

## The Mesonic Auger Effect

G. R. BURBIDGE,\* *University of London Observatory, London, England*

AND

A. H. DE BORDE, *University College, London, England*

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A simplified nonrelativistic theory of the mesonic Auger effect is presented and discussed. Comparison is made between the predicted and observed numbers of Auger electrons emitted in  $\mu^-$  meson capture processes.

### I. INTRODUCTION

NEGATIVE mesons of low energy may be captured by atoms or molecules into stable states of motion by processes of the type

$$A + \mu^- \rightarrow A^* + e, \quad (1)$$

where  $A$  refers to an atom or molecule and  $A^*$  to the same molecule with an electron replaced by the negative meson  $\mu^-$ . The interaction of mesons of low energy with atoms and molecules has been the subject of several theoretical papers.<sup>1-4</sup> Unfortunately the capture process (1) probably occurs at energies where standard approximations are unreliable. The problem has been investigated by one of the authors<sup>5</sup> using both heavy and light particle approximations, but difficulties have arisen and it is hoped to report on this work at a later date.

Owing to its large mass compared with that of an electron, it is anticipated that in general a meson will first be captured into a state of high excitation in the atomic or molecular field. For example, if it is captured into an atomic state with a mean distance from the nucleus equal to that of a  $K$  shell electron, its ionization energy will then be equal to that of a  $K$  electron, but the corresponding total quantum number will be about 15 for a  $\mu$ -meson or 17 for a  $\pi$ -meson.

In such a state, the wave function at the nucleus will be very small, and the probability of interaction with the nucleons, therefore, negligible. The meson, however, can make transitions to states of lower excitation both by radiative and radiationless (Auger) processes, until it eventually reaches its  $1s$  state. Here the meson spends an appreciable fraction of its time within the nucleus,<sup>6</sup> particularly for atoms of medium or high atomic number, so that interaction with nucleons may occur.

Support for this picture of the processes involved in the capture of negative mesons is now available from

several experiments. Thus, Chang<sup>7</sup> and Hincks<sup>8</sup> have observed gamma-rays associated with the capture of  $\mu^-$ -mesons by nuclei. In the latter experiments an estimate of about 9-Mev energy emitted per stopped meson in lead was made, compared with an estimated ionization energy for the  $1s$  level of 9.56 Mev, after allowance for the finite size of the nucleus has been made. This is consistent with the interpretation that the rays arise from the radiative transitions of the captured  $\mu^-$ -meson in the atomic field.

Slow electrons associated with  $\mu$ -meson capture have been observed by Cosyns, Dilworth, Occhialini, Schoenberg, and Page,<sup>9</sup> and by Fry,<sup>10</sup> while similar electrons associated with  $\pi$ -meson capture have been observed by Menon, Muirhead, and Rochat.<sup>11</sup> These have been interpreted as Auger electrons ejected in radiationless transitions in the atomic field.

In the present work, the time required for transitions of the meson from an initial state of high excitation to its  $1s$  state is estimated, as well as the number and energy distribution of the Auger electrons ejected. In making these calculations, pending a fuller analysis of the capture process, some assumption has to be made about the initial state of atomic capture. It has been pointed out by Bohr<sup>12</sup> that for a given total quantum number,  $n$ , the meson is probably most likely to be captured into a circular orbit ( $l=n-1$ ) owing to the greater statistical weight of this orbit. This assumption is made, thereby greatly simplifying the problem as the only significant transitions are then radiative and Auger transitions of the type  $(n, n-1) \rightarrow (n-1, n-2)$ . In the following, transitions of this type, and of the type  $(n, n-2) \rightarrow (n-1, n-2)$  are considered.

For radiative transitions the selection rule  $\Delta l = \pm 1$  applies. For Auger transitions the electron can be ejected into states with  $l=0, 1, 2$ , etc., but estimates of the transition rates for the higher values of  $l$  show these to be appreciably smaller. Throughout these calcula-

\* Present address: Yerkes Observatory, University of Chicago, Williams Bay, Wisconsin.

