tions of i since μ depends on the momentum of the particle. Equation (A11) may also be written as

$$\langle\omega_i^2\rangle_{Av}=\left(1+\frac{2\sigma_1^2R_i^{2\alpha}}{\rho^2}\right)\langle\eta^2\rangle_{Av}\approx\langle\eta^2\rangle_{Av}+2\sigma_1^2R_i^{2\alpha}, \quad (A12)$$

since according to Eqs. (A7) and (38), σ_1 and ρ are independent of i.

Summing over i we have for a group of particles all of which penetrate n plates,

$$\langle\omega^2\rangle_{Av}=\langle\eta^2\rangle_{Av}+2\sigma_1^2\sum_{i=2}^{n}(it)^{2\alpha}/(n-1). \quad (A13)$$

The limits on the summation arise from the fact that we computed values of η for $n\geqslant 2$.

Using 55 values of $\zeta_i-\zeta_i'$ we evaluated σ_1 as 0.4 degrees. Using Eq. (A13) the mass of the proton is

$$m_P=1730m_e. \quad (A14)$$

Since the calculated correction for the noise level scattering depends on σ_1^2, it is quite sensitive to the evaluation of σ_1. In this evaluation our method of measuring angles was not the same as that used in the mass measurements; therefore, the correction is rather uncertain. We have performed the calculation chiefly as an illustration of the methods used.

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Pseudoscalar Matrix Element in Beta-Decay

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The interpretation of the β-spectrum of RaE seems to need a mixture of pseudoscalar and tensor interactions. Estimates of the necessary G_P give $G_P\gg G_T$ to compensate for the small pseudoscalar matrix element. This matrix element is greatly increased if the nucleon is in a potential which strongly mixes free particle states of positive and negative energies even though the diagonal terms are not large. Such an interaction arises from pseudoscalar meson theory. The pseudoscalar matrix element is calculated for the RaE decay assuming that the nucleons interact through pseudoscalar coupled pseudoscalar mesons. Exchange transitions, in which two nucleons exchange the charge and spin given to the electron and neutrino, predominate. The RaE spectrum can be fitted with $G_P\sim -G_T$. Exchange terms alter other momentum type matrix elements appreciably. Such effects are unimportant for gradient coupled pseudoscalar mesons.

I. INTRODUCTION

IN the β-decay of heavy nuclei the perturbation of the electron wave function by the Coulomb field of the nucleus plays a dominant role in determining the spectrum shape. This accounts for the allowed shape associated with most first forbidden transitions. Deviations from the allowed shape are small[1] except for the "unique forbidden" spectra ($\Delta I=2$, yes). The β-spectrum of RaE, assuming it is simple, is very different from the allowed or "unique forbidden" spectra. For many years this was explained as a second forbidden transition[2] ($\Delta I=2$, no).

Recently Petschek and Marshak[3] have pointed out that the shell model predicts unambiguously that the parity of RaE is odd. The final even-even nucleus should have even parity and the RaE β-decay cannot be second forbidden (no). Since a spin change greater than two gives much too large an ft value they attempted to interpret it as a first forbidden transition with $\Delta I=2, 1, 0$. All linear combinations of the β-interactions (S, V, A, T, P) not excluded by the Fierz condition were investigated. They found that the observed spectrum can be understood only if the decay is a $0\rightarrow 0$ transition. It is then possible to cancel those parts of the spectrum which usually dominate and give the allowed shape. The remaining terms give agreement with the measured RaE spectrum. The necessary cancellation is accomplished with a combination of tensor and pseudoscalar interactions such that

$$(G_P/G_T)\left[\int\beta\gamma_5\bigg/\left(\int\beta\sigma\cdot r/R\right)^*\right]\sim-\tfrac{1}{3}. \quad (1)$$

R is the nuclear radius $(e^2/2mc^2)A^{\frac{1}{3}}$. G_P and G_T are the Fermi constants for pseudoscalar and tensor β-interactions.[4] The increase in lifetime which results from the cancellation of the usually dominant terms in first forbidden transitions is enough to explain the large $\log ft$ of RaE.[5]

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[1] All known first forbidden spectra with $\Delta I=1, 0$ have an allowed shape except RaE. This is to be expected only if there is no Fierz type interference between V and T, A and P, S and A. Only STP or VA are compatable with observed allowed and once forbidden spectra. See reference 6.

[2] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **85**, 308 (1952).

