giving the three-dimensional surface with respect to which they are formed every possible orientation.

In our Lagrangian theory, the commutation relations between the y_A and the $\dot{y}_A$ are not determined completely. As a result, the Hamiltonian, though formally it generates the motion, actually does not determine the time derivatives of all field variables. Preliminary examination shows, however, that the variables whose time derivatives remain indeterminate are precisely the ones whose time derivatives are also indeterminate in the classical theory, those which in the Hamiltonian theory are canonically conjugate to the primary constraints. In a nonlinear covariant theory, the separation of those variables whose time derivatives are indeterminate from those which are completely determined is a mathematical task of almost insurmountable difficulty. In all probability, the construction of the Hilbert space of permissible states and legitimate observables will be just as difficult in the Lagrangian formalism as it is in the canonical formalism (see Sec. 4).

In future work we plan to pursue the application of both the Hamiltonian and the Lagrangian theory to actual physical theories and to ascertain the usefulness of either.

PHYSICAL REVIEW VOLUME 89, NUMBER 1 JANUARY 1, 1953

Meson Theory of β-Decay and the ΔL-Forbidden Transitions

SEITARO NAKAMURA
Department of Physics, University of Tokyo, Tokyo, Japan
(Received April 21, 1952)

Existence of the β-meson of vector type which is neither a π- nor μ-meson and which transmits the β-decay of nuclei has been tantatively assumed, the results thus derived being compared with experiments on β-decay. The differences between the β-meson theory and the phenomenological Fermi theory lie in the natural deduction of the vector interaction as well as the introduction of new nuclear matrix elements which are characteristic of the meson dipole. The selection rules for the ΔL-forbidden transitions are involved in these matrix elements, the orders of which are expected to be that of the unfavored parity transitions in each degree of forbidden transitions. The observability of a free β-meson is also discussed.

1. THE MESON THEORY OF β-DECAY

THE theory of β-decay, originally formulated by Yukawa,[1] predicted the virtual emission and reabsorption of charged mesons in the β-decay of nucleons. After the two mesons, π and μ, were discovered, one was not successful in identifying either of them as an intermediary agent of nuclear β-decay, in spite of extensive analyses.[2–5] Recently, Friedman and Rainwater[6] showed that a free π-meson will not decay into an electron and neutrino with a probability more than 1/1419 times the probability of $\pi-\mu$ decay. Sasaki, Hayakawa, and the present author[7] once proposed the existence of a vector meson, which is neither π nor μ, and could be the agent of Yukawa's original idea, that of transmitting the nuclear force and β-decay. We have named it the β-meson. Tanikawa[8] and Caianiello[9] suggested the meson theory of β-decay through a τ-meson, the former assuming it to be pseudoscalar while the latter taking it to be vector. However, it is almost certain that the fate of these theories will primarily be dependent on the β-ray analysis. To clarify this point, we shall derive the β-meson theory in some detail.

For the purpose of this discussion, the β-meson is assumed to be of vector type, as in the Fermi theory the tensor interaction is indispensable in the explanation of the results of the recent experiments on the β-ray spectra and $\beta-\gamma$ angular correlations.[10] Its mass and coupling constants with nucleons and leptons will be tentatively taken arbitrary, not referring to those of any observed meson. Taking Konopinski's[11,12] Hamiltonian for the vector meson theory of β-decay, we have, in the case of an allowed transition, (as regards notation, see refer-

[1] H. Yukawa, Proc. Phys. Math. Soc. Japan **17**, 48 (1935), third series; S. Sakata, Proc. Phys. Math. Soc. Japan **23**, 291 (1941); **24**, 843 (1942); **25**, 86 (1943); H. A. Bethe and L. W. Nordheim, Phys. Rev. **57**, 998 (1940).
[2] Taketani, Nakamura, Ono, and Sasaki, Phys. Rev. **76**, 60 (1949); Nakamura, Fukuda, Ono, Sasaki, and Taketani, Prog. Theor. Phys. **5**, 740 (1950).
[3] H. Yukawa, Revs. Modern Phys. **21**, 474 (1949).
[4] J. Steinberger, Phys. Rev. **76**, 1180 (1949).
[5] M. Ruderman and R. Finkelstein, Phys. Rev. **76**, 1458 (1949).
[6] H. L. Friedman and J. Rainwater, Phys. Rev. **84**, 684 (1951).
[7] Sasaki, Nakamura, and Hayakawa, Prog. Theor. Phys. **3**, 454 (1948).
[8] T. Tanikawa, Prog. Theor. Phys. **3**, 315 (1948).
[9] E. R. Caianiello, Phys. Rev. **81**, 625 (1951).
[10] C. S. Wu, Revs. Modern Phys. **22**, 386 (1950); I. Shaknov, Phys. Rev. **82**, 333 (1951).
[11] E. J. Konopinski, Revs. Modern Phys. **15**, 209 (1945).
[12] E. J. Konopinski and E. Uhlenbeck, Phys. Rev. **60**, 308 (1941).

