important improvements in further work of this type would be the use of a nearly monokinetic electron beam obtained directly from the accelerator or with the aid of a larger analyzer magnet. The electron beam should be well centered in a deeply lighted region with little vertical spread. A long entrance window would minimize scattering of the primary electrons. The chamber should be larger and have a stronger magnetic field applied than in the present experiment if more of the spectrum were to be examined at the same primary energy. In this connection, the problem caused by a wide angular distribution of straggled electrons would become less acute for a given point on the spectrum. The use of higher primary electron energies would permit examination of the high energy end of the spectrum in more detail.

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Effect of Equatorial Ring Current on Cosmic-Ray Intensity*

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The equatorial ring current postulated by Chapman and Ferraro to explain the main phase of terrestrial magnetic storms is analyzed with respect to its effect on the intensity of the cosmic radiation. For mathematical convenience, the ring current is replaced by a current sheet located on the surface of a sphere concentric with the earth, in accordance with a suggestion due to Chapman. A simple expression is then obtained relating the variations in magnetic field at the equator with the corresponding variations to be expected in the intensity of cosmic radiation measured by an arbitrary detector located at any latitude and atmospheric depth.

I. INTRODUCTION

It is well known that terrestrial magnetic storms are due, in part, to current systems located above the surface of the earth. In particular, Chapman and Ferraro[1] have postulated the existence of a westward-flowing ring of current which encircles the earth in the magnetic equatorial plane and which has a radius several times that of the earth. It is supposed that the current decays slowly during the periods between storms but that it is enhanced from time to time by corpuscular beams from the sun, the resulting current variations giving rise to the magnetic disturbances observed on the earth during the main phase of magnetic storms.

Now one can, of course, imagine an infinite number of current systems which could produce the magnetic disturbances observed on the earth's surface. In order to provide an independent test of the ring current theory, therefore, Chapman[2] has suggested that it would be profitable to study the effect of such a current system on the cosmic radiation.

In the following, an attempt is made to determine the variations in cosmic-ray intensity that would be expected to accompany variations in the intensity of the postulated ring current. The calculations are carried out in the approximation corresponding to the Stoermer theory of allowed cones in the field of a simple dipole, i.e., the effect of the ring current on the Stoermer cones is calculated and the assumption is then made that all directions within the modified cones are "allowed." Considerations of the earth's shadow and of the finer details of the Lemaitre-Vallarta theory are neglected. Although we speak here of a "ring" current, the calculations are actually carried out for a simpler current system which approximates the effect of a ring current.

II. DETERMINATION OF THE ALLOWED CONES

We consider the motion of a particle of charge e in the combined magnetic fields of the earth's dipole and of a ring current encircling the earth in the magnetic equatorial plane. The coordinate system is shown in Fig. 1. The positive z axis points toward the earth's magnetic north. The earth's dipole M_e is located at the origin and is directed along the negative z axis. The earth's radius is designated by ρ, and the radius of the ring is taken to be $a\rho$; θ is the angle between the velocity vector of the particle and the meridian plane, where θ is positive if the particle crosses the meridian plane from east to west.

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[1] S. Chapman and V. Ferraro, Terr. Mag. Atmos. Elec. **45**, 245 (1940), and references therein.
[2] S. Chapman, Nature (London) **140**, 423 (1937).

For a magnetic field possessing axial symmetry, as in the case under consideration, the vector potential is independent of the azimuthal angle ω and can be represented by $\mathbf{A}=A\mathbf{i}_\omega$, where $\mathbf{i}_\omega$ is a unit vector pointing from east to west. It is readily shown then that the following expression is an integral of the motion:

$$pr\cos\lambda\sin\theta+A(e/c)r\cos\lambda, \tag{1}$$

where the particle momentum p is, of course, a second integral of the motion.

