Magnetic Moments of Odd-Odd Nuclei and the Strong Spin-Orbit Coupling Shell Model*

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Using the extreme jj coupling model and employing semi-empirical g factors, magnetic moments are computed for those odd-odd nuclei for which I and μ measurements exist (Table I). The results are suggestive as to relevant coupling properties of such nuclei, but as yet not sufficient for definite conclusions. As an aid to possible future work, this note includes a tabulation (Table II) of computed magnetic moments for some odd-odd nuclei for which tentative spin assignments are possible, employing both free nucleon and semi-empirical g factors and based on pure LS and jj two-nucleon states.

THE extreme jj coupling model has been recently applied to the calculation of magnetic moments of odd-odd nuclei by Feenberg,[1] who pointed out its particular success in the case of K^{40}. Since the Schmidt g factors[2] entering in these calculations are known to deviate considerably from the measured values for most nuclei, the question arises as to the scope of validity of such calculations. It has been suggested[3] that in nuclei (such as B^{10}, N^{14}, Na^{22}, V^{50}) whose odd proton and odd neutron belong the the same l, j-orbital, there is an approximate mutual cancellation of the respective deviations from the Schmidt limits. This suggestion is consistent with the following tentative conjecture: The magnetic moments of those odd-odd nuclei (excepting Li^6) which admit of a definite configuration assignment $(l_p, j_p; l_n, j_n)$, are given to a first approximation by the pure jj coupling formula

$$\mu(Z,N)=\frac{1}{2}\left[(g_p+g_n)I+(g_p-g_n)\frac{j_p(j_p+1)-j_n(j_n+1)}{I+1}\right], \quad (1)$$

with g_p and g_n representing the *empirical* g factors of the nuclei $(Z, N-1)$ and $(Z-1, N)$, respectively (and with the customary meaning of the other symbols). Such a result can be expected to have approximate validity if (a) the deviations of odd A magnetic moments from the Schmidt values are due largely to interaction of the odd nucleon with the core, and (b) if jj coupling is in fact an adequate approximation. Conversely, if premise (b) were to become definitely established, then the study of μ for odd-odd nuclei could shed some light on (a).

The results presented in Table I seem to be in accord with the above conjecture.[4] Unfortunately, they are subject to two severe limitations: the number of definitely established μ and I values is quite small, and the number of corresponding g_n is even smaller. In both Tables I and II, some of the empirical g values had to be estimated from measured μ of neighboring nuclei with the same odd nucleon l and j values. An examination of the set of measured odd A magnetic moments lends support to the expectation that in most cases the error committed by such an interpolation is not serious, when precision is not required. Whenever it appeared impossible to make a reliable estimate, the correspond-

TABLE I. Magnetic moments of some odd-odd nuclei for which measurements exist.

Nucleus	I ($\hbar$)	Measured μ (nm)	Configuration[a] proton	neutron	Computed μ using free-nucleon g	Empirical odd nucleon g proton	neutron	Computed μ using available empirical g
B^{10}	3	1.80	$1p_{3/2}^-$	$1p_{3/2}^-$	1.88	(2.0)	−0.786	1.82
N^{14}	1	0.404	$1p_{1/2}^-$	$1p_{1/2}^-$	0.37	(−0.58)	1.40	0.41
K^{40}	4	−1.30[b]	$1d_{3/2}^-$	$1f_{7/2}^+$	−1.70	0.261	···	−1.54
V^{50}	?	0.557I	$1f_{7/2}$	$1f_{7/2}^-$	0.55 I	1.43	···	0.44 I
Co^{60}	(5)	3.0±0.5	$1f_{7/2}^-$	$2p_{3/2}$	3.88	1.33	···	2.73
Rb^{86}	2	(−1.68)	$1f_{5/2}$	$1g_{9/2}^-$	−2.13	0.54	(−0.22)	−1.72
Cs^{134}	4	2.96	$1g_{7/2}$	$2d_{3/2}$	2.18	0.737	(0.55)	2.80
Lu^{176}	≧7	4.2±0.8	$1g_{7/2}$	$1h_{9/2}$	2.85 (I=7) 3.28 (I=8) 3.71 (I=9)	(0.83)	···	3.8 (I=7) 4.5 (I=8) 5.0 (I=9)

[a] The superscripts − and + indicate closed shells or subshells minus or plus one particle, respectively.
[b] Eisinger, Bederson, and Feld, Phys. Rev. 86, 73 (1952).