[3] R. E. Marshak and A. G. Petschek, Phys. Rev. **85**, 698 (1952).

[4] $\gamma_5=i\begin{pmatrix}0 & I\\ I & 0\end{pmatrix}$.

[5] E. J. Konopinski and L. M. Langer, *The Experimental Clarification of the Theory of Beta-Decay*, May, 1952 (to be published).

Ahrens, Feenberg, and Primakoff[6] have estimated the nuclear matrix element for the pseudoscalar interaction in $\Delta I=0$, (yes) transitions. In order to satisfy (1) they find

$$|G_P/G_T|\sim 1836A^{\frac{1}{3}}/Z=133. \qquad (2)$$

This large pseudoscalar coupling constant forced by the interpretation of the RaE spectrum is a peculiar feature. The β-decay of a free neutron is insensitive to the admixture of pseudoscalar interaction given in (2). If ψ_P and ψ_N satisfy Dirac equations for masses M_P and M_N,

$$\int d^4\chi(\psi_P{}^\dagger\gamma_5\psi_N)(\psi_e{}^\dagger\gamma_5\psi_\nu) = \frac{\hbar}{M_P+M_N}\int d^4\chi(\psi_P{}^\dagger\sigma_\mu\psi_N)\frac{\partial}{\partial\chi_\mu}(\psi_e{}^\dagger\gamma_5\psi_\nu). \qquad (3)$$

For N a neutron and P a proton and an electron-neutrino energy of 1.3 Mev, (3) gives 0.1 percent the matrix elements of S, A, V, or T.

The very attractive hypothesis of a universal Fermi interaction among nucleons, electrons, neutrinos, and μ mesons must be abandoned unless $G_P\sim G_T$ or smaller. For the decay $\mu\rightarrow e+\nu+\nu$ according to (3) the matrix element of γ_5 is comparable to that of the other four interactions. Although the large pseudoscalar term of (2) does not affect the free neutron decay, its inclusion in μ decay gives a lifetime 10^5 shorter than observed.[7]

The average potential in which a nucleus moves inside of a nucleus is small next to the nucleon mass. It does not follow that the nucleon can be treated as almost free. Matrix elements of the potential off the energy shell (in the plane wave representation) can be much larger than the diagonal terms which yield the average. This is the case, for example, if a significant part of the nuclear force arises from pseudoscalar mesons with pseudoscalar coupling. The matrix element of γ_5 is then much greater than the estimate which leads to (2) and the otherwise weak pseudoscalar β-decay is greatly magnified. Cooperative transitions in which two nucleons exchange charge and momentum, which are in turn transferred to the electron and neutrino, predominate.

II. THE PSEUDOSCALAR BETA-DECAY INTERACTION

The Hamiltonian for pseudoscalar coupled mesons and nucleons interacting with the electron-neutrino field is

$$H=\int d\mathbf{x}\sum_{\alpha=1}^{3}\Big\{(\pi_\alpha)^2+(\nabla\varphi_\alpha)^2+(\varphi_\alpha)^2+\psi^*(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta M)\psi \qquad (4a)$$

$$+g\varphi_\alpha(\psi^*\beta\gamma_5\tau_\alpha\psi) \qquad (4b)$$

$$+G_P\Phi\left(\psi^*\beta\gamma_5\left[\frac{\tau_1+i\tau_2}{2}\right]\psi\right), \qquad (4c)$$

where

$$\Phi=(\psi_e{}^*\beta\gamma_5\psi),\quad \hbar=c=\mu=1, \qquad (5)$$

and

$$[\pi(x),\varphi(x')]_-=-i\delta(x-x'),$$
$$[\psi_i{}^*(x),\psi_j(x')]_+=\delta(x-x')\delta_{ij}.$$

Charge symmetric coupling is taken for the meson-nucleon interaction. Only pseudoscalar β-decay is considered here. To extract the major part of the β-decay matrix element is is useful to invoke the canonical transformation[8–10]

$$U=\exp(iS),\quad S=i\int(\psi^*\gamma_5\tau_\alpha\psi g\varphi_\alpha/2M)d\mathbf{x}, \qquad (6)$$

so that

$$\bar{H}=U^{-1}HU$$

explicitly exhibits the nonrelativistic features of (4a) and (4b).

