Letters to the Editor

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F-Center Wave Functions and Electronic *g*-Values in KCl Crystals*

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RECENTLY Hutchison and Noble[1] have observed electronic spin resonance in a KCl crystal colored with excess K. The resonance is believed to be associated with *F*-centers, but one cannot without confirmatory experiments exclude the possibility of contributions from other types of centers.[2] They observed the splitting factor $g=1.995\pm0.001$, which differs from the free electron $g=2.0023$ by $\Delta g=-0.007\pm0.001$. This shift, although small, has significant implications for the angular dependence of the ground state of the *F*-center electron. We show below that it is not possible to account for the order of magnitude of the shift using the standard theoretical model[3] of an *S*-electron in a central field. The shift Δg may, however, be explained approximately if the ground state has predominantly *G*-character, or if we use a molecular orbital wave function representing the excess electron as shared among the cations which bound the vacancy. The two models are closely related, but the molecular orbital viewpoint is perhaps more attractive on general grounds.

The spin-orbit interaction to the first order of perturbation theory gives $\Delta g=0$ if the ground state has *S*-character. The nonspherical components of the potential energy arise in large part from the charges of the six K^+ ions adjacent to the vacancy; these components mix the ground state with functions having $L=4$ and higher. Writing the additions as $\psi_G{}^M$, an improved ground-state function may be written

$$\psi'=\psi_S'+b(\psi_G{}^4+\psi_G{}^{-4}), \tag{1}$$

and the lowest excited state of interest here is expected to have the form

$$\psi''=\psi_S''+c(\psi_G{}^4-\psi_G{}^{-4}). \tag{2}$$

We neglect $\psi_G{}^0$ and terms with $L>4$. Both ψ' and ψ'' have $\Delta g=0$.

The spin-orbit interaction $\lambda\mathbf{L}\cdot\mathbf{S}$ will mix some ψ'' in ψ'. If for simplicity we set $b=c$, then for the perturbed ground state

$$\Delta g\cong-\frac{64b^2}{(1+2b^2)^2}\cdot\frac{\lambda}{\Delta}, \tag{3}$$

where Δ is the energy separation between the two states considered. We may safely assume that Δ is of the order of 1 ev, but it is more difficult to estimate λ. If we suppose that the $\psi_G{}^4$ function is chiefly concentrated about a circle passing through four of the K^+ ions surrounding the vacancy, we estimate $\lambda\approx10^{-17}$ erg, where the effective potential is taken as e^2/r with r measured from the center. Then $\lambda/\Delta\approx10^{-5}$, and in the limit $b\gg1$ we have $\Delta g\approx-2\times10^{-4}$. For $b=0.1$, $\Delta g\approx-6\times10^{-8}$, which is much too small.

We must thus suppose that there is a strong mixture of *G*-function in the ground state; there is no other possibility of accounting for the observed Δg value. The difference between the calculated maximum $\Delta g\approx-2\times10^{-4}$ and the observed $\Delta g\approx-7\times10^{-3}$ may not be significant in view of the considerable uncertainties surrounding the estimate of λ made above. The estimate using Eq. (7) below is closer, however.

It thus appears that we might use a *G*-function as a starting approximation rather than an *S*-function. The lowest energy *G*-function[4] in the cubic potential of the six K^+ ions is nondegenerate and transforms as $x^4+y^4+z^4$, as does also the molecular orbital,

$$\Psi=\sum_{i=1}^{6}\psi_i, \tag{4}$$

proposed by Muto,[5] and Inui and Uemura,[6] for the *F*-center ground state. Here ψ_i is the wave function of the valence electron when on the ith of the six neighboring K^+ ions.

On the molecular orbital model the important spin-orbit effects arise through the electrostatic polarization of the K atoms by the vacancy. The ground state of the valence electron on the K atom is normally pure $4s$, but the unsymmetrical electrical field associated with the vacancy will polarize the K atom, thus mixing a large amount of $4p$ function into the ground state. The polarized ground-state function for the atom on the positive x side of the vacancy is

$$\psi_1=\psi_S-(\epsilon/\sqrt{2})(\psi_P{}^1-\psi_P{}^{-1}). \tag{5}$$

The axis of quantization is taken along the z axis. The other ψ_i will be composed of similar mixtures of $4s$ and $4p$ functions, taken in such a way as to possess electric dipole moments in the direction of the vacancy. Spin-orbit interaction with the other $4p$ states will partially lift the quenching of the angular momenta of the atoms in the $x-y$ plane. In the presence of spin-orbit interaction $\lambda\mathbf{L}\cdot\mathbf{S}$, we have to first order

$$\psi_1=\psi_S-\frac{\epsilon}{\sqrt{2}}\psi_P{}^1\left(1+\frac{\lambda}{2\Delta}\right)+\frac{\epsilon}{\sqrt{2}}\psi_P{}^{-1}\left(1-\frac{\lambda}{2\Delta}\right). \tag{6}$$