<sup>1</sup> B. Ferretti, *Nuovo cimento* **5**, 325 (1948).

<sup>2</sup> R. Huby, *Phil. Mag.* **40**, 685 (1949).

<sup>3</sup> R. L. Rosenberg, *Phil. Mag.* **40**, 759 (1949).

<sup>4</sup> A. S. Wightman, *Phys. Rev.* **77**, 521 (1950).

<sup>5</sup> G. R. Burbidge, Ph.D. thesis, London (1951), (unpublished).

<sup>6</sup> J. A. Wheeler, *Revs. Modern Phys.* **21**, 133 (1949).

<sup>7</sup> W. Y. Chang, *Revs. Modern Phys.* **21**, 166 (1949).

<sup>8</sup> E. P. Hincks, *Phys. Rev.* **81**, 313 (1951).

<sup>9</sup> Cosyns, Dilworth, Occhialini, Schoenberg, and Page, *Proc. Phys. Soc. (London)* **A62**, 801 (1949).

<sup>10</sup> W. F. Fry, *Phys. Rev.* **83**, 594 (1951).

<sup>11</sup> Menon, Muirhead, and Rochat, *Phil. Mag.* **41**, 583 (1950).

<sup>12</sup> N. Bohr, quoted by B. Bruno, *Arkiv Mat., Astron. Fysik*, **36A**, No. 8, 44 (1948).

tions the mass of the  $\mu$ -meson has been put equal to  $210m_e$ .

## II. AUGER TRANSITION PROBLEM

The initial state of the system consists of an electron in its ground state  $\psi_i$  and a meson in an excited state  $\chi_i$  (quantum numbers,  $n_1, l_1, m_1$ ). The final state consists of an electron in a positive energy state  $\psi_f$ , (quantum numbers,  $l, m$ ) and a meson in a lower excited state  $\chi_f(n_2, l_2, m_2)$ . Hydrogen-like wave functions will be assumed (the Bohr radius  $a_0$  being replaced by  $a' = a_0/\mu$  for the meson, where  $\mu$  is the meson mass in atomic units).

We have<sup>13</sup>

$$\begin{aligned} \psi_f &= \left\{ \frac{m_e}{\hbar} \frac{2l+1}{4\pi} \frac{(l-|m|)!}{(l+|m|)!} \right\}^{\frac{1}{2}} 2^{l+1} \kappa^{l+\frac{1}{2}} e^{i\kappa r_2} r_2^l \\ &\quad \times {}_1F_1(l+1-iy; 2l+2, -2i\kappa r_2) |\Gamma(l+1-iy)| e^{\frac{1}{2}\pi y} \\ &\quad \times P_l^{|m|}(\cos\theta_2) e^{im\varphi_2} / (2l+1)!, \\ \chi_i &= \left\{ \frac{2l_1+1}{4\pi} \frac{(l_1-|m_1|)!}{(l_1+|m_1|)!} \frac{(n_1-l_1-1)!}{2n_1[(n_1+l_1)!]^3} \right\}^{\frac{1}{2}} \left( \frac{2\mu Z_1}{n_1 a_0} \right)^{l_1+\frac{1}{2}} \\ &\quad \times r_1^{l_1} \exp\left(-\frac{\mu Z_1}{n_1 a_0} r_1\right) L_{n_1+l_1}^{2l_1+1}\left(\frac{2\mu Z_1}{n_1 a_0} r_1\right) \\ &\quad \times P_{l_1}^{|m_1|}(\cos\theta_1) e^{im_1\varphi_1}, \end{aligned}$$

$\chi_f$  being the same as  $\chi_i$  apart from change of suffix. Here  $y = Z/\kappa a_0$ ,  $m_e$  is the electronic mass,  $Z$  and  $Z_1$  are the effective nuclear charges experienced by the electron and meson, respectively, and  $\kappa$  is the momentum of the ejected electron. The associated Legendre polynomials are those defined by Rodrigue's formula. The meson and electron coordinates are, respectively,  $(r_1, \theta_1, \varphi_1)$ ,  $(r_2, \theta_2, \varphi_2)$ .