The vector potential due to the earth's dipole moment is given by

$$A_e=M_e\cos\lambda/r^2. \tag{2}$$

The vector potential due to a ring of current is a complicated expression and depends on the assumed cross-sectional radius. However, if we assume that the latter is small compared to $a\rho$, then we can write

$$(r\ll a\rho)\quad A_r=M_r r\cos\lambda/(a\rho)^3, \tag{3}$$

$$(r\gg a\rho)\quad A_r=M_r\cos\lambda/r^2, \tag{3'}$$

where M_r is the equivalent dipole moment of the ring. We now make the approximation, suggested by Chapman,[2] that these limiting equations hold for all values of r, in the first case for $r\leqslant a\rho$ and in the second case for $r\geqslant a\rho$. This is equivalent to replacing the ring by a current sheet located on the surface of a sphere of radius $a\rho$, where the current density is proportional to $\cos\lambda$. We will continue to speak of a "ring" current, however, as a matter of convenience.

Introduce the Stoermer variable R, defined by the equation

$$R=r(cp/eM_e)^{\frac{1}{2}}. \tag{4}$$

The earth's radius, in Stoermer units, is given by

$$R_e=\rho(cp/eM_e)^{\frac{1}{2}}; \tag{5}$$

and the radius of the ring, in Stoermer units, is

$$R_a=aR_e. \tag{6}$$

From Eq. (1) we then obtain the following relations:

$$(R\leqslant R_a)\quad R\cos\lambda\sin\theta+\cos^2\lambda/R+KR^2\cos^2\lambda/R_a^3=\alpha, \tag{7}$$

$$(R\geqslant R_a)\quad R\cos\lambda\sin\theta+(1+K)\cos^2\lambda/R=\alpha, \tag{7'}$$

where α is the constant of the motion and $K\equiv M_r/M_e$.

One can now proceed in the same way as in the derivation of the allowed Stoermer cones in the field of a simple dipole. (Notice that these equations reduce to the Stoermer equation when K vanishes.) For any choice of α one can plot the allowed regions in the meridian $(R-\lambda)$ plane from the requirement $|\sin\theta|\leqslant 1$. When α is larger than a certain value $\bar{\alpha}$, the allowed region breaks up into two parts, the outer part extending to infinity, the inner part being insulated from infinity by forbidden regions. The qualitative features are the same as in ordinary Stoermer theory; but $\bar{\alpha}$ now depends on

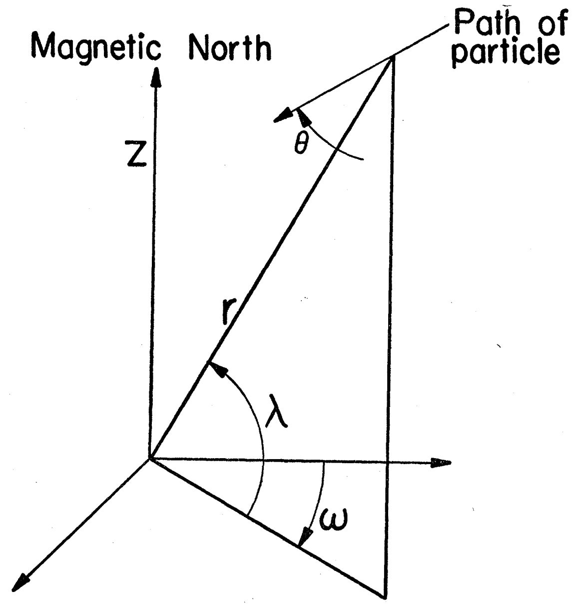

FIG. 1. Illustration of coordinate system used.

the parameters K and R_a, whereas in ordinary Stoermer theory it has the fixed value two.