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[1] E. Feenberg, Phys. Rev. 76, 1275 (1949).
[2] T. Schmidt, Z. Physik 106, 358 (1937). This paper employs the free nucleon model.
[3] I. Talmi, Phys. Rev. 83, 1248 (1951).
[4] The experimental μ were taken from the compilation by P. F. Klinkenberg [Revs. Modern Phys. 24, 63 (1952)], in which μ-values obtained from optical hyperfine structure are corrected for the effect of finite nuclear size. For V^{50} and Cs^{134}, see Walchi, Leyshon, and Scheitlin, Phys. Rev. 86, 618 (1952), and Jaccarino, Bederson, and Stroke, Phys. Rev. 87, 676 (1952).

TABLE II. Computed magnetic moments for some odd-odd nuclei for which measurements do not now exist.

Nucleus	Assigned I ($\hbar$)	Configuration[a] proton	neutron	LS states	$\mu(LS)$ (nm)	$\mu(jj)$ j_pj_n	$j_p'j_n$[b]	j_pj_n'	$j_p'j_n'$	$\mu(jj)$ using available empirical g
P^{32}	1	$2s_{1/2}{}^+$	$1d_{3/2}$	^3D	-0.440	-0.440	...	...	...	-0.12
K^{42}	2	$1d_{3/2}{}^-$	$1f_{7/2}$	$^1D, {}^3P, {}^3D, {}^3F$	$0, -0.120, 0.293, 0.080$	-1.72	-1.50	1.02	2.46	
$Ga^{66, 68}$	1	$(2p_{3/2}{}^+$	$2p_{3/2})$	$^1P, {}^3S, {}^3P, {}^3D$	$\frac{1}{2}, 0.88, 0.69, 0.310$	0.627	-1.46	2.84	0.373	
$Ga^{66, 68}$ [c]	1	$(2p_{3/2}{}^+$	$1f_{5/2})$	^3D	-0.94	-0.94	...	...	...	
As^{72}	2	$(1f_{5/2}$	$1g_{9/2})$	$^1D, {}^3P, {}^3D, {}^3F$	$-0.334, -0.620, 0.015, -0.141$	-2.13	-1.89	0.864	2.08	
As^{72}	2	$(1f_{5/2}$	$2p_{1/2})$	$^1D, {}^3D, {}^3F$	2.67, 2.51, 1.86	0.379	3.88	0.420	6.24	
Br^{80*} (4.4 hr)	5	$2p_{3/2}$	$1g_{9/2}$	$^1H, {}^3G, {}^3H$	1, 1.08, 1.11	0.091	-2.18	5.28	...	0.142
Y^{90}	2	$2p_{1/2}$	$2d_{5/2}{}^+$	$^1D, {}^3P, {}^3D, {}^3F$	0.334, 0.380, 0.572, 0.301	-1.61	-0.981	0.883	3.29	-0.84
In^{114*} (50 days)	5	$(1g_{9/2}{}^-$	$3s_{1/2})$	^3G	4.88	4.88	...	...	...	4.87
In^{114*}	5	$(1g_{9/2}{}^-$	$2d_{3/2})$	$^1H, {}^3G, {}^3H, {}^3I$	3.67, 4.28, 3.69, 3.16	6.99	2.86	4.89	0.045	
Sb^{124}	3	$1g_{7/2}{}^+$	$3s_{1/2}$	^3G	3.09	3.09	...	...	...	3.13
$Au^{192,194}$, $Au^{196,198}$	2	$2d_{3/2}$	$3p_{1/2}$	$^1D, {}^3P, {}^3D, {}^3F$	1.67, 2.38, 1.68, 1.19	0.762	4.05	-1.19	3.30	0.836
Tl^{204}	2	$(3s_{1/2}{}^-$	$3p_{3/2})$	^3P	0.88	0.88	...	...	...	1.05
Tl^{204}	2	$(3s_{1/2}{}^-$	$2f_{5/2})$	^3F	-0.59	-0.59	...	...	...	
Tl^{204}	2	$(2d_{3/2}$	$3p_{1/2})$	$^1D, {}^3P, {}^3D, {}^3F$	1.67, 2.38, 1.68, 1.19	0.762	4.05	-1.19	3.30	
Bi^{204}	3	$(1h_{9/2}{}^+$	$3p_{3/2})$	^3G	3.85	3.85	...	...	...	
RaE^{210}	2	$(1h_{9/2}{}^+$	$3d_{5/2}{}^+)$	^3F	3.43	3.43	...	...	...	