The number of transitions per second is given by<sup>14</sup>

$$P_A = \left| \frac{1}{\hbar} \int \int \chi_f^* \psi_f^* \frac{\epsilon^2}{r_{12}} \chi_i \psi_i d\tau_1 d\tau_2 \right|^2. \quad (2)$$

The angular integrations are easily performed by expanding  $1/r_{12}$  in spherical harmonics:

$$\begin{aligned} \frac{1}{r_{12}} &= \sum_{N=0}^{\infty} \sum_{M=-N}^N \frac{(N-|M|)!}{(N+|M|)!} P_N^{|M|}(\cos\theta_1) \\ &\quad \times P_N^{|M|}(\cos\theta_2) e^{iM(\varphi_2-\varphi_1)} \begin{cases} r_1^N/r_2^{N+1} & (r_2 \geq r_1) \\ r_2^N/r_1^{N+1} & (r_1 \geq r_2). \end{cases} \quad (3) \end{aligned}$$

Since the total transition rate must be independent of  $m_1$ , the integration is simplified by choosing  $m_1 = 0$ . Performing the integrations, squaring, and summing over final values of  $m_2$ , we get the following contribu-

tions from all angular parts:

$S$  transitions ( $l=0, l_1=l_2$ ): 1

$P$  transitions ( $l=1, l_1=l_2+1$ ):  $l_1/3(2l_1+1)$ .

The radial integrals are best performed separately for the  $S$  and  $P$  transitions.

### A. $P$ Transitions

The only nonvanishing term of (3) is the term  $N=1$ . For  $l_1 = n_1 - 1$ , we have

$$L_{n_1+l_1}^{2l_1+1} = -(2n_1-1)!, \quad L_{n_2+l_2}^{2l_2+1} = -(2n_1-3)!.$$

Hence, apart from a numerical factor, the required integral is

$$\begin{aligned} I &= \int_0^\infty dr_2 {}_1F_1(2+iy; 4; 2i\kappa r_2) e^{-Br_2} \\ &\quad \times \left\{ \int_0^{r_2} dr_1 r_2 r_1^{2n_1} e^{-Ar_1} + \int_{r_2}^\infty dr_1 r_2^4 r_1^{2n_1-3} e^{-Ar_1} \right\}, \end{aligned}$$

where

$$A = \frac{\mu Z_1}{a_0} \left( \frac{1}{n_1} + \frac{1}{n_1-1} \right), \quad B = \frac{1}{a_0} (Z + i\kappa a_0).$$

The integration over  $r_1$  may be carried out by parts, and the integrations over  $r_2$  (following Burhop<sup>15</sup>) by use of the relation

$$\int_0^\infty r^p e^{-qr} {}_1F_1(a; b; cr) dr = \frac{\Gamma(p+1)}{q^{p+1}} {}_2F_1(a, p+1; b; c/q).$$

We find  $I = I_1 + I_2 + I_3$ , where  $I_1, I_2$  are finite series,

$$\begin{aligned} I_1 &= \sum_{N=1}^{2n_1+1} \frac{(2n_1)! N}{(A+B)^{N+1} A^{2n_1+2-N}} \\ &\quad \times {}_2F_1\left(2+iy, N+1; 4; \frac{2i\kappa}{A+B}\right), \end{aligned}$$

$$\begin{aligned} I_2 &= \sum_{N=1}^{2n_1-2} \frac{(2n_1-3)! N(N+1)(N+2)(N+3)}{(A+B)^{N+4} A^{2n_1-1-N}} \\ &\quad \times {}_2F_1\left(2+iy, N+4; 4; \frac{2i\kappa}{A+B}\right), \end{aligned}$$

and

$$I_3 = \frac{(2n_1)!}{\kappa^2 A^{2n_1+1}} \frac{1}{(i+y)^2} {}_2F_1\left(2+iy, 2; 4; \frac{2}{1-iy}\right).$$

Since  $|A| \gg |B|$ , the major contribution to  $I$  arises from  $I_3$ . This may be evaluated explicitly by successive

<sup>13</sup> W. Gordon, Z. Physik 48, 180 (1928).