Our object is to find the minimum momentum for which a particle coming from infinity can arrive at the earth's surface at latitude λ and at angle θ with respect to the meridian plane. Setting $R=R_e$ in Eq. (7), we can solve for R_e in terms of λ and θ for any choice of α. If α is larger than $\bar{\alpha}$, the smaller of the two roots puts the earth's surface in the inner allowed region, and the earth cannot be reached by particles coming from infinity. In this way we find the minimum value of R_e, hence the minimum momentum, for the arrival of a particle from infinity. Denoting this minimum by $\bar{R}_e$, we find

$$\bar{R}_e=\frac{2(1+K/a^3)\cos^2\lambda}{\bar{\alpha}+[\bar{\alpha}^2-4(1+K/a^3)\cos^3\lambda\sin\theta]^{\frac{1}{2}}}. \tag{8}$$

There remains now to determine $\bar{\alpha}$ as a function of the parameters K and R_a. Let $\sin\theta=\cos\lambda=1$ in Eqs. (7) and (7′); and denote the resulting functions of R on the left-hand sides of these equations by $F_1(R)$ and $F_2(R)$, respectively. Consider the equation

$$F(R)=\alpha, \tag{9}$$

where

$$F(R)=\begin{cases}F_1=R+1/R+KR^2/R_a^3, & \text{for } R\leqslant R_a;\\ F_2=R+(1+K)/R, & \text{for } R\geqslant R_a.\end{cases} \tag{10}$$

As in ordinary Stoermer theory, $\bar{\alpha}$ is that value of α for which Eq. (9) has only a single positive root; since for larger values of α the equation clearly has two positive roots and the allowed region in the meridian plane then splits up into two parts, the inner part being insulated

TABLE I. Maximum values of $(-k)$.

λ	10°	20°	30°	40°	50°
$-k$	3.7	2.4	2.6	3.8	7.8

TABLE II. Maximum values of $(-H\delta C/C\delta H)$ for total cosmic-ray intensity at sea level.

λ	10°	20°	30°	45°
$-H\delta C/C\delta H$	0.4	0.4	1.0	~ 0

from infinity by forbidden regions. In effect then, $\bar{\alpha}$ is equal to the smallest value of $F(R)$ for positive values of the argument R.

Let α_1 and α_2 be the respective minimum values of $F_1(R)$ and $F_2(R)$; and let R_1 and R_2 be the values of R which produce these minima: $F_1(R_1)=\alpha_1$, $F_2(R_2)=\alpha_2$. It can be seen from Eq. (10) that $R_1<R_2$. Hence, (a) if $R_a<R_1$, the smallest value of $F(R)$ is given by $\bar{\alpha}=\alpha_2$; (b) if $R_a>R_2$, then $\bar{\alpha}=\alpha_1$; (c) if $R_1<R_a<R_2$ then $\bar{\alpha}$ is equal to the smaller of the two minima, α_1 and α_2.

From Eq. (10) we find

$$\alpha_2=2(1+K)^{\frac{1}{2}};\quad R_2=(1+K)^{\frac{1}{2}};\tag{11}$$

and if we make the reasonable assumption that K is small compared to unity, then for $R_a\gtrsim 1$ it is easily shown that

$$\alpha_1\approx 2(1+K/2R_a^3);\quad R_1\approx(1+2K/R_a^3)/(1+3K/R_a^3).\tag{12}$$

Recalling our assumption that K is small, we see that $\alpha_1=\alpha_2$ when $R_a\approx 1$. From the discussion of the preceding paragraph it therefore follows that

$$\bar{\alpha}=\begin{cases}\alpha_2, & \text{for } R_a\lesssim 1;\\ \alpha_1, & \text{for } R_a\gtrsim 1.\end{cases}\tag{13}$$

The allowed cones in the presence of the "ring" current can now be obtained by substituting these results into Eq. (8).

