That the variational determination of σ, Eq. (19), is actually superior to σ_B can be seen by reference to Table II. This table includes a few values calculated from zero- and first-order phase shifts obtained numerically by Chandrasekhar and Breen.[7] It is also interesting to observe that $\sigma_{\text{var}} \rightarrow \sigma_{\text{Born}}$ as $k_0 \rightarrow \infty$. We may therefore conclude on the basis of the discussion of the cross-section theorem that the forward amplitude as obtained variationally with the undisturbed wave as the trial solution is indeed better than that determined from the second Born approximation for all incident energies.

SUMMARY

(a) Good estimates of the of the phase shift may be conveniently made variationally with polynomial type wave functions.

(b) For a given trial solution the Schwinger formalism appears to furnish estimates of the phase shift superior to the Hulthén-Kohn method.

(c) Of particular advantage, in comparison with the Hulthén method, is the fact that in the Schwinger principle the trial wave functions need not be normalized to the correct asymptotic form.

(d) For the undisturbed wave as trial solution, the forward scattering amplitude is determined more accurately variationally for all incident energies than by use of the second Born approximation.

(e) The improved estimate of the total cross section, σ_{var}, still underestimates the correct value at the low incident energies and approaches the Born approximation for the higher energies in a continuous manner.

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The Absolute Determination of Resonant Energies for the Radiative Capture of Protons by Boron, Carbon, Fluorine, Magnesium, and Aluminum in the Energy Range below 500 kev

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An absolute electrostatic analyzer of deflecting angle $\pi/2\sqrt{2}$ radians has been used to measure and stabilize the energy of the proton beam from an air insulated electrostic generator. The following resonant energies for proton capture by some of the light elements have been measured:

$B^{11}(p,\gamma)C^{12}$	163.8±0.3 kev.
$C^{12}(p,\gamma)N^{13}$	456.8±0.5 kev.
$F^{19}(p,\gamma)Ne^{20*}$	224.4±0.4 kev.
$Mg^{26}(p,\gamma)Al^{27}$	314.8±0.5, 338.5±0.5, 389.4±0.5, 436.5±0.4, 454.2±0.3, 484.0±1.0 kev.
$Mg^{24}(p,\gamma)Al^{25*}$	418.0±0.5 kev.
$Al^{27}(p,\gamma)Si^{28}$	226.3±1.5, 294.1±0.5, 325.6±0.4, 404.7±0.4, 438.5±0.5, 504.0±0.6 kev.

The high energy resolution of one thousand obtained has made accurate determination of the half-widths of these resonances possible.

INTRODUCTION

THE construction of the 63.6° electrostatic analyzer and its use to measure absolutely and stabilize the energy of the beam from an electrostatic generator have been described in a previous paper.[1]

The uncertainty of the measurements is about one part in a thousand except for the very weak resonances.

The agreement of the absolute values obtained for the strong lithium resonance at 441.5 kev and the fluorine resonances at 340.4 and 483.1 kev[1] with recent precise determinations by other workers[2–4] confirms the claim for high absolute accuracy.

The resonances investigated in the present work have not, in general, been measured to this accuracy previously, and it is hoped that precise determinations of nuclear resonances, and hence of nuclear energy level spacing, will be of use in testing theories on nuclear structure, whilst for closely spaced levels precise energy determinations are necessary in order to provide definite identification.

Because of the high energy resolution and stability of the beam, it has been possible to measure the natural half-widths of many resonances to an accuracy not previously obtained.

EXPERIMENTAL DETAILS

The energy of the beam could be varied continuously and target currents up to 25 microamperes were obtained. These were fed into a beam current integrator and timing unit which stopped the γ-ray counts when the target had received a predetermined charge.

[1] S. E. Hunt Proc. Phys. Soc. (London) **A65**, 982 (1952).
[2] E. L. Hudspeth and C. P. Swann, Phys. Rev. **75**, 1272 (1949).
[3] W. A. Fowler and C. C. Lauritsen, Phys. Rev. **76**, 314 (1949).
[4] A. H. Morrish, Phys. Rev. **76**, 1651 (1949).

