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Study of Elastic Pion-Hydrogen Scattering in Photographic Emulsions*†

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Pion-hydrogen elastic scattering events were observed in photographic emulsions exposed to the 75±5 Mev negative and positive pion beams of the Nevis synchrocyclotron. The methods of identification, based on the energy momentum conservation in the interaction, are described. The cross section measured for the elastic scattering of negative and positive pions from hydrogen at 75±5 Mev are ~3 mb and 41±15 mb, respectively.

THE elastic scattering of π-mesons from hydrogen has been studied in photographic emulsions. The events can be identified because of energy-momentum conservation in the interaction.

The elastic $\pi+H$ scattering events (Fig. 1) are characterized by the following four criteria:

1. Coplanarity.
2. Angular correlation between scattered meson and recoil proton.
3. Range-angle correlation for the recoil proton (Fig. 3).
4. Identification of the grey prong (Fig. 1) as a meson by grain density measurements.

Ilford G5 emulsions were exposed to the 75±5 Mev negative and positive pion beams of the Nevis synchrocyclotron.

The emulsions were scanned "by area" (10×45 oil objective). The total path length in hydrogen was calculated from the total area scanned, the hydrogen concentration and a sampling of the meson track density (correcting for beam contamination).[1–3] To insure a high scanning efficiency for the one-prong events, each proton beginning in the emulsion was examined closely to determine whether a meson was associated with it. There is reasonable agreement between the total number of meson interactions found and the number calculated from the total pion path length and the known cross sections[2–4] when corrections are made for scanning efficiency ($e=0.85$) and the types of interaction missed by area scanning.[5]

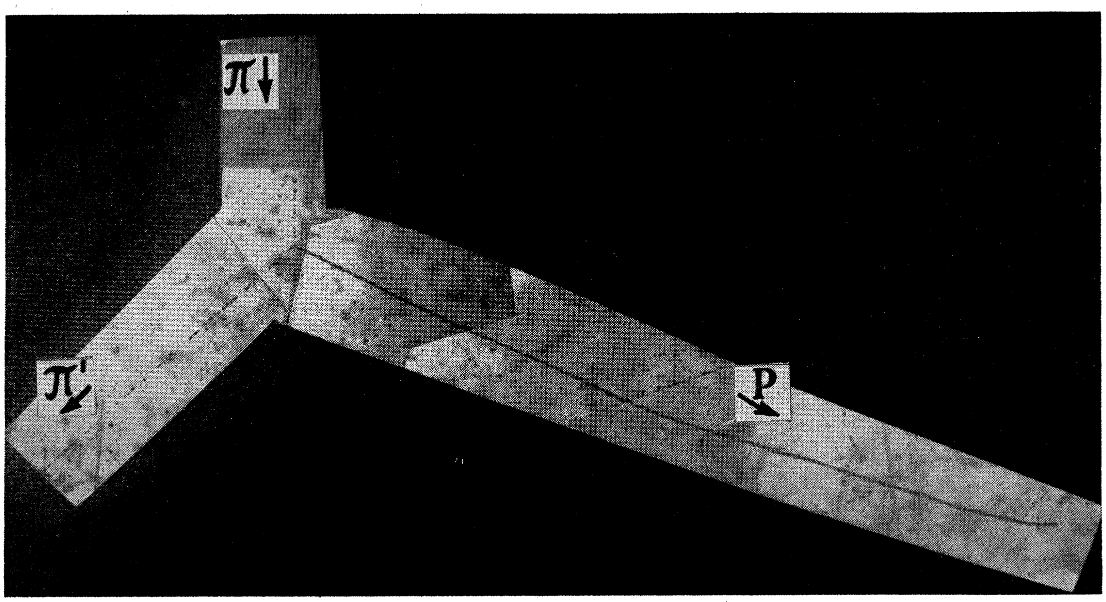

FIG. 1. Microphotograph of a $\pi^{+}+H$ scattering event. The recoil proton range is 370μ (observer, Mrs. E. Arase).

* A preliminary report of this work was presented at the 1952 Washington meeting of the American Physical Society [Phys. Rev. 87, 222 (1952)].
† This research was supported by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commision.
1 Bernardini, Booth, Lederman, and Tinlot, Phys. Rev. 82, 105 (1951).
2 Bernarding, Booth, and Lederman, Phys. Rev. 83, 1075 (1951).
3 G. Bernardini and F. Levy, Phys. Rev. 84, 610 (1951).
4 Chedester, Isaacs, Sachs, and Steinberger, Phys. Rev. 82, 958 (1951).
5 Stops, elastic scatterings, inelastic scatterings without heavy prongs, stars with very short prongs (<20μ), stars with one single grey prong.

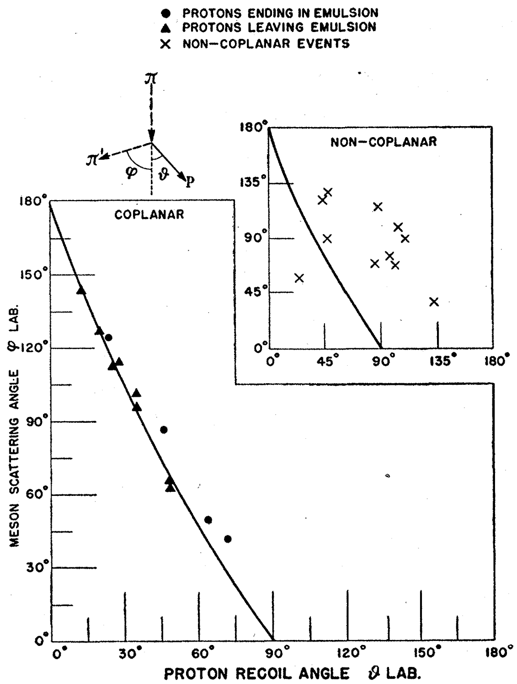

FIG. 2. The correlation between meson scattering angle $\phi_{\rm lab}$ and recoil proton angle $\theta_{\rm lab}$ evaluated for $\pi+H$ scattering at 75 Mev. The coplanar $\pi^+ + H$ events are given on the main plot. The noncoplanar events on the inset.