<sup>14</sup> P. A. M. Dirac, Proc. Roy. Soc. (London) A114, 243 (1927).

<sup>15</sup> E. H. S. Burhop, Proc. Roy. Soc. (London) A148, 272 (1935).

use of the relations<sup>16</sup>

$$\begin{aligned} & \frac{\Gamma(a)\Gamma(b)}{\Gamma(c)} {}_2F_1(a, b; c; z) \\ &= \frac{\Gamma(a)\Gamma(b-a)}{\Gamma(c-a)} (-z)^{-a} {}_2F_1(a, 1-c+a; 1-b+a; z^{-1}) \\ &+ \frac{\Gamma(b)\Gamma(a-b)}{\Gamma(c-b)} (-z)^{-b} {}_2F_1(b, 1-c+b; 1-a+b; z^{-1}), \\ & \quad (|\arg(-z)| < \pi), \end{aligned}$$

and

$${}_2F_1(a, b; c; z) = (1-z)^{c-a-b} {}_2F_1(c-a, c-b; c; z),$$

which give

$$I_3 = \frac{3}{\kappa^2} \frac{(2n_1)!}{A^{2n_1+1}} \frac{1}{1+y^2} \exp[y(2 \tan^{-1}y - \pi)].$$

An approximate evaluation of the series terms  $I_1$  and  $I_2$  may be made since  $|2i\kappa/(A+B)| \ll 1$  and, usually,  $|B| < |A|$ , by putting each hypergeometric function equal to 1, and ignoring  $B$  by comparison with  $A$ . The series may then be summed to give

$$I_1 = -C_1(2n_1+2)!/2, \quad I_2 = +C_2(2n_1+2)!/5,$$

where  $C_1, C_2$  are correction factors of order 1. A graphical method of determining  $C_1, C_2$  has been described in the thesis of one of the authors.<sup>5</sup> In general  $C_1, C_2$ , have been found to be less than 1, and a convenient method of estimating the error involved in the approximation is by putting both  $C_1$  and  $C_2$  equal to 1 and 0 in turn. The correct value should lie between these two limits.

### B. S Transitions

The particular transition considered is that in which  $l_1=l_2=n_1-2, n_2=n_1-1$ . For this case we have

$$L_{n_1+l_1^{2l_1+1}} = (2n_1-2)! \left\{ \frac{2Z_1\mu r_1}{n_1 a_0} - (2n_1-2) \right\}.$$

The integrations over  $r_1$  and  $r_2$  may be performed as before. It is found however that the two terms arising at the lower limit of the integration over  $r_1$  cancel. These are the equivalent of  $I_3$  in the  $P$  transitions integral, and in the absence of cancellation would be expected to give the major contribution. Thus, the  $S$  transition rates are, in general, smaller than the corresponding rates for  $P$  transitions.

After some simplification the remaining terms in the

radial integral give

$$\begin{aligned} I &= \sum_{N=1}^{2n_1-2} \frac{(2n_1-2)!}{2n_1-1} \frac{N(N+1)}{A^{N+2}(A+B)^{2n_1-1-N}} \\ &\quad \times {}_2F_1\left(1+iy; N+1; 2; \frac{2i\kappa}{A+B}\right) \\ &= C_3 \frac{(2n_1-2)!(2n_1-2)(2n_1)}{3} \frac{1}{A^{2n_1+1}}, \end{aligned}$$

where  $C_3$  is a correction factor of the same type as  $C_1, C_2$ .

### C. Radiative Transitions

The radiative transition rate is given by

$$P_R = \frac{4}{3} \frac{(2\pi\nu)^4 \epsilon^2}{h\nu c^3} \left| \int \chi_f^* \mathbf{r}_1 \chi_i d\tau_1 \right|^2,$$

where  $\nu$  is the frequency of the emitted quantum.