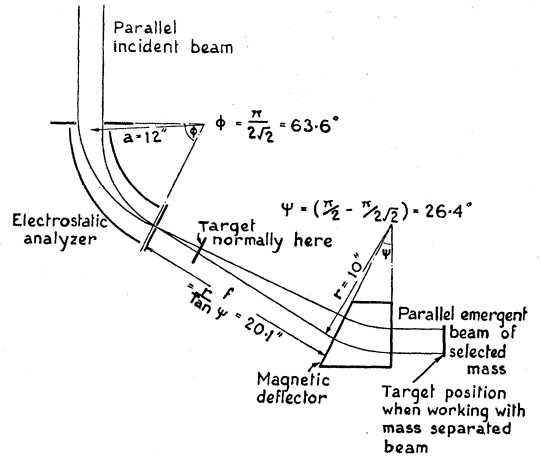

FIG. 1. Diagrammatic representation of the electrostatic analyzer and deflector magnet.

In normal operation the target was placed a short distance from the exit slit of the electrostatic analyzer to allow the finely focused beam to diverge and prevent damage resulting from local overheating of the target surface. When there was any uncertainty as to which mass component of the beam produced an observed resonance, mass separation was effected by placing a small triangular magnet after the electrostatic analyzer. The first focal point of the magnet was made to coincide with the exit slit of the analyzer so that the two formed a telescopic system, and the deflecting angle of the magnet was complementary to that of the analyzer to produce a horizontal emergent beam. A Mu metal shield was fitted over the analyzer for these exposures to reduce the stray magnetic field from the magnet.

The system is illustrated diagrammatically in Fig. 1.

It was possible to observe the 224.4-kev fluorine resonance using the mass 1 and mass 2 components of the beam and the 163.8-kev boron resonance using mass 1, mass 2, and mass 3 components.

ABSOLUTE ACCURACY

The errors associated with the determinations have been discussed in detail elsewhere.[1] It has been estimated that systematic errors associated with geometrical measurements on the electrostatic analyzer and the measurement of the voltages applied to the deflector plates are less than five parts in ten thousand. The main source of random error was the uncertainty in estimating exactly the point of maximum yield from a curve drawn through the experimental points. This varied considerably, being about 5 parts in 10^4 for the single determination of a fairly strong, sharp resonance to about 3 parts in 10^3 for that of a weak, broad resonance. The errors were approximately half this when scintillation counters were used for detection. They were further reduced by making between five and ten independent determinations of each resonance. This had the advantage that other small random errors associated with the estimation of target thickness and the measurement of deflector voltages also tended to be minimized. The results quoted are the mean values of these several determinations and the uncertainties were obtained by adding the standard deviation of this mean value to the estimated systematic error.

TARGETS

The target materials were deposited on carefully cleaned and polished copper backings. These were heated to a temperature of between 200 and 250°C during irradiation and were situated close to a liquid nitrogen cooled baffle. This has been found to be effective in stopping the contamination of the target surface by condensed oil from the diffusion pumps.[1]

Boron and separated magnesium isotope targets were deposited by the Isotope Division A.E.R.E., Harwell, using a mass spectrograph. Natural magnesium and aluminum were deposited by vacuum evaporation of the metal from molybdenum boats and tungsten filaments, respectively.

Some boron targets were also prepared by evaporation from graphite boats. The target thickness was estimated from the total amount of boron plus boron carbides evaporated.[5]

Fluorine targets were prepared by evaporating calcium, lithium, and magnesium fluorides from molybdenum boats.

In all evaporations the target thickness in micrograms per square centimeter was estimated from the total weight of material evaporated and the geometry of the evaporation. The accuracy of this method was tested by evaporating aluminum on to a larger surface and estimating the thickness of the deposit by direct weighing of the backing using a microbalance. The two estimates of target thickness agreed to within two percent.