In particular, we want to find the minimum value of R_e for arrival of a particle from the vertical direction ($\sin\theta=0$). This minimum is designated by $\bar{R}_{ev}$. From Eqs. (6), (8), and (13) we find

$$(\bar{R}_{ev}\lesssim 1/a)\quad \bar{R}_{ev}=\cos^2\lambda(1+K/a^3)/2(1+K)^{\frac{1}{2}};\tag{14}$$

$$(\bar{R}_{ev}\gtrsim 1/a)\quad \bar{R}_{ev}=\cos^2\lambda(1+K/a^3)/2(1+K/2a^3\bar{R}_{ev}^3).\tag{14'}$$

Equation (14′) has yet to be solved for $\bar{R}_{ev}$. Since $K/a^3\ll 1$, however, a very good approximation is obtained by setting $\bar{R}_{ev}=\frac{1}{2}\cos^2\lambda$ on the right-hand side of Eq. (14′).

The minimum momentum for vertical arrival, designated by $\bar{p}_v$, can now be found from Eq. (5). Neglecting terms higher than the first order in $K=M_r/M_e$, we arrive at the following results:

$$(\cos^2\lambda\lesssim 2/a)\quad \bar{p}_v=B\cos^4\lambda[M_e-(1-2/a^3)M_r];\tag{15}$$

$$(\cos^2\lambda\gtrsim 2/a)\quad \bar{p}_v=B\cos^4\lambda[M_e-(4\cos^{-6}\lambda-1)(2M_r/a^3)];\tag{15'}$$

where $B=e/4c\rho^2$.

III. RELATION BETWEEN COSMIC-RAY AND MAGNETIC VARIATIONS

Equations (15) and (15′) give the vertical cutoff momentum as a function of geomagnetic latitude λ. Consider now a cosmic-ray detector located at λ and at a depth x below the top of the atmosphere; and for simplicity, suppose that it is a "vertical" detector; i.e., that it responds only to particles which are incident from the vertical direction within a small solid angle. The counting rate C arises from the primary radiation, either directly or through secondary particles. If we assume that the latter maintain the direction of travel of the primaries, then the counting rate will be a function of the vertical flux of primaries, hence a function of $\bar{p}_v$:

$$C=C(\bar{p}_v).\tag{16}$$

If the detector is a nondirectional device (e.g., an ionization chamber), the vertical flux of radiation can be obtained from the observed counting rate by a Gross transformation.

Let H be the total magnetic field intensity at the geomagnetic equator. This contains a term H_e due to the earth's dipole and a term H_r due to the extraterrestrial current system, where

$$H_e=M_e/\rho^3;\quad H_r=-2M_r/(a\rho)^3.\tag{17}$$

Suppose now that M_r changes by δM_r. This produces directly a change in H; and indirectly, by inducing currents below the earth's surface, it produces effectively a change δM_e in the earth's dipole moment.[2] Assuming that a fraction f of the observed change in H is due to this secondary process, we can write

$$f\delta H=\delta H_e=\delta M_e/\rho^3;\quad (1-f)\delta H=\delta H_r=-2\delta M_r/(a\rho)^3.\tag{18}$$

From Eq. (16) we obtain the relation

$$\partial C/\partial H=(\partial C/\partial\lambda)\frac{\partial\bar{p}_v/\partial H}{\partial\bar{p}_v/\partial\lambda}.\tag{19}$$

Evaluating the derivatives in Eq. (19), and making use of the fact that $H\approx H_e\gg H_r$, we obtain the following results:

$$\frac{\delta C}{C}=\frac{1}{C}\left(\frac{\delta C}{\delta\lambda}\right)k\frac{\delta H}{H};\tag{20}$$

where

$$(\cos^2\lambda \lesssim 2/a) \quad k = -[(a^3-2)(1-f)+2f]/8\tan\lambda; \quad (20')$$

$$(\cos^2\lambda \gtrsim 2/a) \quad k = -[f+(1-f)(4\cos^{-6}\lambda-1)]/4\tan\lambda. \quad (20'')$$

These equations give the changes in cosmic-ray intensity that would be expected to accompany changes in the intensity of the extra-terrestrial current system. It should be pointed out that the derivative $\delta C/\delta\lambda$ involves the measurement of λ in radians. The discontinuity at the value $\cos^2\lambda \approx 2/a$ arises from the peculiarity of the current system considered. For a true ring current the function k would show a smooth transition at $\cos^2\lambda = 2/a$. The results obtained here are fairly exact (within the limits of the Stoermer approximation) for the current sheet that has been considered. They are only approximate if the current system is more nearly represented by a ring, although the approximation improves as $\cos^2\lambda$ becomes increasingly larger or smaller than $2/a$.