ences 11 and 12),

$$P(W)dW=\left(\frac{|\int 1|^2}{T_1}+\frac{|\int\beta\boldsymbol{\sigma}|^2}{T_2}+\frac{|\mathbf{D}^{(0)}|^2}{T_2'}\right)\times F(\pm Z,W)(W_0-W)^2(W^2-1)^{\frac{1}{2}}WdW, \quad (1)$$

$$\frac{1}{T_1}=\frac{8}{\pi}\left(\frac{m}{m_\beta}\right)^5\left(\frac{g_1g_1'}{\hbar c}\right)^2\frac{m_\beta c^2}{\hbar}$$

$$\frac{1}{T_2}=\frac{32}{9\pi}\left(\frac{m}{m_\beta}\right)^5\left(\frac{f_1f_1'}{\hbar c}\right)^2\frac{m_\beta c^2}{\hbar}$$

$$\frac{1}{T_2'}=\frac{8}{\pi}\left(\frac{m}{m_\beta}\right)^5\left(\frac{f_1f_1'}{\hbar c}\right)^2\frac{m_\beta c^2}{\hbar}$$

$$\mathbf{D}^{(0)}=\frac{1}{\rho^2}\int\left\{\beta(\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r}\right\}\times(m_\beta\gg m). \quad (2)$$

Here m_β, m are the rest mass of the β-meson and electron, respectively; g_1 and f_1 are the nuclear coupling constants, g_1' and f_1' the coupling constants with leptons; ρ is the nuclear radius. $\int 1$ and $\int\beta\boldsymbol{\sigma}$ are the well-known nuclear matrix elements, the former corresponding to the Fermi selection rule, the latter to the Gamow-Teller selection rule. $\mathbf{D}^{(0)}$ is an entirely new matrix element, the selection rule of which is given as follows:

$\Delta J=0, \pm 1$ (no $0\rightarrow 0$),

$\Delta L=0, \pm 1, \pm 2$ (no $0\rightarrow 0$, $1\leftrightarrow 0$), parity change: no.

It is interesting to note that $\mathbf{D}^{(0)}$ directly allows the so-called ΔL-forbidden, ΔJ-allowed transition. The corresponding β-ray spectrum is of the allowed shape, as is seen in Eq. (1). The correction factors of the meson theory of β-decay in the forbidden transitions can be derived in the same way as in the case of the Fermi theory. The results are identical with those of the linear combination theory of the vector, V, and the tensor, T, Fermi-interactions, computed by Smith,[13] except for the following terms:

$$\left(\frac{1}{12}K^2L_0+\frac{3}{4}L_1\right)\sum_{ij}|D_{ij}^{(1)}|^2 \quad (3)$$

in the first forbidden transision, and

$$\frac{1}{72}\left(\frac{1}{15}K^4L_0+2K^2L_1+15L_2\right)\sum_{ijk}|D_{ijk}^{(2)}|^2 \quad (4)$$

in the second forbidden transition. Here

$$D_{ij}^{(1)}=\frac{1}{\rho^2}\int\beta(\boldsymbol{\sigma}\cdot\mathbf{r})(x_ix_j-\tfrac{1}{3}\delta_{ij}r^2), \quad (5)$$

$$D_{ijk}^{(2)}=\frac{1}{\rho^2}\int\beta(\boldsymbol{\sigma}\cdot\mathbf{r})\left(x_{(i}x_jx_{k)}-\frac{6}{5}\delta_{(ij}x_{k)}r^2\right). \quad (6)$$

[13] A. M. Smith, Phys. Rev. **82**, 955 (1951).