The change in the g-factor for the *F*-center will then be

$$\Delta g=-\frac{4}{3}\frac{\lambda}{\Delta}\cdot\frac{\epsilon^2}{1+\epsilon^2}. \tag{7}$$

From the $4p$ doublet splitting of the K atom we obtain $\lambda=38\text{ cm}^{-1}$. For Δ we use the $4s-4p$ separation, which is close to the *F*-band energy. With hydrogenic wave functions we estimate $\epsilon=0.9$, which is consistent with an estimate on the basis of polarizability of a K atom. We find $\Delta g=-1.7\times10^{-3}$, of the correct order of magnitude, but somewhat too small. It would be raised somewhat by considering higher states.

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A Suggested Scheme for Meson Production

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A MASS of facts[1-8] is rapidly accumulating concerning the existence of two classes of particles, the first with masses intermediate between the π-meson and the proton, and the second with masses greater than the proton. Unsystematized, these facts tend to present a confusing appearance to a casual inspection. The purpose of this note is to suggest a classification of all observed particles, based on the properties of first-order wave equations, and the existence, already suggested from several quarters,[9] of isobaric excited states of the nucleon, and to propose a model for the structure of the nucleon.

The meson field (φ) for particles with integral spin is supposed to be determined by a wave equation of the type

$$(i\beta^{\lambda}\partial_{\lambda}-m)\varphi=\text{const}\times\Sigma_{i,\,j}(\bar{\Psi}_i\Psi_j+\bar{\Psi}_j\Psi_i), \tag{1}$$

where the Ψ_i are nucleon wave functions, and m is a multiple of thc π-meson mass. Bhabha and others[10–13] have shown that relativistie restrictions imposed on the matrices β^{λ} require that they should satisfy

$$[\beta^{\lambda},[\beta_{\mu},\beta_{\nu}]]=\delta_{\mu}{}^{\lambda}\beta_{\nu}-\delta_{\nu}{}^{\lambda}\beta_{\mu}. \tag{2}$$

The masses of particles derived from nucleon interactions would then be simple multiples of the π-meson mass. The lowest spin state of such particles would be spin 0. These particles of zero spin may decay by neutrino emission to particles of spin $\frac{1}{2}$, of which the μ-meson is the representative of lowest (nonvanishing) mass. Another mode of decay is possible in which one or more mesons of spin 0 are created, with a Q-value which is either very small, or a multiple of m_{π}.

The classification of mesons with mass less than the proton mass is therefore as follows:

	Charged	Uncharged	Spin $\frac{1}{2}$
1.	$\pi^{\pm}\rightarrow\mu^{\pm}+\nu$	$\pi^0\rightarrow 2\gamma$	$\mu^{\pm}\rightarrow e^{\pm}+2\nu$
2.	$\zeta^{\pm}\rightarrow\pi^{\pm}+\pi^0$	$\zeta^0\rightarrow\pi^{+}+\pi^{-}$	
3.	$\tau^{\pm}\rightarrow\pi^{+}+\pi^{-}+\pi^{\pm}(?)$	$\tau^0(V_2{}^0)\rightarrow\pi^{+}+\pi^{-}$ or $\zeta^{\pm}+\pi^{\mp}$ or $\pi^{+}+\pi^{-}+\pi^0$	
4.	$\psi^{\pm}\rightarrow\pi^{\pm}+\zeta^0(?)$		$\kappa^{\pm}\rightarrow\mu^{\pm}+\pi^{+}+\pi^{-}$ or $\mu^{\pm}+2\nu$
5.	$\chi^{\pm}\rightarrow\pi^{\pm}+\tau^0$	$\chi^0\rightarrow\pi^{+}+\pi^{-}$ or $\psi^{\pm}+\pi^{\mp}$	

Most of these reactions are given in the Report of the International Physics Conference, Copenhagen, June, 1952, except $V^{\pm}$, as it was called there, has been identified with the spin-$\frac{1}{2}$ $\kappa^{\pm}$, since its mass would otherwise not fall in with this scheme.