$$\bar{H}_{a+b}=\int d\mathbf{x}\sum_{\alpha=1}^{3}\Big\{(\pi_\alpha)^2+(\nabla\varphi_\alpha)^2+(\varphi_\alpha)^2 +\psi^*(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta M)\psi \qquad (7a)$$

$$+\frac{g}{2M}\nabla\varphi_\alpha\cdot(\psi^*\tau_\alpha\boldsymbol{\sigma}\psi)+\frac{g^2}{2M}(\varphi_\alpha)^2(\psi^*\psi)\Big\}. \qquad (7b)$$

Terms in higher powers of g have been omitted. The effect of all the omitted parts of (7) seems to be renormalization[11,12] of the pair coupling term of (7b),

$$g^2\rightarrow\lambda g^2, \qquad (8)$$

as long as only low energy meson-nucleon interactions are considered. The β coupling (4c) is also altered by (6).

$$\bar{H}_c=G_P\int d\mathbf{x}\Phi\left\{\left(\psi^*\beta\gamma_5\left[\frac{\tau_1+i\tau_2}{2}\right]\psi\right)\left(1-\frac{\lambda g^2\varphi^2}{2M^2}\right)\right. \qquad (9a)$$

$$+\frac{\lambda gG_P}{2M}(\psi^*\beta\psi)(\varphi_1+i\varphi_2). \qquad (9b)$$

(9b) does not contain the nuclear matrix element of γ_5 and can be much larger than (9a). The nucleon which is coupled to the electron-neutrino pair through (9b) does not change its charge. Rather the charge is taken from the meson field. The sources of this field, the neighboring nucleons, recoil to supply the spin, charge, and momentum carried off by the electron and neutrino.

In the interaction representation the lowest order (in λg^2) contribution to the β-decay with no real mesons

[6] Ahrens, Feenberg, and Primakoff, Phys. Rev. **87**, 663 (1952).

[7] The presence of pseudoscalar beta-interaction causes difficulty in understanding the absence of $\pi\rightarrow e+\nu$ [M. Ruderman and R. Finkelstein, Phys. Rev. **76**, 1458 (1949); M. Ruderman, Phys. Rev. **85**, 157 (1952)]. See, however, R. J. Finkelstein, Phys. Rev. **88**, 555 (1952).

[8] J. V. Lepore, Phys. Rev. **88**, 750 (1952).

[9] F. J. Dyson, Phys. Rev. **73**, 929 (1948).

[10] L. I. Foldy, Phys. Rev. **84**, 168 (1951).

[11] S. Drell and E. Henley, Phys. Rev. **88**, 1053 (1952).

[12] M. Ruderman, University of California Radiation Laboratory report 1876 July 1, 1952.

absorbed or emitted is the irreducible interaction.

$$H_\beta \equiv \frac{i}{2}\int dt'\epsilon(t,t')[\bar{H}_c(t), \bar{H}_b(t')]. \quad (10)$$

The transition amplitude for β-decay is then the matrix element of

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} dt H_\beta.$$

The nonrelativistic limit of (10) is

$$\lambda G_P(g^2/4\pi)(\mu/2M)^2[\Phi(\mathbf{r}_2)(\tau_1+i\tau_2)_1(\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})_1 \times Y(r_{12})(1)_2+\Phi(\mathbf{r}_1)(\tau_1+i\tau_2)_2(\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})_2 Y(r_{12})(1)_1], \quad (11)$$

where

$$Y(r_{12})=\exp(-\mu r_{12})/\mu^2 r_{12}. \quad (12)$$

The infinite correction to the matrix element for the decay of a single free nucleon, which is contained in (10), is included by renormalizing G_P in (9a) and will be ignored.[13]

III. ESTIMATE OF THE MATRIX ELEMENT FOR RaE

According to the shell model RaE has a neutron and a proton outside of a closed shell of 82 protons and 126 neutrons. Any changes in the structure of the core during the β-decay are neglected. The two nucleons outside of the core are assumed to be too far apart (about $A^{\frac{1}{3}}\hbar/\mu c$) to interact strongly with each other. It is in the spirit of the shell model to describe RaE by a Hartree wave function. The proton and neutron states of the core are the orthonormal sets

$$\psi_m{}^P \quad m=1,2,\cdots,82; \quad \psi_n{}^N \quad n=1,2,\cdots,126.$$

The beta-transition is the transfer of the extra core neutron from the state $\psi_I{}^N$ to the extra core proton state $\psi_F{}^P$. The matrix element of (11) for this transition is

