all nuclei with higher proton numbers are also K-capturing or β^+-active.

If now we have a stable nucleus (N, Z) for a certain odd value of the mass number, then $(N+1, Z-1)$ must be β^--active and $(N-1, Z+1)$ either K-capturing or β^+-active. From what has already been pointed out, it is clear that all nuclei on the higher neutron number side of (N, Z) will then be β^--active and all nuclei on the higher proton number side of (N, Z) will be K-capturing or β^+-active. Thus for a particular odd mass number we shall have only one stable nucleus, all other isobars being unstable.

Let us now consider what happens if the above argument is applied to the case of an even mass nucleus. For such nuclei we can distinguish between the even-even and odd-odd nuclei. Let (N, Z) be an even-even nucleus which is β^--active. Then we have

$$B_N(N, Z)-B_Z'(N-1, Z+1)<M_N-M_H. \quad (8)$$

Now since,

$$B_N'(N+1, Z-1)=B_N(N, Z)-\delta+B_{NN}-B_{NZ},$$
$$B_Z(N, Z)=B_Z'(N-1, Z+1)+\delta-B_{ZZ}+B_{NZ},$$

we have

$$B_N'(N+1, Z-1)-B_Z(N, Z)=B_N(N, Z)-B_Z'(N-1, Z+1)$$
$$-2\delta+B_{NN}+B_{ZZ}-2B_{NZ}<M_N-M_H, \quad (9)$$

because δ is positive and $(B_{NN}+B_{ZZ}-2B_{NZ})$ is negative. Thus the odd-odd nucleus $(N+1, Z-1)$ must be β^--active. But this same argument applied now to the even-even nucleus $(N+2, Z-2)$ leads to a different result. For $B_N(N+2, Z-2)$ and $B_Z'(N+1, Z-1)$ we write

$$B_N(N+2, Z-2)=B_N'(N+1, Z-1)+\delta+B_{NN}-B_{NZ},$$
$$B_Z'(N+1, Z-1)=B_Z(N, Z)-\delta-B_{ZZ}+B_{NZ}.$$

Thus we get

$$B_N(N+2, Z-2)-B_Z'(N+1, Z-1)=B_N'(N+1, Z-1)$$
$$-B_Z(N, Z)+2\delta+B_{NN}+B_{ZZ}-2B_{NZ}. \quad (10)$$

Since δ is positive and $(B_{NN}+B_{ZZ}-2B_{NZ})$ is negative, we cannot conclude from the above equation that the left-hand side will be less than M_N-M_H. It may or may not be, depending on the relative magnitudes of δ and $(B_{NN}+B_{ZZ}-2B_{NZ})$. This shows that the even-even nucleus $(N+2, Z-2)$ may be stable in spite of the fact that the odd-odd nucleus $(N+1, Z-1)$ is β^--active, and thus the existence of stable isobars for even mass nuclei becomes possible.

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Crystallization of Silicon from a Floating Liquid Zone

PAUL H. KECK AND MARCEL J. E. GOLAY
Signal Corps Engineering Laboratories, Fort Monmouth, New Jersey
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CRYSTALLIZATION of silicon from a melt is of great interest as an initial step in the preparation of transistor devices. Since molten silicon is very active, there is no crucible material known of which one could be sure that no impurities are being introduced during the melting process. Fused silica, which is considered the best refractory material for melting silicon, is slightly reduced to silicon monoxide by the melt. The extent to which these reduction products affect the electrical properties of silicon is not known. This situation lends attractiveness to the method described below, which permits crystallization of silicon from the melt without having recourse to any crucible material.