The effect on the cosmic radiation of a true ring current (taken to have a negligibly small cross-sectional radius) has been calculated by Hayakawa *et al.*[3] However, these authors have considered the effects only for a particular primary momentum (15 Bev/c) at a particular latitude (0°).

Following Chapman,[2] we assume that the fraction f has the value one-third. Equations (20′) and (20″) then reduce to the following:

$$(\cos^2\lambda \lesssim 2/a) \quad k = -(a^3-1)/12\tan\lambda; \quad (21')$$

$$(\cos^2\lambda \gtrsim 2/a) \quad k = -(8\cos^{-6}\lambda-1)/12\tan\lambda. \quad (21'')$$

The most significant feature of these results is the algebraic sign of the cosmic-ray effect. It is seen that a decrease in magnetic field intensity at the equator should correspond to an increase in cosmic-ray intensity, contrary to some of the qualitative arguments which have appeared in the literature. If the radial distance of the "ring" current is small, k is given by Eq. (21′), and it is seen that $|k|$ decreases rapidly with decreasing radius. For sufficiently large values of a, however, k is given by Eq. (21″), and it attains its maximum value, independent of a. In Table I we list the maximum value of $|k|$ for several latitudes.

[3] Hayakawa, Nishimura, Nagata, and Sugiura, J. Sci. Res. Inst. Tokyo **44**, 121 (1950).

In order to indicate the magnitude of the cosmic-ray variations to be expected, we can take as an example the total intensity of ionizing radiation at sea level, using for the latitude effect, $\delta C/\delta\lambda$, the values given in the paper by Johnson.[4] The maximum value of $|H\delta C/C\delta H|$ is given in Table II. For the nucleonic component, which shows a larger latitude effect,[5] these values would be considerably larger. During large magnetic storms, $\delta H/H$ may be as large as $\frac{1}{2}-1$ percent. Thus, unless the radius of the ring is assumed to be very small, the cosmic-ray variations should be detectable at intermediate latitudes. Above the knee of the latitude curves, however, the effect should vanish. Finally, it should be observed that our results cannot be used at very small latitudes, because of the inadequacy there of the Stoermer approximation. In this approximation k approaches infinity as λ becomes very small, but $\delta C/\delta\lambda$ approaches zero; and the problem becomes indeterminate.

It is perhaps most often the case that magnetic storms are not accompanied by detectable cosmic-ray disturbances, or that if they are, the cosmic-ray and magnetic variations do not show a detailed correspondence in time.[6] If the theory of the ring current is to be retained, it is necessary to suppose that the radius of the ring at such times is very small—and possibly the currents actually flow in the ionosphere. In other cases there does exist a detailed correlation between cosmic-ray and magnetic field variations, but the algebraic sign of the effect is the opposite of that predicted by the ring current theory.[7,8] Correlations with the proper algebraic sign and reasonable order of magnitude have, however, been reported[9]; but the statistical uncertainties make it difficult to determine whether or not the cosmic-ray and magnetic field variations show the detailed correspondence in time which would be required by the ring current theory.

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[4] T. H. Johnson, Terr. Mag. Atmos. Elec. **43**, 1 (1938).
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[6] S. E. Forbush, Terr. Mag. Atmos. Elec. **43**, 203 (1938); J. A. Simpson (to be published).
[7] S. E. Forbush, Phys. Rev. **51**, 1108 (1937).
[8] V. F. Hess and A. Demmelmair, Nature (London) **140**, 316 (1937).
[9] See, e.g., Hess, Demmelmair, and Steinmaurer, Terr. Mag. Atmos. Elec. **43**, 7 (1938).