$$2\lambda G_P(g^2/4\pi)(\mu/2M)^2\int\int d\mathbf{r}_1 d\mathbf{r}_2[\Phi(\mathbf{r}_2)\rho(\mathbf{r}_2) \times\langle\psi_F{}^{P*}(\mathbf{r}_1)\boldsymbol{\sigma}\psi_I{}^N(\mathbf{r}_1)\rangle\cdot\boldsymbol{\nabla}_1 Y(r_{12}) \quad (13a)$$

$$-\sum_{n=1}^{126}\Phi(\mathbf{r}_1)\langle\psi_n{}^{N*}(\mathbf{r}_1)\psi_I{}^N(\mathbf{r}_1)\rangle \times\langle\psi_F{}^{P*}(\mathbf{r}_2)\boldsymbol{\sigma}\psi_n{}^N(\mathbf{r}_2)\rangle\cdot\boldsymbol{\nabla}_2 Y(r_{12}) \quad (13b)$$

$$-\sum_{m=1}^{82}\Phi(\mathbf{r}_1)\langle\psi_F{}^{P*}(\mathbf{r}_1)\psi_m{}^P(\mathbf{r}_1)\rangle \times\langle\psi_m{}^{P*}(\mathbf{r}_2)\boldsymbol{\sigma}\psi_I{}^N(\mathbf{r}_2)\rangle\cdot\boldsymbol{\nabla}_2 Y(r_{12}), \quad (13c)$$

where

$$\rho(r)=\rho_P(r)+\rho_N(r),$$

$$\rho_P(r)=\sum_{m=1}^{82}\psi_m{}^{P*}(r)\beta\psi_m{}^P(r)=(Z/A)\rho(r), \quad (14)$$

$$\rho_N(r)=\sum_{n=1}^{126}\psi_n{}^{N*}(r)\beta\psi_n{}^N(r)=(A-Z/A)\rho(r).$$

Equation (14) implies a uniform proton and neutron density throughout the core. Since $R\sim A^{\frac{1}{3}}\hbar/\mu c$,

$$\rho(r)=3\mu^3/4\pi, \quad r\leq R; \qquad =0, \quad r>R. \quad (15)$$

To evaluate (13b, c) it is necessary to know ψ^P and ψ^N but an estimate, independent of these functions, is obtained by the approximation

$$Y(r)\sim(4\pi/\mu)\delta(\mu r). \quad (16)$$

Equation (16) is certainly valid as long as the momentum transfers between nucleons 1 and 2 in (13b, c) are small next to μc. Substituting (16) in (13) and performing a partial integration, (13b) and (13c) become

$$-8\pi\lambda G_P(g^2/4\pi)(\mu/2M)^2\Phi(R) \times\int d\mathbf{r}[\tfrac{1}{2}\langle\psi_F{}^{P*}(\mathbf{r})\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\psi_I{}^N(\mathbf{r})\rangle\rho_N(r) \quad (17a)$$

$$-\tfrac{1}{2}\langle\psi_F{}^{P*}(\mathbf{r})\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\psi_I{}^N(\mathbf{r})\rangle\rho_P(r) \quad (17b)$$

$$-\langle\psi_F{}^{P*}(\mathbf{r})\boldsymbol{\sigma}\cdot\sum_{n=1}^{126}\boldsymbol{\nabla}\psi_n{}^N(\mathbf{r})\rangle\langle\psi_n{}^{N*}(\mathbf{r})\psi_I{}^N(\mathbf{r})\rangle \quad (17c)$$

$$\times\langle\psi_F{}^{P*}(\mathbf{r})\sum_{m=1}^{82}\boldsymbol{\nabla}\psi_m{}^P(r)\rangle\langle\psi_m{}^{P*}(\mathbf{r})\cdot\boldsymbol{\sigma}\psi_I{}^N(\mathbf{r})\rangle]\mu^{-4}. \quad (17d)$$

The variation of $\Phi(r)$ over the nucleus gives a much smaller contribution than (17) and has been neglected. (17c) and (17d) vanish if the core is everywhere isotropic. When (14) and (15) are used, (17a) and (17b) give

$$-3\lambda G_P(g^2/4\pi)(\mu/2M)^2 \times\Phi(R)\left(\frac{A-2Z}{A}\right)\int d\mathbf{r}\langle\psi_F{}^{P*}\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\psi_I{}^N\rangle\mu^{-1}. \quad (18)$$

The matrix element in (18) has been investigated by Ahrens and Feenberg.[14] They find

$$\int\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}=\frac{Me^2Z\Lambda}{2\hbar}\int\frac{\boldsymbol{\sigma}\cdot\mathbf{r}}{R}. \quad (19)$$