The suspected high value of the surface tension of liquid silicon pointed to the possibility that a suitably large zone of molten silicon could be held stable between two vertically aligned, solid rods, and this surmise was verified indeed by experiment. A piece of silicon which had the shape of a rod was held vertically and clamped at both ends. A heater element consisting of a short tantalum cylinder was mounted around the center portion of the rod. Heating this cylinder to incandescence caused a short section of the silicon to melt. The molten zone which developed and assumed the shape of a pear was held stable between the solid parts of the silicon rod by surface tension. The molten zone was then caused to travel slowly along the silicon rod by moving downwards or upwards the bracket to which the upper and lower clamps holding the silicon piece were attached. This constitutes a method of vertical zone melting with a floating liquid zone requiring no container for its support. The silicon rod shown in Fig. 1 was recrystallized by this method. Another possible application of this method is the growth of a single crystal from a polycrystalline mass. For this a seed crystal is placed against one end of the polycrystalline rod, and an initial floating zone of liquid silicon is formed at the joint. Then the seed crystal together with the silicon rod is slowly moved while the crystal grows from the melt and the floating liquid zone travels along the silicon piece. Thus the polycrystalline mass can be converted into a single crystal almost to its support.

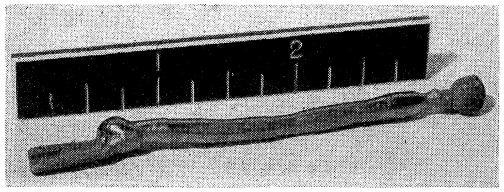

FIG. 1. Silicon recrystallized from a floating liquid zone. The comparison scale is in inches.

The new method is not limited to silicon; it may find useful applications for a number of materials.

We wish to express our appreciation for fruitful discussions of this work with Mr. E. L. Manning and Dr. B. Bradshaw. Moreover, it is a pleasure to acknowledge the enthusiastic help of Lt. W. Van Horn and Mr. J. Soled in experimentation.

Ferroelectricity in Oxides of Face-Centered Cubic Structure

W. R. COOK, JR., AND HANS JAFFE
Brush Development Company, Cleveland, Ohio
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IN a recent letter[1] we reported on dielectric and x-ray observations on cubic cadmium niobate ($Cd_2Nb_2O_7$) and lead niobate ($Pb_2Nb_2O_7$) to which we ascribed the fluorite structure. B. C. Frazer of Brookhaven National Laboratory has pointed out to us that the structure of $Cd_2Nb_2O_7$ was studied by Byström[2] in 1944 who found it to be of the pyrochlore ($NaCaNb_2O_6F$) type. This structure type is face-centered cubic with dimensions double that of the fluorite structure. We have confirmed the presence of additional faint lines of a doubled cell for $Cd_2Nb_2O_7$.

The pyrochlore structure differs from the fluorite structure in the following ways:

There is a regular alternation of cations in the (fluorite) cation positions and a regular omission of one of the eight oxygens in a manner which leaves the larger cation surrounded by eight oxygens but reduces the coordination around the smaller cation to six. These six oxygens are equally displaced from the symmetrical fluorite positions towards the smaller cation.

We have now found a strictly cubic pattern of unit cell 10.561 $\pm$0.001A for a lead niobate which is deficient in lead, approximating the composition $Pb_{1.5}Nb_2O_{6.5}$. The diffraction line in-

tensities are very similar to those of $Cd_2Nb_2O_7$. The dielectric constant is 200 at room temperature and rises to about 500 at -196°C.

The interesting ferroelectric properties found in $Cd_2Nb_2O_7$ are variable. The dielectric constant and Curie point vary markedly in disks from the same mix, with the highest Curie point at -103°C on a disk with a dielectric constant of 450 at room temperature. The remanent charge density of the latter sample as shown by a hysteresis loop is 3.2 microcoulomb per centimeter[2] near -196°C.

Further work to determine accurate locations of the oxygens in $Cd_2Nb_2O_7$ and $Pb_2Nb_2O_7$ is underway.