TABLE I. Selection rule derived from the meson dipole interaction.

Matrix elements	ΔJ	ΔL	Parity change	Spectrum correction factors
$\mathbf{D}^{(0)}$	$0, \pm 1$ (no $0\rightarrow 0$)	$0, \pm 1, \pm 2$ (no $0\rightarrow 0$ $1\leftrightarrow 0$)	no	1
$D_{ij}^{(1)}$	$\pm 1, \pm 2$ (no $1\leftrightarrow 0$)	$\pm 1, \pm 2, \pm 3$ (no $1\leftrightarrow 0$ $2\leftrightarrow 0$)	yes	'a' type
$D_{ijk}^{(2)}$	$\pm 2, \pm 3$ (no $2\leftrightarrow 0$)	$\pm 2, \pm 3, \pm 4$ (no $2\leftrightarrow 0$ $3\leftrightarrow 0$)	no	D_2 type

The selection rules of $D_{ij}^{(1)}$ and $D_{ijk}^{(2)}$ and the corresponding forbidden spectrum correction factors are listed in Table I.

Among the first forbidden, ΔL-forbidden cases, Pr^{142} [14] and Sb^{124} [15] are reported to show the 'a' type forbidden spectra. The orbital assignments, $(d_{5/2}, h_{9/2})$ for Pr^{142} and $(g_{7/2}, s_{1/2})$ for Sb^{124} are, however, not conclusive. If the super-allowed beta-decay $O^{14}\rightarrow N^{14*}$ is really a $0\rightarrow 0$ (no) transition,[16] the vector or scalar interaction which enables us to derive the necessary matrix element $\int 1$ should be applied.

2. DISCUSSIONS

In comparing the results with experiments, no quantitatively definite conclusions could be obtained. But the following features are noted:

(1) The beta-decay in the ΔL-forbidden transitions, classified on the current version of the shell model by Mayer, Moszkowski, and Nordheim,[17] can be directly explained by the matrix elements $\mathbf{D}^{(0)}$ (ΔJ-allowed), $D_{ij}^{(1)}$ (ΔJ-first forbidden) and $D_{ijk}^{(2)}$ (ΔJ-second forbidden). There is some evidence that the ΔL-forbidden, ΔJ-allowed case has the allowed shape, and the ΔL-forbidden, ΔJ-forbidden case has 'a' type for the corresponding β-ray spectra.

Following Mayer, Moszkowski, and Nordheim,[14] $\log ft$ for the transition, $\Delta J=\pm 1$, $\Delta L=\pm 2$, no, covers the range from 4.9 up to 9.0 and the mean is about 6. From Eq. (2), we may expect that the order of the matrix element $\mathbf{D}^{(0)}$ is about that of the allowed unfavored parity transition. In this case, the higher orbital change might reduce the overlapping of the radial parts of the wave functions before and after the decay, compared to the case of the normal allowed transition. This may account for the above figures concerning the ft values. It must be borne in mind, however, with Brysk,[18] that the ΔL-forbidden transition may also be ascribed to the peculiar character of the nuclei. It may

[14] L. W. Nordheim, Revs. Modern Phys. **23**, 327 (1951), Table II, reference *hh*.

[15] L. M. Langer, Phys. Rev. **84**, 1059 (1951); Nakamura, Umezawa, and Takebe, Phys. Rev. **83**, 1273 (1951).

[16] J. D. Seagrave, Phys. Rev. **85**, 197 (1952).

[17] Mayer, Moszkowski, and Nordheim, Revs. Modern Phys. **23**, 315 (1951); L. W. Nordheim, Revs. Modern Phys. **23**, 322 (1951).

[18] H. Brysk, Phys. Rev. **84**, 362 (1951).

be worthwhile to mention that the final odd mass number nuclei of the ΔL-forbidden β-decay are almost about half-way between the two Schmidt limits.[19] As was suggested by Feenberg and Hammack,[20] a small admixture of the state having the orbital angular momentum which makes the transition in the allowed category for $\int \beta \boldsymbol{\sigma}$, $(\Delta L = \pm 1,\ 0)$, would be enough to explain the larger ft values of the ΔL-forbidden case. Emphasis should be laid on the fact that, in this case, one could expect, in principle, clear-cut distinction between meson theory and Fermi theory. Indeed, $\mathbf{D}^{(0)}$, $D_{ij}^{(1)}$ and $D_{ijk}^{(2)}$ cannot be derived from any of the five Fermi interaction or their linear combinations. However, the calculation of the nuclear matrix elements depends on the particular theory of nuclear structure. It may, therefore, be hard to draw any final conclusion.