It was not found possible to evaporate carbon, and it was considered undesirable to dilute the target by evaporating a carbon compound. Carbon targets were, therefore, prepared by depositing soot from a Calor gas flame on to a copper backing, on which a very thin (~100 ev) calcium fluoride target had been previously deposited. It was then possible to estimate the thickness of the carbon targets in energy units directly from the observed displacements of the fluorine resonances.

The thickness of other targets in energy units was estimated from published stopping power curves, when available, and from Bethe's formula:[6]

Tangen[7] and Morrish[4] have shown that the true resonant energy is given by $E_0 = E - \Delta E/2$, where E is the observed energy of maximum yield and ΔE is the thickness of the target in energy units. In order to

[5] N. V. Sidgwick, *Chemical Elements and Their Compounds* (Clarendon Press, Oxford, 1950), Vol. 1, p. 368.

[6] M. S. Livingston and H. A. Bethe, Revs. Modern Phys. **9**, 263 (1937).

[7] R. Tangen, Kgl. Norske Videnskab. Selskabs, Skrifter No. 1 (1946).

minimize the uncertainty in ΔE targets were kept as thin as possible, so far as this was consistent with obtaining measurable yields. Thicknesses between 100 ev and 2 kev were used for most of the determinations in the present work, except in the case of carbon, for which uncertainties in the stopping power of the target material were not relevant.

The thickness of the targets in energy units was further checked by rotating them through an angle of 60° with respect to the proton beam. This effectively doubled the target thickness and the resonance peak was displaced by a further energy of $\Delta E/2$.

Tangen[7] has shown that if Γ is the experimentally observed half-width of a resonance, then Γ_0, the true half-width, is given by $\Gamma_0=(\Gamma^2-\Delta E^2)^{\frac{1}{2}}$. This correction applies only when ΔE is small compared with Γ_0. Estimates of half-width were therefore made only when this condition obtained. When the observed half-width did not greatly exceed the thickness of the thinnest target on which it was observed, this was given as an upper limit of the possible natural half-width.

Resonances were also observed using thick targets in the case of carbon and aluminium. The resonant energy was taken as the point of inflection in the experimental curve. The values were in good agreement with those obtained from the thin target yield curves, although the points of inflection could not be located with the same accuracy as could the peaks of thin target yield curves. Since thick target yield curves represent the integral of thin target yield curves they were, however, useful in confirming that no measurable resonance had been omitted.

DETECTION

Initially the γ-rays were detected by using an end window type Geiger-Müller counter with a lead radiator. The solid angle subtended by the counter at the target was arranged to be as large as possible.

In order to rotate the target through 60° with respect to the incident beam, a brass tube with flanges inclined at an angle of 60° was interposed between the analyzer and target flange. This rotated both target and detector without affecting the geometrical efficiency of detection.

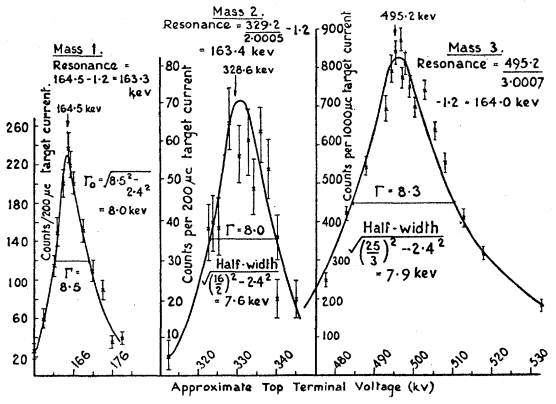

FIG. 2. The boron resonance observed using mass 1, mass 2, and mass 3 components of the beam.