The integral has been evaluated in general form by Gordon.<sup>17</sup> The only amendments required are a summation over all possible final states, the substitution of the mesonic mass for the electron mass, and the substitution of the Bohr value for the frequency of the emitted quantum,

$$\nu = \frac{2n_1-1}{n_1^2(n_1-1)^2} \frac{Z_1^2 \epsilon^2 \mu}{2a_0 h}.$$

### III. RESULTS AND DISCUSSION

Collecting together all factors, the following formulas are found to the relevant transition rates.

*P Auger Transitions:*  $(n_1, n_1-1) \rightarrow (n_1-1, n_1-2)$

$$\begin{aligned} P_A &= \frac{1}{3} \left( \frac{Z}{Z_1} \right)^2 C^2 \frac{\pi m_e \epsilon^4}{\mu^2 \hbar^3} \frac{2^{4n_1+6} n_1^{2n_1+2} (n_1-1)^{2n_1+4}}{(2n_1-1)^{4n_1+2} (1+y^2)} \\ &\quad \times \frac{y^2 \exp\{y(4 \tan^{-1}y - \pi)\}}{\sinh \pi y}, \quad (4) \end{aligned}$$

where

$$\begin{aligned} C &= 1 - \left( \frac{Z}{Z_1} \right)^2 \frac{(2n_1+1)(2n_1+2)n_1^2(n_1-1)^2(1+y^2)}{3(2n_1-1)^2 \mu^2 y^2 \exp\{y(2 \tan^{-1}y - \pi)\}} \\ &\quad \times \left( \frac{C_1}{2} - \frac{C_2}{5} \right). \end{aligned}$$

*S Auger Transitions:*  $(n_1, n_1-2) \rightarrow (n_1-1, n_1-2)$ .

$$P_A = \left( \frac{Z}{Z_1} \right)^4 \frac{\pi m_e \epsilon^4}{\mu^4 \hbar^3} \frac{2^{4n_1+6} n_1^{2n_1+4} (n_1-1)^{2n_1+5}}{9(2n_1-1)^{4n_1+2}} \frac{e^{\pi y}}{\sinh \pi y} C_3^2. \quad (5)$$

<sup>16</sup> E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis* (The Macmillan Company, New York, 1946), fourth edition, Chapter 14.

<sup>17</sup> W. Gordon, *Ann. Physik* 2, 1031 (1929).

TABLE I. Auger and radiative transition rates/sec ( $P_A$  and  $P_R$ ) for elements in photographic emulsions together with energy,  $E$ , of emitted Auger electron in kev.