It remains to account for those particles whose masses are greater than the proton mass. It seems that there is a "mass barrier" at the proton mass which is necessary to ensure the stability of the proton, and prevents the decay of isobaric states to particles whose masses are *all* below the proton mass. We assume the nucleon may be regarded as consisting of two particles, one a massive core of spin zero and the other of spin one-half, bound together by the meson forces, which lead to the existence of one or more energy levels, excited states corresponding to the possibly observed isobaric states of the nucleon.[1, 6] To account for the existence of charged as well as neutral mesons of zero spin, it is necessary to suppose that the core (denoted by F) may be positively charged or neutral. If the spin-$\frac{1}{2}$ particle is identified with either an electron, or a neutrino, a simple explanation of β decay can be given:

$$N=F^0+\nu\rightarrow F^0+\nu+e^{+}+e^{-}\rightarrow P+\nu+e^{-}\quad (P=F^0+e^{+})$$

or

$$N=F^{+}+e^{-}\rightarrow F^{+}+e^{-}+\nu+\nu\rightarrow P+\nu+e^{-}\quad (P=F^{+}+\nu).$$

In the real excited state recognized as a $V_1{}^0$ particle, a μ^{-} meson may replace the electron.

Thus, the structure of the nucleon is in some ways comparable with that of the hydrogen atom, but in this instance there are no definite selection rules and the quanta may correspond to mesons of various masses. In the interaction of two nucleons, either or both may transit to excited states, through the exchange of spin-$\frac{1}{2}$ particles. These excited states may be real or virtual. As an example of a process in which virtual intermediate states are important, we cite the interaction of a π^{-} with a proton:

$$\begin{aligned} P+\pi^{-}&=(F^{+}+\nu)+\pi^{-}\\ &\rightarrow(F^0+\nu)\text{ excited}\\ &\rightarrow(F^0+\nu)+(\pi^0\text{ or }\gamma)\\ &\rightarrow N+(\pi^0\text{ or }\gamma). \end{aligned}$$

We do not, however, suppose that nuclear forces are due directly to meson coupling, but rather to exchange between the spin-$\frac{1}{2}$ particles, which are actually coupled to the core by the meson field.

Most of the real excited states have short lives, decaying to the ground state with meson emission. In case both nucleons are excited, as is likely at high energies, two but not more than two mesons of the various species will be created in a nucleon-nucleon interaction. At low energies only π-mesons could be emitted; but at higher energies, there is an increasing probability for mesons of greater mass to result; and, as the experimental evidence suggests,[5, 8] the proportion of heavier mesons may be quite high. This model of meson production is intermediate between the multiple[14–16] and plural[17] models which have hitherto been advanced. Obviously a succession of real transitions between excited states is possible. Possible observed reactions[1, 6, 7] are

$$V_1{}^0\rightarrow P+\pi^{-},$$
$$V_1{}^{\pm}\rightarrow V_1{}^0+\pi^{\pm}.$$

A further feature of this model is the resolution of the anomaly of the copious production of $v_1{}^0$ mesons in spite of their relatively long lifetimes. Since they mostly appear in real states as the result of nuclear collisions and not through the absorption of mesons, (though this is possible in principle), the coupling constant which characterizes the decay does not have to be as large as other theories would require. The interaction of the spin-zero mesons with the nucleon must clearly be quite strong, at least at low energies, on this model; however, as has been shown, there is no need to postulate any additional coupling to account for β-decay.

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Interpretation of Isomeric Transitions of Electric Quadrupole Type

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IN the recent classification of nuclear isomers,[1] the transitions of electric quadrupole type are unique in possessing several examples of lifetimes appreciably shorter than predicted on the basis of the shell model. In some cases, the predicted lifetimes are more than a factor of a hundred too long. It appears evident that we have here the effect of some type of cooperative nuclear motion.[1]

A natural interpretation of these transitions is obtained in the model describing the nucleus in terms of the coupled single particle motion and nuclear surface oscillations.[2, 3] According to such a model, the low-lying states of the nucleus arise either by excitation of the particle structure with an accompanying readjustment of the surface, or by an excitation of the surface without a change of the particle quantum numbers. In many cases, the first few excited states are of the former character and can therefore be classified by means of the shell model. The readjustment of the surface implies, however, that transition probabilities between two such states will