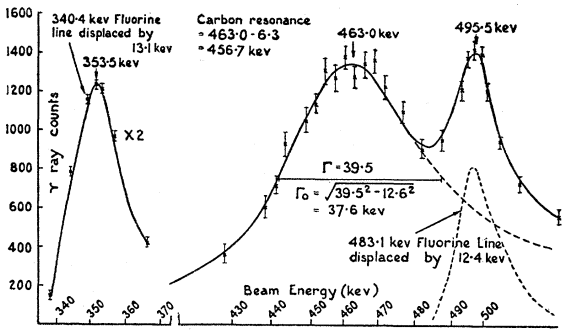

FIG. 3. The carbon resonance observed using a carbon target deposited on a 100-ev thick calcium fluoride target.

In later exposures the sensitivity of detection was improved by using a scintillation counter consisting of a thallium activated sodium iodide crystal and an E.M.I. type 5311 photomultiplier.

When using both Geiger counters and scintillation counters, background counts were reduced as far as possible by using thick lead shielding. The background was estimated by placing a clean brass surface in front of the target and repeating the exposures. The estimated background has been subtracted from all experimental points shown in the figures.

DISCUSSION OF RESULTS

Boron. $B^{11}(p,\gamma)C^{12*}$

The mean value of ten independent determinations of the energy of the boron resonance was 163.8 kev with a standard deviation of 0.2 kev. The total uncertainty after allowing for possible systematic errors is ± 0.3 kev. A mean value of 7.3 ± 0.5 kev was obtained for the natural half-width of the resonance. Typical curves obtained using mass 1, 2, and 3 components of the beam are shown in Fig. 2.

Another precise determination of this resonance has been made by Morrish[4] using an electrostatic analyzer calibrated by deflecting a low energy electron beam. He quotes a value of 162.8 kev with an uncertainty of ± 0.2 kev, though it appears that this latter figure refers to the internal consistency of this results, as the probable errors associated with the calibration of the apparatus are quoted as 0.3 percent.[4,8] His value for the natural half-width was 4.5 ± 1.5 kev. The disagreement of both these measurements with those obtained in the present work is somewhat greater than the combined errors.

Earlier and less precise determinations made by other workers have been listed by Morrish.

[8] Allison, Frankel, Hall, Montague, Morrish, and Warshaw, Rev. Sci. Instr. **20**, 735 (1949).

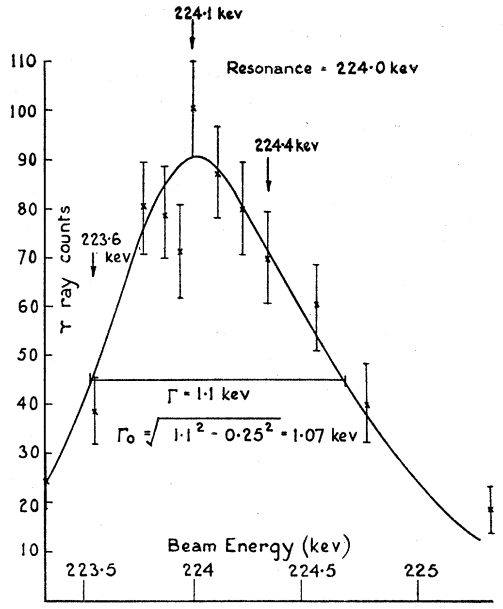

FIG. 4. The weak fluorine resonance at 224.4 kev observed on a calcium fluoride target of thickness 250 ev.

Carbon. $C^{12}(p,\gamma)N^{13}$

The mean value of five determinations, using targets of thickness in energy units between 5 and 15 kev deposited over thin calcium fluoride targets, was 456.8 kev, the total uncertainty was ± 0.5 kev, and the mean half-width was 39.5 ± 1.0 kev. A typical curve is shown in Fig. 3. It will be seen that the fluorine resonance at 340.4 kev was displaced by 13.1 kev and the peak at 483.1 kev by 12.4 kev. The thickness of the carbon target at the energy of the carbon resonance was therefore taken as 12.6 kev. This represents a mean value for the thickness since it is seen from the broadening of the fluorine lines that the carbon deposition was not uniformly thick.