(2) The need for the Fermi interaction $\int 1$ in addition to the Gamow-Teller interaction $\int \boldsymbol{\sigma}$ was discussed by Moszkowski,[21] and Horie and Umezawa,[22] in evaluating the matrix elements for light nuclei.[23] In the β-meson theory, the above is deduced as a matter of course.

(3) Apart from the ΔL-forbidden group as mentioned above, the spectrum correction factors in the forbidden transitions derived from the β-meson theory are essentially identical with the linear combination theory of V and T, except that the relative weight of V and T is now determined in (1). It is clear that, in a spectrum analysis of this type, we have to meet the most crucial test of the theory, since the difference between the meson and the Fermi theory is evident in the shape of the spectrum. Of course, it is easily seen that remarkable features of the forbidden spectra, i.e., 'a' type, D_2 type, etc., which revealed the success of the tensor Fermi interaction would not be devalued by our theory. However, the results will, in general, depend on the unknown ratios of the several matrix elements, which are understood from the equations of Konopinski and Uhlenbeck,[11,12] and Smith.[13]

For instance, it can be shown[24] that the β-meson theory predicts the approximate allowed spectrum or some appreciable deviation from it in the first forbidden transition, $\Delta J = \pm 1$, 0, yes, depending on the unknown ratios of the several nuclear matrix elements. (See Appendix.) It must be admitted that there remain ambiguities concerning these ratios of the matrix elements.[25] Only systematic experimental investigations[23,26] will clarify the validity of the theory with regards to the spectrum shape.

(4) While the free β-meson is to be expected to decay into an electron and a neutrino in our theory, there has been no evidence to indicate its existence. However, if we take into account the fact that the predicted lifetime, τ_β,[27,28] for the β-decay of the β-meson is much shorter than that of any charged meson so far observed, it seems not at all absurd to deduce, with Caianiello, that the free β-meson might escape detection in the experiments now available.

The author would like to take this opportunity to thank Professor H. Yukawa for his kind information and encouraging discussions from abroad. He would like to thank Dr. M. Taketani and Messrs. M. Sasaki and M. Umezawa for their kind discussions throughout this work. The author is greatly indebted to Professors E. T. Konopinski, L. M. Langer, M. G. Mayer, S. A. Moszkowski, and L. W. Nordheim, for sending him their manuscripts before publication.

APPENDIX

If one puts

$$\int \boldsymbol{\alpha} = i\Lambda_1 \frac{\alpha z}{2\rho} \int \mathbf{r}, \qquad \int \beta \boldsymbol{\alpha} = i\Lambda_1{}^{\beta} \frac{\alpha z}{2\rho} \int \mathbf{r},$$

$$\int \beta \boldsymbol{\sigma} \times \mathbf{r} = ik^{\beta} \int \mathbf{r}, \qquad \int \beta\gamma_5 = i\Lambda_2{}^{\beta} \frac{\alpha z}{2\rho} \int \beta \boldsymbol{\sigma}\cdot\mathbf{r}, \tag{7}$$

the spectrum correction factors, $C'(z, W)$, for the first forbidden transition derived from the vector and the

[19] Mayer, Moszkowski, and Nordheim, Revs. Modern Phys. **23**, 315 (1951), Table II; L. W. Nordheim, Phys. Rev. **75**, 1894 (1949), Figs. 1 (a) and (b).

[20] E. Feenberg and K. C. Hammack, Phys. Rev. **75**, 1877 (1949).

[21] S. A. Moszkowski, Phys. Rev. **82**, 118 (1951); see also reference 21.

[22] H. Horie and M. Umezawa, Phys. Rev. **83**, 1253 (1951).

[23] In an article by E. J. Konopinski and L. M. Langer, in *Annual Reviews of Nuclear Science*, Vol. 2, (Annual Reviews, Inc., Stanford) in press, all the possible linear combinations of the five Fermi interactions have been pinned down in the light of the newest spectrum analysis. Following them, the combination (V, T) should be ruled out by the evidence that the precision measurements of the first forbidden spectra give the allowed type, whereas the main terms in the (V, T) combination give spectrum correction factors which are energy dependent.