Element	$n_1=2$	3	4	5	6	7	8
Transition $(n_1, n_1-1) \rightarrow (n_1-1, n_1-2)$							
C	$\begin{cases} P_A & 2.07 \times 10^{11} \\ P_R & 1.71 \times 10^{14} \\ E & 76.6 \end{cases}$	$\begin{cases} 6.08 \times 10^{12} \\ 1.76 \times 10^{13} \\ 14.0 \end{cases}$	$\begin{cases} 3.88 \times 10^{13} \\ 3.76 \times 10^{12} \\ 4.8 \end{cases}$	$\begin{cases} 1.31 \times 10^{14} \\ 1.53 \times 10^{12} \\ 2.1 \end{cases}$	$\begin{cases} 3.15 \times 10^{14} \\ 4.48 \times 10^{11} \\ 1.0 \end{cases}$	$\begin{cases} 6.11 \times 10^{14} \\ 2.02 \times 10^{11} \\ 0.6 \end{cases}$	$\begin{cases} 1.02 \times 10^{15} \\ 1.01 \times 10^{11} \\ 0.3 \end{cases}$
N	$\begin{cases} P_A & 2.24 \times 10^{11} \\ P_R & 3.16 \times 10^{14} \\ E & 104.3 \end{cases}$	$\begin{cases} 6.54 \times 10^{12} \\ 3.26 \times 10^{13} \\ 19.1 \end{cases}$	$\begin{cases} 4.12 \times 10^{13} \\ 6.96 \times 10^{12} \\ 6.5 \end{cases}$	$\begin{cases} 1.39 \times 10^{14} \\ 2.15 \times 10^{12} \\ 2.9 \end{cases}$	$\begin{cases} 3.33 \times 10^{14} \\ 8.30 \times 10^{11} \\ 1.4 \end{cases}$	$\begin{cases} 6.43 \times 10^{14} \\ 3.74 \times 10^{11} \\ 0.7 \end{cases}$	$\begin{cases} 1.07 \times 10^{15} \\ 1.88 \times 10^{11} \\ 0.4 \end{cases}$
O	$\begin{cases} P_A & 2.38 \times 10^{11} \\ P_R & 5.39 \times 10^{14} \\ E & 136.2 \end{cases}$	$\begin{cases} 6.89 \times 10^{12} \\ 5.57 \times 10^{13} \\ 24.9 \end{cases}$	$\begin{cases} 4.33 \times 10^{13} \\ 1.19 \times 10^{13} \\ 8.5 \end{cases}$	$\begin{cases} 1.45 \times 10^{14} \\ 3.66 \times 10^{12} \\ 3.7 \end{cases}$	$\begin{cases} 3.47 \times 10^{14} \\ 1.42 \times 10^{12} \\ 1.8 \end{cases}$	$\begin{cases} 6.66 \times 10^{14} \\ 6.38 \times 10^{11} \\ 0.9 \end{cases}$	$\begin{cases} 1.11 \times 10^{15} \\ 3.21 \times 10^{11} \\ 0.5 \end{cases}$
Br	$\begin{cases} P_A & 3.18 \times 10^{11} \\ P_R & 1.98 \times 10^{17} \\ E & 2601 \end{cases}$	$\begin{cases} 9.02 \times 10^{12} \\ 2.04 \times 10^{16} \\ 471 \end{cases}$	$\begin{cases} 5.53 \times 10^{13} \\ 4.35 \times 10^{15} \\ 156.8 \end{cases}$	$\begin{cases} 1.81 \times 10^{14} \\ 1.34 \times 10^{15} \\ 65.9 \end{cases}$	$\begin{cases} 4.23 \times 10^{14} \\ 5.19 \times 10^{14} \\ 30.0 \end{cases}$	$\begin{cases} 7.90 \times 10^{14} \\ 2.34 \times 10^{14} \\ 13.1 \end{cases}$	$\begin{cases} 1.27 \times 10^{15} \\ 1.17 \times 10^{14} \\ 4.0 \end{cases}$
Ag	$\begin{cases} P_A & \begin{cases} 3.25 \times 10^{11} - \\ 3.29 \times 10^{11} \end{cases} \\ P_R & \begin{cases} 6.43 \times 10^{17} \\ 6.63 \times 10^{16} \end{cases} \\ E & \begin{cases} 4698 \\ 848 \end{cases} \end{cases}$	$\begin{cases} 9.24 \times 10^{12} - \\ 9.40 \times 10^{12} \\ 6.63 \times 10^{16} \end{cases}$	$\begin{cases} 5.63 \times 10^{13} - \\ 5.78 \times 10^{13} \\ 1.41 \times 10^{16} \end{cases}$	$\begin{cases} 1.84 \times 10^{14} - \\ 1.92 \times 10^{14} \\ 4.36 \times 10^{15} \end{cases}$	$\begin{cases} 4.28 \times 10^{14} - \\ 4.58 \times 10^{14} \\ 1.69 \times 10^{15} \end{cases}$	$\begin{cases} 7.97 \times 10^{14} - \\ 8.86 \times 10^{14} \\ 7.60 \times 10^{14} \end{cases}$	$\begin{cases} 1.27 \times 10^{15} - \\ 1.48 \times 10^{15} \\ 3.82 \times 10^{14} \end{cases}$

## Radiative Transitions:

$$P_R = \left( \frac{e^2}{\hbar c} \right)^6 \frac{\mu m_e c^3}{3e^2} Z_1^4 \frac{2^{4n_1} n_1^{2n_1-4} (n_1-1)^{2n_1-2}}{(2n_1-1)^{4n_1-1}}. \quad (6)$$

A factor of 2 has been incorporated in the Auger transition rates to allow for the 2 electrons in the  $K$  shell. In numerical work, since  $C$  is almost independent of  $Z$ , it has been computed for one element only. Values of the  $K$  shell ionization potential from the tables of Compton and Allison<sup>18</sup> have been taken, the ionization potential corresponding to  $Z=Z_1-1$  being used to allow for screening by the meson. In Table I, Auger and radiative rates for some elements are presented, together with the energy of the ejected electron.