[a] See footnote a to Table I.
[b] A dash signifies reversal of l, s orientation.
[c] See reference 10.

ing free-nucleon g factors were employed. If in case of V^{50} and Co^{60} we employ g_n corresponding to $\mu \sim -1$, a mean value fitted to the diagram of measured μ-values, one finds $\mu(V^{50}) = 0.57I$, and $\mu(Co^{60}) = 3.65$. With similar g_n, one finds μ $(K^{42}) = -1.0$ (see Table II).

Lu^{176}, whose spin is still unknown, is included in Table I because of the indication that it is possible to have here a normal configuration assignment[5] and a spin close to the experimental lower bound. A discussion leading to different conclusions has been published recently.[6]

Table II lists a few odd-odd nuclei, of half-lives exceeding one hour, for which spin assignments can now be made, at least tentatively. These assignments are based on Nordheim's rules[7] and on an analysis of beta-decay schemes compiled by King.[8] Because of existing uncertainty as to the actual coupling schemes that obtain for the ground and first excited states of various nuclei, computed values for the two-particle LS and jj coupling type states corresponding to the given configurations and I, are also included in this table.[9] At the same time only those nuclei are listed for which the computed μ are either the same for all states, or else are sufficiently distinct for the LS and jj states to present the possibility of μ measurement discrimination between them. Although the latter group do not comprise any examples strictly similar to K^{40}, for which μ for the *expected* jj coupling scheme is definitely distinguished (by its sign) from those given by the other coupling possibilities, it will be observed that such a situation does obtain in the case of $Ga^{66,68}$, Br^{80*} (4.4 hr), $Au^{192,194,196,198}$, and Tl^{204} if the configurations which belong to them are, respectively, $p_{1/2}-p_{3/2}$, $p_{1/2}-g_{9/2}$, $d_{3/2}-p_{3/2}$, $d_{3/2}-p_{3/2}$ instead of those listed in the table.

In both Tables I and II parentheses enclosing a quantity indicate some uncertainty regarding it. A few cases with uncertainty in the configuration assignment are included in the tabulation.[10] These indicate a possibility of resolving such uncertainties when the corresponding I and μ measurements become available.

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[5] See entry for Lu^{176} in Table II, L. W. Nordheim, Revs. Modern Phys. **23**, 322 (1951), p. 326.
[6] P. F. Klinkenberg, Physica **17**, 715 (1951).
[7] L. W. Nordheim, Phys. Rev. **78**, 294 (1950). These rules appear to admit of some exceptions.
[8] Richard King (unpublished).
[9] A similar tabulation for Y^{90} is given in reference 1, to which the reader is referred for relevant discussion.
[10] Recent Ga^{66} and Ga^{68} disintegration measurements [A. Mukerji and P. Preiswerk, Helv. Phys. Acta **25**, 387 (1952)] support the following respective configuration assignments: $p_{3/2}f_{5/2}$, $p_{3/2}p_{3/2}$.