Λ is a nuclear structure factor, estimated to be 1 for a heavy nucleus. Pursey,[15] using a specific model, deduces

[13] The mesic corrections to free nucleon beta-decay have been investigated by Kotani, Machida, Nakamura, Takebe, Umezawa, and Yoshimura. Prog. Theor. Phys. **6**, 1007 (1951). The correction to the allowed shape is of order $(\Delta p/M)^2(g^2/4\pi)(\mu/2M)$, which is quite negligible. The change in magnitude of the Fermi constant cannot be calculated. One could, in this way, get a large G_P for nucleons without spoiling the possibility of a universal Fermi interaction, but the mesic radiative corrections to the other nuclear beta-interactions could not change their order of magnitude.

[14] T. Ahrens and E. Feenberg, Phys. Rev. **86**, 64 (1952).

[15] D. L. Pursey, Phil. Mag. **42**, 1193 (1951).

(19) with Λ equal to 2. The estimate (19) makes it possible to compare (13b) and (13c) with (1) without knowledge of the initial and final nucleon states. The evaluation of (13a), however, needs the amplitudes of the extra core states near the boundary of the core. (13a) has been integrated for $\psi_F{}^P$ and $\psi_I{}^N$ solutions of the wave equation in a spherical well of radius R. For a $0\to 0$ transition for the RaE decay the shell model predicts for $\psi_F{}^P$ an $h_{9/2}$ state and for $\psi_I{}^N$ a first excited (one node) $g_{9/2}$ state. With these wave functions and the definition (12), (13a) gives

$$-(1.7)\lambda G_P(g^2/4\pi)(\mu/2M)^2\Phi(R)\int \boldsymbol{\sigma}\cdot\mathbf{r}/R. \quad (20)$$

IV. FERMI CONSTANT FOR PSEUDOSCALAR BETA-DECAY

From (1), (18), (19), and (20)

$$G_P\sim -G_T\left(\frac{4\pi}{\lambda g^2}\right)(0.02\Lambda+0.04)^{-1}. \quad (21)$$

To fit the low energy properties of the two-nucleon system Lévy[16] uses a value of 12 for $(\lambda g^2/4)$. This results in $G_P\sim -1.4G_T$, a decrease of 10^2 from (2). Although the numerical results cannot be taken very seriously, it does appear possible to increase the pseudoscalar matrix element by several orders of magnitude over the results obtained from (3). If the nuclear force is due to gradient coupled pseudosclar mesons it is again necessary for $G_P\gg G_T$.

It is apparent from (18) that the ratio of pseudoscalar to $\int\boldsymbol{\sigma}\cdot\mathbf{r}$ will decrease for lighter nuclei. (18) vanishes for a symmetric nucleus. (13a) and (20) are significant only when ψ_F and ψ_I are states of high orbital momenta.

V. EXCHANGE MATRIX ELEMENTS FOR OTHER BETA-INTERACTIONS

Other beta-interactions are affected by the transformation (6), although none as much as the pseudoscalar. The first forbidden tensor which also gives $0\to 0$ (yes) transitions becomes

$$\tfrac{1}{2}G_T\{\Phi(1)(\beta\boldsymbol{\sigma}\cdot\mathbf{r})_1(\tau_1+i\tau_2)_1 +\Phi(2)(\beta\boldsymbol{\sigma}\cdot\mathbf{r})_2(\tau_1+i\tau_2)_2\} \quad (22a)$$

$$-iG_T(g^2/4\pi)(\mu/2M)^2\{\Phi(1)(\tau_1+i\tau_2)_1(\boldsymbol{\sigma})_1 \cdot\boldsymbol{\nabla}_1Y(1,2)(\beta\boldsymbol{\alpha}\cdot\mathbf{r})_2+\Phi(2)(\tau_1+i\tau_2)_2(\boldsymbol{\sigma})_2 \cdot\boldsymbol{\nabla}_2Y(2,1)(\beta\boldsymbol{\alpha}\cdot\mathbf{r})_1\}. \quad (22b)$$

[16] Maurice M. Lévy, Phys. Rev. 88, 725 (1952).