Table I enables an estimate of the total time of transition from a level  $n_1=8$  to the  $1s$  state to be made for a  $\mu$ -meson. For bromine the estimated time is  $4 \times 10^{-15}$  sec, while for carbon the time is  $9 \times 10^{-14}$  sec. The time for carbon may be lengthened if the atom becomes completely ionized through earlier Auger transitions, but allowing for radiative transitions only, the time required is as short as  $2 \times 10^{-11}$  sec. These times are all very much less than the life time of a  $\mu$ -meson,  $2.15 \times 10^{-6}$  sec, and strongly confirm previous conclusions that the majority of  $\mu^-$  meson decays observed will take place when the meson has spent almost its entire life in a  $1s$  state. The weakness of the interaction between  $\mu$ -mesons and nucleons is thus demonstrated.

As the  $P$  transitions are apparently more important than the  $S$  transitions, no  $S$  transitions have been calculated. The exceptional case of the  $2s \rightarrow 1s$  transition, where the only other possible transition is the  $2s \rightarrow 2p$  radiative transition calculated by Wheeler,<sup>6</sup> for

which the rate is very small for light elements, has been considered. It will be seen that for  $\gamma$  small, (4) reduces to the formula quoted by Wheeler except for the  $Z$  dependence. The argument of Wheeler that the  $S$  transitions should be more probable than the  $P$  transitions, in view of the greater magnitude of the  $S$  electronic wave functions in the region of energy transfer, does not appear to be valid.

## IV. EXPERIMENTAL COMPARISON

Auger electrons accompanying stopped  $\mu$ -mesons have been observed by Cosyns *et al.*,<sup>9</sup> 1949, and Fry,<sup>10</sup> 1951, and the number of electrons per stopped meson deduced. The results for the heavy elements, silver and bromine, may be separated from those for the lighter elements by the use of Wheeler's  $Z^4$  law for nuclear absorption by which decay electrons should rarely be emitted for elements of atomic number appreciably greater than 11. Cosyns deduced a frequency of 20 percent for Auger electrons of energy greater than 20 kev in silver bromide, but commented that the rigorous criterion adopted for the recognition of Auger electrons may have excluded as many as had been accepted. No conclusions were reached concerning the lighter elements but out of 53 negative meson decays he observed two cases in which both Auger and decay electrons were emitted. This suggests an Auger frequency of the order of 4 percent.

The experiments of Fry were based on a somewhat smaller number of tracks and gave a frequency for emission of Auger electrons in silver bromide of energy between 15 and 70 kev of  $0.34 \pm 0.06$  when appropriately weighted cases of double emissions were included.

The predicted and experimental results for the heavy elements (assuming the probability of capture independent of  $Z$ ) are shown in Table II. The predicted

<sup>18</sup> A. H. Compton and S. K. Allison, *X-Rays in Theory and Experiment* (D. Van Nostrand and Company, Inc., New York, 1935).

results are rather high, even after allowance has been made for the fact that the observations must be regarded as a lower limit. It is possible that the discrepancy may be removed by consideration of transitions between noncircular orbits. Calculations concerning such transitions and those involving the emission of  $L$  shell electrons are proceeding. Also no allowance has been made for the reduction of the Auger transition rate caused by the ejection of  $K$  shell electrons by earlier Auger transitions.

For the light elements, assuming relative abundances of H:C:N:O of 340, 170, 36, 90, the computed Auger frequency is 1.6 percent. However, since almost every meson passing through the  $2s$  state of carbon, nitrogen, and oxygen might be expected to emit an Auger electron, this figure is certainly an underestimate. To account for the observed number, a very small proportion of mesons would be required to pass through the  $2s$  state.