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Variational Methods for Periodic Lattices and Artificial Dielectrics*

CARSON FLAMMER
Stanford Research Institute, Stanford, California
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IN a recent paper, Kohn[1] has given a valuable variational principle for the determination of the propagating solutions of the Schrödinger equation within a unit cell of a periodic lattice. Of late, the cellular method has also been found useful in the analysis of artificial dielectrics which consist of arrays of conducting obstacles embedded in a dielectric medium.[2,3] Such artificial dielectrics are useful in the form of lenses at microwave frequencies. It is of interest, therefore, to obtain the electromagnetic analog of Kohn's variational principle.

The electromagnetic problem is the following:

$$(\nabla\times\nabla\times-k^2)\begin{matrix}\mathbf{E}(\mathbf{r})\\ \mathbf{H}(\mathbf{r})\end{matrix}=0, \quad \mathbf{r} \text{ within } V, \tag{1}$$

$$\mathbf{n}\times\mathbf{E}(\mathbf{r})=0, \quad \mathbf{r} \text{ on } \sigma, \tag{2}$$

$$\mathbf{n}\times\begin{matrix}\mathbf{E}(\mathbf{r}')\\ \mathbf{H}(\mathbf{r}')\end{matrix}=-\exp(i\boldsymbol{\kappa}\cdot\boldsymbol{\tau}_r)\mathbf{n}\times\begin{matrix}\mathbf{E}(\mathbf{r})\\ \mathbf{H}(\mathbf{r})\end{matrix}, \quad \mathbf{r} \text{ on } \Sigma. \tag{3}$$

Here σ denotes the surface of the object within a unit cell, while V and Σ denote the volume and surface of the unit cell; $\mathbf{n}$ is the outward normal at the designated point; $\boldsymbol{\kappa}$ is the wave-number vector; and $\boldsymbol{\tau}_r$ is the vector which translates a given face of the unit cell into coincidence with its conjugate, i.e., $\boldsymbol{\tau}_r=\mathbf{r}'-\mathbf{r}$. Only the transverse components need be subjected to the boundary conditions on Σ, because the normal components then automatically satisfy them in consequence of the Maxwell equations $\nabla\times\mathbf{E}=ik\sqrt{(\mu/\epsilon)}\mathbf{H}$, $\nabla\times\mathbf{H}=-ik\sqrt{(\epsilon/\mu)}\mathbf{E}$.

With the aid of the vector analog of Green's theorem, the first variation of the functional,

$$I=[ik\sqrt{(\mu/\epsilon)}]^{-1}\int_V \mathbf{E}^*\cdot(\nabla\times\nabla\times-k^2)\mathbf{E}dV, \tag{4}$$

is easily shown to be

$$\delta I=\int_{\sigma+\Sigma}(\mathbf{n}\times\mathbf{E}^*\cdot\delta\mathbf{H}-\mathbf{n}\times\delta\mathbf{E}\cdot\mathbf{H}^*)dS, \tag{5}$$

if $\nabla\times\delta\mathbf{E}=ik\sqrt{(\mu/\epsilon)}\delta\mathbf{H}$. Transforming the part of Eq. (5) over Σ in the manner of Kohn, we find that the desired variational principle is

$$\mathrm{Re}\Big\{[ik\sqrt{(\mu/\epsilon)}^{-1}]\int_V \mathbf{E}^*\cdot(\nabla\times\nabla\times-k^2)\mathbf{E}dV+\int_\sigma \mathbf{n}\times\mathbf{E}\cdot\mathbf{H}^*dS + \int_\Sigma \mathbf{n}\times\mathbf{E}(\mathbf{r})\cdot\mathbf{H}^*(\mathbf{r}')\exp(i\boldsymbol{\kappa}\cdot\boldsymbol{\tau}_r)dS\Big\}=\text{stationary}. \tag{6}$$

If the trial functions $\mathbf{E}$, $\mathbf{H}$ satisfy the vector wave Eq. (1) and the boundary condition of Eq. (2), the variational principle becomes

$$\mathrm{Re}\int_\Sigma \mathbf{n}\times\mathbf{E}(\mathbf{r})\cdot\mathbf{H}^*(\mathbf{r}')\exp(i\boldsymbol{\kappa}\cdot\boldsymbol{\tau}_r)dS=\text{stationary}. \tag{7}$$