Unlike (9) the exchange term does not dominate. In the representation (6) nucleons 1 and 2 are moving in weak potentials so that

$$2M(\beta\boldsymbol{\alpha}\cdot\mathbf{r})_{1,2}\sim -i(3+\boldsymbol{\sigma}\cdot\mathbf{L})_{1,2}. \quad (23)$$

With the approximations which lead to (18), (22b) gives

$$G_T(g^2/4\pi)(\mu/2M)^2A^{-\frac{1}{3}} \times\int\boldsymbol{\sigma}\cdot\mathbf{r}\left[\Lambda\alpha Z\left(\frac{21}{4}-\frac{3Z}{A}\right)+5.1\left(\frac{\mu}{2M}\right)\right] \sim 0.04G_T\int\boldsymbol{\sigma}\cdot\mathbf{r}. \quad (24)$$

$\int\beta\boldsymbol{\sigma}\cdot\mathbf{r}$ is only slighly altered by the inclusion of (22b) and the estimate (21) is virtually unchanged. The exchange terms associated with momentum type matrix elements are more important. Corresponding to (21a) and (21b) these interactions become

$$\tfrac{1}{2}iG_T\{\Phi(1)(\tau_1+i\tau_2)_1(\beta\boldsymbol{\alpha})_1+\Phi(2)(\tau_1+i\tau_2)_2(\beta\boldsymbol{\alpha})_2\} \quad (25a)$$

$$+G_T(g^2/4\pi)(\mu/2M)^2\{\Phi(1)(\tau_1+i\tau_2)_1(\boldsymbol{\sigma})_1\cdot\boldsymbol{\nabla}_1Y(1,2)(\boldsymbol{\sigma})_2 +\Phi(2)(\tau_1+i\tau_2)_2(\boldsymbol{\sigma})_2\cdot\boldsymbol{\nabla}_2Y(1,2)(\boldsymbol{\sigma})_1\}, \quad (25b)$$

$$\tfrac{1}{2}iG_P\{\Phi(1)(\tau_1+i\tau_2)_1(\gamma_5)_1+\Phi(2)(\tau_1+i\tau_2)_2(\gamma_5)_2\} \quad (26a)$$

$$+iG_P(g^2/4\pi)(\mu/2M)^2\{\Phi(1)(\tau_1+i\tau_2)_1(\boldsymbol{\sigma})_1 \cdot\boldsymbol{\nabla}_1Y(1,2)(\tau_3)_2+\Phi(2)(\tau_1+i\tau_2)_2(\boldsymbol{\sigma})_2\cdot\boldsymbol{\nabla}_2Y(1,2)(\tau_3)_1 -\Phi(1)(\boldsymbol{\sigma}\tau_3)_1\cdot\boldsymbol{\nabla}_1Y(1,2)(\tau_1+i\tau_2)_2 -\Phi(2)(\boldsymbol{\sigma}\tau_3)_2\cdot\boldsymbol{\nabla}_2Y(1,2)(\tau_1+i\tau_2)_1\}, \quad (26b)$$

$$\tfrac{1}{2}G_V\{\Phi(1)(\tau_1+i\tau_2)_1(\boldsymbol{\alpha})_1+\Phi(2)(\tau_1+i\tau_2)(\boldsymbol{\alpha})_2\} \quad (27a)$$

$$-iG_V(g^2/4\pi)(\mu/2M)^2\{\Phi(1)(\tau_1+i\tau_2)_1(\boldsymbol{\sigma})_1 \cdot\boldsymbol{\nabla}_1Y(1,2)(\boldsymbol{\sigma}\tau_3)_2+\Phi(2)(\tau_1+i\tau_2)_2(\boldsymbol{\sigma})_2 \cdot\boldsymbol{\nabla}_2Y(1,2)(\boldsymbol{\sigma}\tau_3)_1-\Phi(1)(\tau_3\boldsymbol{\sigma})_1\cdot\boldsymbol{\nabla}_1Y(1,2) \times(\tau_1+i\tau_2)_2(\boldsymbol{\sigma})_2-\Phi(2)(\tau_3\boldsymbol{\sigma})_2 \cdot\boldsymbol{\nabla}_2Y(1,2)(\tau_1+i\tau_2)_1(\boldsymbol{\sigma})_1\}. \quad (27b)$$

For $(g^2/4\pi)(\mu/2M)^2\sim 1/12$, (a) and (b) can be comparable.

This investigation originated from a remark of Dr. S. Moszkowski on the possible importance of velocity ($\boldsymbol{\alpha}$) dependent potentials on the pseudoscalar matrix element. It is a pleasure to thank him and Professor Robert Serber for helpful discussion.