These results are also based on a hydrogenic approximation for the atom and have been extrapolated to include the heavy elements of the emulsion. The effect of the finite size of the nucleus on the energy levels, as discussed by Wheeler,<sup>6</sup> has been ignored.

The simple picture must be qualified by further consideration.<sup>5</sup> In atoms containing many electrons, a large number of Auger electrons are probably emitted in the first transitions, and the atom may become highly ionized. Inside the orbit defined for the  $\mu$ -meson by  $n=15$ , the  $K$  electrons will first be ejected, and the rate of Auger transition may then depend upon the rate at which the  $K$  shell is replenished from the outer shells. The alternative to this is the direct Auger ejection of outer shell ( $L$ ,  $M$ , etc.) electrons, and these rates are probably much smaller than the  $K$  shell ejection rates. It may be, however, that the atom has been completely stripped of electrons by the first transitions—this is clearly possible for light atoms. In this case the rate of descent of the meson towards the nucleus will depend on the rate of recombination of the system, and the radiative transition rate. The possibility of electron recombination can be ruled out for hydrogen by the calculations of Wightman<sup>4</sup> and Fermi and Teller,<sup>19</sup> which show that an electron cannot be bound to the proton-meson system which has a separation less than  $0.64 a_0$ , corresponding to a meson orbit  $n \approx 12$ . For nuclei of higher atomic number than hydrogen the electron will have stable orbits whatever the value of the nucleus-meson separation. Thus electron recombination is possible in principle even for close orbits. From Table I, however, it is seen that for close orbits radiative transitions dominate, so this effect would not be important. The calculation of the critical separation for the meson-proton system leads to an interesting conclusion. The loss of energy of such a system to sur-

rounding atoms or molecules will depend on the velocity of the system and the material through which it is moving. But the time taken for the meson to descend by radiative transitions to its  $K$  orbit, if there is no interaction with other particles, is calculated to be  $2 \times 10^{-7}$  sec,<sup>5</sup> which is about one-tenth of the natural lifetime of the  $\mu$ -meson. Thus, if there are conditions in which collisional de-excitation is negligible, it is possible that in some cases a  $\mu$ -meson may decay before reaching the  $K$  orbit.

Fermi and Teller and Frank<sup>20</sup> pictured the meson-proton system in its ground state as effectively neutral. In this state Frank suggested that the system would not have any interaction with electrons, and that, consequently, it would be able to penetrate easily the electron shells of a heavy atom, perhaps Ag, in an emulsion. If the system could get inside the atomic  $K$  orbit and still remain bound, it would appear that the meson might be absorbed by the nucleus and the proton repelled by it. Alternatively, there should be a finite probability that the proton will be captured by the nucleus. Frank showed that capture of the proton is

TABLE II. Predicted and observed numbers of slow electrons per negative  $\mu$ -meson stopped in silver bromide.

	Energy range						
	15-70 kev	10-20 kev	20-40 kev	40-60 kev	60-100 kev	>100 kev	>20 kev
Predicted	0.64	0.39	0.48	0.10	0.06	0.03	0.67
Observed by Cosyns <i>et al.</i>	...	...	...	...	...	...	0.20
Observed by Fry	0.34	...	...	...	...	...	...

probable if the nucleus is a deuteron but not if it is a heavy nucleus. For such a heavy nucleus it is to be expected that when the proton is ejected electrons will be scattered out. In the light of later calculations<sup>4,5</sup> it would appear that the dipole system of meson and proton will not be neutral with respect to electrons in the shells. In this case the initial stages in the process of Frank would not take place; the meson-proton system might dissociate in the outer electronic shells, or, much more probably, remove electrons. The absorption of the meson and the repulsion of the proton scattering out more electrons could then take place. The calculation of the energies of the ejected electrons using this mechanism would be extremely complicated.

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<sup>19</sup> E. Fermi and E. Teller, Phys. Rev. **72**, 399 (1947).

<sup>20</sup> F. C. Frank, Nature **160**, 525 (1947).