Equation (7) may be cast into the form

$$\mathrm{Re}\int_\Sigma \delta\mathbf{E}(\mathbf{r})\cdot[\mathbf{n}\times\mathbf{H}^*(\mathbf{r})+\mathbf{n}\times\mathbf{H}^*(\mathbf{r}')\exp(i\boldsymbol{\kappa}\cdot\boldsymbol{\tau}_r)]dS + \mathrm{Re}\int_\Sigma \delta\mathbf{H}(\mathbf{r})\cdot[\mathbf{n}\times\mathbf{E}^*(\mathbf{r})+\mathbf{n}\times\mathbf{E}^*(\mathbf{r}')\exp(i\boldsymbol{\kappa}\cdot\boldsymbol{\tau}_r)]dS=0, \tag{8}$$

by adding two terms which vanish by virtue of the vector Green's theorem.

In the same way that the method of Slater[4] follows from the equation analogous to Eq. (8) in the atomic case, as shown by Kohn, so the electromagnetic extension of Slater's method follows from Eq. (8). In this method, the boundary conditions of Eq. (3) are satisfied at only a finite number of points. When series expansions of vector functions, containing as many arbitrary coefficients as there are equations, are substituted for $\mathbf{E}$ and $\mathbf{H}$, a compatibility equation is obtained which yields the desired relation between $\boldsymbol{\kappa}$ and k. The surface $k=$constant in $\boldsymbol{\kappa}$-space is the analog of the index surface in crystal optics and is, in general, two-sheeted. When the wave propagating in the artificial dielectric is approximately plane, the relevant refractive index is $|\boldsymbol{\kappa}|/k$ to a first approximation and is a function of the direction of the wave normal and of the polarization of the wave.

As a case in point, we have applied Slater's method to the case when the objects are infinitesimally thin, circular disks and the unit cell is a rectangular parallelepiped. There are two independent field distributions to be considered. Using series of oblate spheroidal vector wave functions which satisfy the boundary conditions on the disk, as in the recently obtained vector wave function solution of the diffraction of electromagnetic waves by circular disks and apertures,[5] we have satisfied the boundary conditions at the midpoints of the faces of the unit cell. The resulting surfaces $k=$constant in $\boldsymbol{\kappa}$-space are prolate spheroids at low frequencies. The employment of the variational principle of Eq. (7) should lead to more accurate forms of the surfaces at higher frequencies.

The general form of the variational principle, Eq. (6), may be expected to be useful in the case of objects for which there are available no vector wave functions that can be used to satisfy completely the boundary conditions on the object within a unit cell.

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A General Relativistic Approach to Short-Range Forces

HENRI AMAR
Lafayette College, Easton, Pennsylvania
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ORDINARY Newtonian gravitational forces are far too small to account for nuclear interactions. However, it is possible to modify the law of gravitation in such a way as to get unusually strong gravitational attractions in the very small forces. To do so we propose the following generalized law of gravitation

$$R_{\mu\nu}= \quad =(\Lambda g\cdots)_{\mu\nu}, \tag{1}$$

where $g_{\alpha\beta}$ is the metric tensor, $R_{\mu\nu}$ the Riemann-Christoffel tensor, and $\Lambda\ldots^{\cdots}$ some multiplicative tensor such that the right side of (1) is covariant of second rank. The first simplest cases are

$$R_{\mu\nu}=\Lambda g_{\mu\nu} \tag{1a}$$

and

$$R_{\mu\nu}=\Lambda_\mu{}^\sigma g_{\sigma\nu}. \tag{1b}$$

Professor Bargmann and Professor Einstein in a private communication gave us the simple proof that in (1a) Λ is necessarily