One of the thin calcium fluoride targets was irradiated before the deposition of the soot target, and the yield from the two fluorine resonance was found to be negligible between 400 and 475 kev. It was, therefore, reasonable to assume that the yield from these resonances did not affect the position of the carbon peak observed in the composite targets. Some uncertainty was introduced in the half-width measurements since the yield from the upper fluorine resonance started to be appreciable at about 480 kev. It was possible, however, to analyze the combined curve with a fair degree of accuracy by assuming that the upper fluorine resonance was broadened by the same amount as the lower one. Additional irradiations were also made of carbon targets deposited directly on the copper backings. The exact shape of the carbon resonance could then be followed in the absence of the fluorine lines. An estimate of the target thickness was necessary to obtain the natural half-width from the experimentally observed half-width. This was taken as twice the displacement of the carbon peak itself from the mean corrected value of the resonance previously determined.

The values of the half-width so obtained were in good agreement with those obtained from the composite targets.

Previous determinations of this resonance have been summarized by Hornyak, Lauritsen, Morrison, and Fowler.[9] The highest precision previously quoted being for a determination by Fowler and Lauritsen[3] of 456 ± 2 kev with a half-width of 35 kev; this measurement was made relative to the 441.5-kev-lithium resonance.

Fluorine. $F^{19}(p,\gamma)Ne^{20*}$

The measurement of the strong resonances at 340.4 ± 0.5 and 483.1 ± 0.5 kev has been reported previously.[1] A resonance of approximately one three-hundredth of the yield of the 340.4 line has been observed. The mean of ten determinations was 224.4 ± 0.4 kev, and the half-width of the resonance, observed on a target of thickness in energy units 0.25 kev, was 1.1 kev. The natural half-width of the resonance is therefore estimated to be 1.0 ± 0.3 kev.

A typical resonance curve is shown in Fig. 4. This resonance has been observed previously by Tangen[7] who measured the resonant energy as 222 ± 2 kev and gave the half-width as less than 2 kev, which was the thickness in energy units of the target used.

Magnesium

Magnesium occurs in three isotopic forms Mg^{24}, Mg^{25}, and Mg^{26}. The compound nuclei Al^{25} and Al^{26} produced on proton capture by the two lower isotopes decay by positron emission, while Mg^{26} produces the stable isotope Al^{27}.

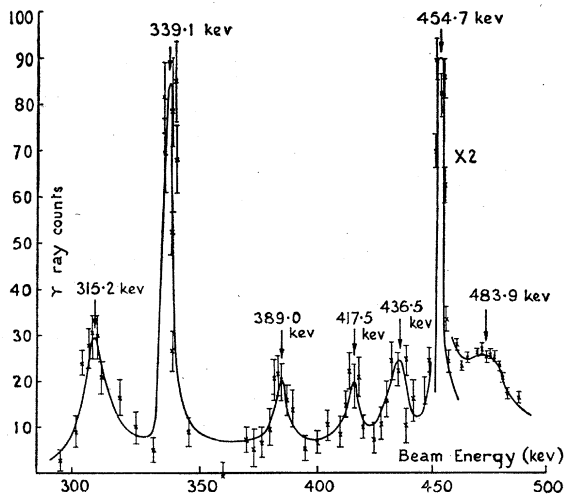

FIG. 5. The magnesium resonances observed on a target of thickness 1.1 kev.

[9] Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. 22, 291 (1950).

On bombarding metallic magnesium targets, (p,γ) resonance were observed at the following energies: 314.8±0.5, 338.5±0.5, 389.4±0.5, 418.0±0.5, 436.5 ±0.4, 454.2±0.3, and 484.0±1.0 kev, with respective half-widths of: 4.0±1.0, 2.5±1.0, 4.0±1.0, 4.0±0.5, 4.0±1.0, <1.0, and 10±2 kev. A typical experimental curve is shown in Fig. 5.