This conclusion is of great importance to our theory. However, if the following assumption is valid; (see Appendix)

$$\Lambda_1 = 1,\ \Lambda_1{}^{\beta} \neq k^{\beta} \quad \text{or} \quad \Lambda_1{}^{\beta} = k^{\beta},\ \ \Lambda_1 \neq 1,$$

it may be possible to expect the main terms of the correction factor to be energy independent by cancelling out the $1/W$ term accidentally, as is mentioned in the text.

[24] S. Nakamura, Prog. Theor. Phys. (to be published).

[25] T. Ahrens and E. Feenberg, Phys. Rev. **86**, 64 (1952).

[26] If the pseudoscalar (P) Fermi interaction is really necessary, as has recently been shown in the attempt by Petschek and Marshak, to explain the first forbidden spectrum of RaE, we have to assume the pseudoscalar β-meson (rather than the π-meson), in addition to the vector meson. In this case, the spectrum formula (1) should be augmented by the $\int \beta\gamma_5$ interaction only, as was proved by Nelson, Miyazima, and Dyson. See A. G. Petschek and R. E. Marshak, Phys. Rev. **85**, 698 (1952); E. C. Nelson, Phys. Rev. **60**, 830 (1941); T. Miyazima, Proc. Phys. Math. Soc. Japan (in Japanese) **16**, 340 (1941); and F. J. Dyson, Phys. Rev. **73**, 929 (1948).

[27] $\tau_\beta \cong 3\times10^{-11}$ sec, provided $m_\beta = 1200$ m, $T_2 = 3\times10^3$ sec. $f_1{}^2/\hbar c = 0.5$ and $g_1 = 0$. (See reference 9, and also H. A. Bethe and L. W. Nordheim, Phys. Rev. **57**, 998 (1940).)

[28] Gamma-instability of the free β-meson, that is, $\beta \rightarrow \pi +$ photon, should be taken into consideration. See R. J. Finkelstein, Phys. Rev. **72**, 415 (1947); Fukuda, Hayakawa, and Miyamoto, Prog. Theor. Phys. **5**, 283, 352 (1950); Ozaki, Oneda, and Sasaki, Prog. Theor. Phys. **5**, 25, 165 (1950).

pseudoscalar β-meson theory will be formally expressed in terms of descending powers of $\alpha z/2\rho$.

$$C^1(Z,W)=\left(\frac{\alpha z}{2\rho}\right)^2(C_{V,T}+C_{T,P})-\frac{\alpha z}{2\rho}(C_{V,T}'+C_{T,P}')$$
$$+(C_{V,T}''+C_{T,P}''+C_a), \tag{8}$$

$$C_{V,T}=\left|\int \mathbf{r}\right|^2[(1-\Lambda_1)^2/T_1+(\Lambda_1^\beta-k^\beta)^2/T_2$$
$$-\{(1-\Lambda_1)(\Lambda_1^\beta-k^\beta)/T_1T_2\}\cdot(2/W)], \tag{9}$$

$$C_{T,P}=\left|\int \beta\boldsymbol{\sigma}\cdot\mathbf{r}\right|^2(1/T_2-\Lambda_2^\beta/T_3)^2, \tag{9'}$$

$$C_{V,T}'=\left|\int \mathbf{r}\right|^2\{[(1-\Lambda_1)/T_1+k^\beta(\Lambda_1^\beta-k^\beta)/T_2]\tfrac{2}{3}$$
$$\cdot(K+P^2/W)+([(\Lambda_1^\beta-k^\beta)$$
$$-k^\beta(1-\Lambda_1)]/T_1T_2)\cdot(2K/3W)\}, \tag{10}$$

$$C_{T,P}'=\left|\int \beta\boldsymbol{\sigma}\cdot\mathbf{r}\right|^2(1/T_2)$$
$$\cdot(1/T_2-\Lambda_2^\beta/T_3)\cdot\tfrac{2}{3}\cdot(K-P^2/W), \tag{10'}$$

