=2}\rightarrow(j^2)_{J=0}$ would give slightly larger S-values. For the fast transitions, the intrinsic quadrupole moments, calculated from the observed lifetimes and energies by means of formula (3), are listed in the last column.

The values obtained for Q_0 are just of the magnitude encountered in the measured quadrupole moments[6] for this region of elements and in the values derived from anomalies in isotope shifts.[7]

While low-lying rotational levels are expected in regions of high deformability, in regions of very low deformability, as near shell closings, the first excited states should correspond to excitation of the particle structure. Thus, the very long lifetime of Pb^{204} is associated with its proximity to shell closings at $Z=82$ and $N=126$. The reason for its S-value being smaller than unity may derive from the circumstance that the excitation is presumably of the neutrons, the protons forming a closed shell structure.

Further measurements of the lifetimes of the $I=2$ first excited state in even-even nuclei would be of interest in establishing the rotational or particle character of these states.

From the measured lifetimes and energies of the rotational levels one can also determine the mass parameter B. For the levels listed in Table I, one obtains values significantly larger than those estimated from the liquid drop model of nuclear deformations.[8] Also the analysis of other nuclear properties, such as static magnetic dipole and electric quadrupole moments, gives evidence of a similar deviation from the simple hydrodynamical approximation.

A more detailed discussion of these problems will be shortly forthcoming[3] in connection with a fuller account of the coupled surface, single particle model and its consequences for various nuclear properties.

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Comparison of Slater's and Peckar's Treatments of Perturbed Periodic Potentials

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SLATER'S[1] and Peckar's[2] treatments of the perturbed periodic potential problem have the obvious difference that Peckar limited himself to quadratic terms in the expansion of the energy of the unperturbed problem as a function of crystal momentum, whereas Slater allowed this energy to be an arbitrary function of crystal momentum. Except for this difference, the fundamental equations of the two methods [Slater's equation (6) and Peckar's equation (4)] are identical. Clearly there must be a close relationship between the two methods, but its nature appears to have escaped mention in the literature. In fact, statements[3] can be found that imply important differences between the two treatments.

We start with Slater's form of solution, and from it derive Peckar's. From Slater's Eq. (5),

$$\psi_k(\mathbf{q})=\Sigma_n\,\Psi_k(\mathbf{Q}_n)a(\mathbf{q}-\mathbf{Q}_n). \tag{1}$$

Replacing the factor $a(\mathbf{q}-\mathbf{Q}_n)$ by Slater's equation (1A) and rearranging terms, we obtain,

$$\psi_k(\mathbf{q})=\sum_{p_0} N^{-\frac{1}{2}}\sum_n \Psi_k(\mathbf{Q}_n)\exp(-i\mathbf{p}_0\cdot\mathbf{Q}_n/\hbar)u_{p_0}(\mathbf{q}). \tag{2}$$

Let $b_k(\mathbf{p}_0)$ be given by Eq. (3), where $\mathbf{p}_0$ is an allowed crystal momentum from the central zone:

$$N^{-\frac{1}{2}}\,\Sigma_n\,\Psi_k(\mathbf{Q}_n)\exp(-i\mathbf{p}_0\cdot\mathbf{Q}_n/\hbar)=b_k(\mathbf{p}_0). \tag{3}$$

$$\psi_k(\mathbf{q})=\sum_{p_0} b_k(\mathbf{p}_0)u_{p_0}(\mathbf{q}). \tag{4}$$

Except for obvious differences in notation, Eq. (4) is identical with Peckar's equation (10), which gives the perturbed wave function as an expansion in terms of the unperturbed functions. However, our approach makes the rationale of Peckar's method quite clear. To determine the coefficients $b_k(\mathbf{p}_0)$, Peckar first finds the function $\Psi_k(\mathbf{Q}_n)$ as a solution of his reduced mass equation (4) or, more generally, as a solution of Slater's Eq. (6). He could then have used our Eq. (3) to find $b_k(\mathbf{p}_0)$; but actually he expanded Ψ_k as a Fourier series, the coefficients of which are $b_k(\mathbf{p}_0)$. In fact, by inverting our Eq. (3), we obtain

$$\Psi_k(\mathbf{Q}_n)=N^{-\frac{1}{2}}\sum_{p_0}\exp(i\mathbf{p}_0\cdot\mathbf{Q}_n/\hbar)b_k(\mathbf{p}_0), \tag{5}$$

which is Peckar's equation (6) in our notation, except for the restriction of the arguments $\mathbf{Q}_n$ to lattice points.