It was thought that the formation of oxide films on the metallic target surfaces might increase their effective thicknesses considerably and thus produce errors in the target correction. The displacement of the strong 454.2-kev line on rotating the targets through 60° indicated that any oxide layers were of negligible thickness. This was also confirmed by observing this resonance on magnesium fluoride targets, when the absolute determinations and measurements relative to the fluorine lines were in excellent agreement with those obtained using metallic targets.

Tangen has observed all the above resonances to an accuracy of about 1 percent,[7] and the present observations are mainly within the limits of experimental error which he quoted, or sufficiently near to his values to allow positive identification, except for the weak broad resonance at 484±1.0 kev. The discrepancy between the present determination and Tangen's value of 494±5 kev was thought to be sufficiently serious to produce doubts on the identification of this resonance, but no other resonance was observed up to 510 kv so that it seems probable that the two measurements refer to the same resonance.

From the absence of positron emission Tangen ascribed all but the 418.0-kev resonance to the Mg^{26} isotope. This he originally ascribed to Mg^{25}, although Grotdal[10] later ascribed it to the Mg^{24} isotope. The resonance has been observed using separated Mg^{24} in the present work, thus confirming Grotdal's result.

Aluminum. $Al^{27}(p,\gamma)Si^{28}$

Aluminum resonances have been observed at the following energies using thin targets: 294.1±0.5, 325.6 ±0.4, 404.7±0.4, 438.5±0.5, and 504.0±0.6, and using a thick aluminum disk as a target a further weak resonance has been detected at 226.3±1.5 kev.

The half-widths of the resonances observed on a 1.0-kev thick target were only slightly greater than the thickness of the target. From thick target yield curves the width of the stronger 504.0- and 404.7-kev lines were both estimated to be 0.7±0.3 kev, while the weaker resonances all had a half-width of about or slightly less than 1 kev. Typical resonance curves are shown in Fig. 6.

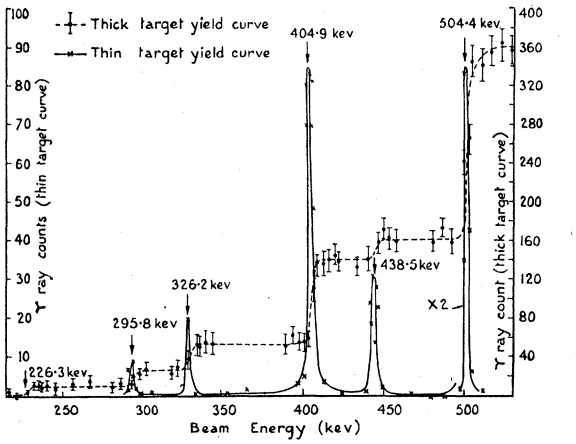

FIG. 6. Thick and thin target resonance curves for aluminum.

The aluminum resonances in this energy range have been investigated previously by Gentner[11] who reported a sharp resonance at 425 kev and by Tangen who reported resonances at 225±3, 295±3, 325±3, 404±3, 443±5, and 503±5 kev. The present results are in good agreement with those of Tangen. However, it is interesting to note that from Tangen's results the levels of Si^{28} can be arranged into a system of doublets of separation 34.0 kev, the spacing between the doublets being twice this. The errors associated with the fit are within the experimental errors quoted by Tangen. It was hoped that more precise determinations would demonstrate the regularity more convincingly, but the errors associated with the fitting of a regular doublet system to the present results are far greater than the experimental errors.

The only half-width measurement quoted previously is Tangen's value of less than 0.5 kev for the strong 404.7-kev line.

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Thanks are also due Dr. T. E. Allibone, F.R.S., for permission to publish this paper.

[10] Grotdal, Lönjö, Tangen, and Bergström, Phys. Rev. **77**, 296 (1950).

[11] W. Gentner, Z. Physik. **107**, 354 (1937).