$$C_{V,T}''=\left|\int \mathbf{r}\right|^2(1/T_1)$$
$$\cdot[K^2/3+P^2/3+(2/9)\cdot KP^2/W]$$
$$+\left|\int \beta\boldsymbol{\sigma}\times\mathbf{r}\right|^2(1/T_2)$$
$$\cdot[K^2/6+P^2/6+(2/9)\cdot KP^2/W], \tag{11}$$

$$C_{T,P}''=\left|\int \beta\boldsymbol{\sigma}\cdot\mathbf{r}\right|^2(1/T_2)$$
$$\cdot[K^2/9+P^2/9-(2/9)\cdot KP^2/W], \tag{11'}$$

$$C_a=(\textstyle\sum|B_{ij}|^2\cdot 1/T_2$$
$$+\textstyle\sum|D_{ij}^{(1)}|^2\cdot 1/T_2')(K^2/12+P^2/12). \tag{12}$$

(This expression is a good approximation for the case $\alpha z\ll 1$.) Here T_3 is the characteristic time of the β-decay deduced from the pseudoscalar β-meson theory.

Except for the case, $\Delta J=\pm 2$, yes, the correction factors given by (8) depend upon the ratios, (Λ_1, Λ_1^β, $\Lambda_2^\beta)\alpha z/2\rho$, k^β, of the several matrix elements. (The phase of (7) was taken following the approximation of Ahrens and Feenberg.[22]) If $\alpha z/2\rho\gg 1$, one can easily infer the simple results, provided that the following special assumptions are valid.

(I) $(1-\Lambda_1)(\Lambda_1^\beta-k^\beta)\neq 0$.

The correction factors in the leading terms (9) and (9′) will take the form of $1-\mathcal{L}/W$, where

$$\mathcal{L}=\frac{2(1-\Lambda_1)(\Lambda_1^\beta-k^\beta)}{(1-\Lambda_1)^2/T_1+(\Lambda_1^\beta-k^\beta)^2/T_2+(1/T_2-\Lambda_2^\beta/T_3)^2\cdot|\int\beta\boldsymbol{\sigma}\cdot\mathbf{r}|^2/|\int\mathbf{r}|^2}.$$

(II) (a) $\Lambda_1=1$, and $\Lambda_1^\beta\neq k^\beta$ (or $1/T_2\neq\Lambda_2^\beta/T_3$),
(b) $\Lambda_1^\beta=k^\beta$, and $\Lambda_1\neq 1$ (or $1/T_2\neq\Lambda_2^\beta/T_3$).

The $1/W$ term in (9) will be canceled out, and the correction factors in (9) and (9′) become independent of w.

(III) $\Lambda_1=1$, $k=\Lambda_1^\beta$, and $\Lambda_2^\beta/T_3=1/T_2$.

The terms containing the factors, $(\alpha z/2\rho)^2$, and $(\alpha z/2\rho)$, in (8) will be all canceled out by themselves, and the smallest terms, (11), (11′), and (12), will determine the spectrum shape.

If $\mathcal{L}>0$ in the case (I), a downward deviation of the Fermi plot is expected at the low energy region. Some indications of such a deviation were reported in the experimental spectra of Th^{233} [29] and Ru^{103}.[30] However, the evidence seemed not altogether unobjectionable. The attempts[31,32] to explain the first forbidden β-ray spectra by means of the large cancellation of the correction factors by assuming a suitable ratio of matrix elements should be compared to the case (III). For instance, in our theory, the spectrum of Tm^{170} [31] ($\Delta J=\pm 1$ ($3\rightarrow 2$), yes), is to be explained by (11) and that of RaE[32] ($\Delta J=0$ ($0\rightarrow 0$), yes) by (11′).

Actually, in the cases of Ru^{103}, Tm^{170}, RaE, and Th^{233}, the assumption $\alpha z\ll 1$ does not hold, and more elaborate discussions are required, which will be published later,[33] with the investigation of the second forbidden transition.

[29] Bunker, Langer, and Moffat, Phys. Rev. **80**, 468 (1950).
[30] E. Kondaiah, Arkiv Fys. **4**, No. 2 (1951).
[31] Nakamura, Umezawa, and Takebe, Phys. Rev. **84**, 865 (1951).
[32] A. G. Petschek and R. E. Marshak, Phys. Rev. **85**, 698 (1952).
[33] Taketani, Umezawa, Nakamura, and Ono, Prog. Theor. Phys. (to be published).