For negative pions, ordinary and water-loaded[6] emulsions were exposed and scanned. In scanning an area of emulsion corresponding to a traversal of π^- mesons through 630 ± 100 g/cm^2 of hydrogen, only one $\pi^- + H$ scattering event was found. This corresponds to a cross section of ~ 3 mb. Such a low cross section for the elastic scattering is consistent with the measurements by Shutt and co-workers.[7]

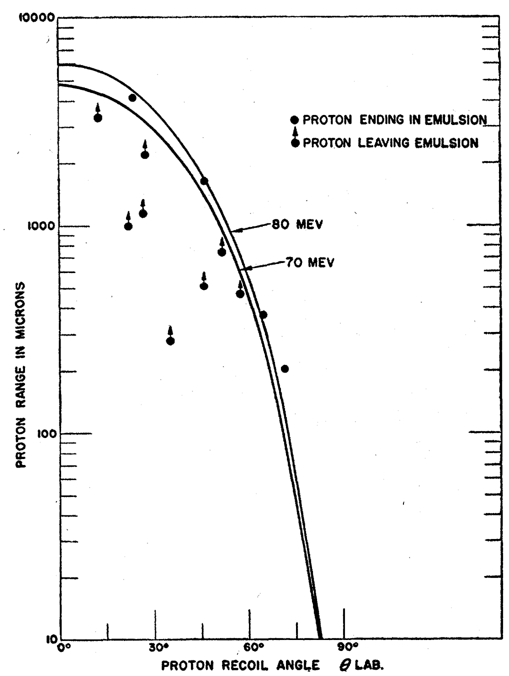

FIG. 3. The correlation between recoil proton range and angle ($\theta_{\rm lab}$) evaluated for $\pi+H$ scattering in Ilford emulsions at 70 and 80 Mev.

[6] S. Goldhaber and G. Goldhaber, Phys. Rev. 87, 185 (1952).

For positive pions, ordinary emulsions were used, and an area of emulsion corresponding to a traversal of π^+ mesons through 580 ± 100 g/cm^2 of hydrogen was scanned. In this area 234 meson interactions were found, of which 23 were one-prong scattering events. Of these, 12 were identified as $\pi^+ + H$ scatterings. The other 11 one-prong scattering events were inelastic scatterings from other nuclei in the emulsion.

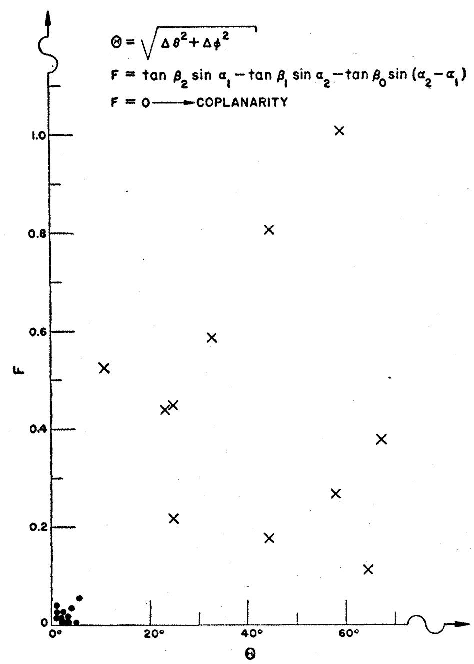

FIG. 4. The deviation from coplanarity $|F|$ plotted *versus* $|\Theta|$, the deviation from the angular correlation for $\pi^+ + H$ scattering. The black dots represent the identified elastic scattering events; the crosses represent the inelastic scatterings.

When $\phi_{\rm lab}$ the meson scattering angle is plotted *versus* $\theta_{\rm lab}$ the proton recoil angle, all the coplanar scattering events lie close to the calculated angular correlation curve for $\pi+H$ scattering (Fig. 2). The noncoplanar scattering events show no such clustering around the calculated curve (inset Fig. 2). Figure 3 gives calculated correlation curves between recoil proton range and angle, as well as the points corresponding to the coplanar π^+ scattering events. Within the experimental limits the points agree with the calculated range for events with recoil protons ending in the emulsion, or lie below the calculated range for protons leaving the emulsion. The condition for coplanarity of three prongs (incoming meson, scattered meson, and recoil proton)

[7] Shutt, Fowler, Miller, Thorndike, and Fowler, Phys. Rev. 84, 1247 (1951).

is given by $F=0$, where $F=\tan\beta_2 \sin\alpha_1-\tan\beta_1 \sin\alpha_2 -\tan\beta_0 \sin(\alpha_2-\alpha_1)$. Here α_1, α_2 are the projected angles of the two outgoing prongs in the plane of the emulsion, β_1, β_2, are the corresponding dip angles, and β_0 is the dip angle of the incoming meson.[8] The deviation from coplanarity $|F|$ is plotted (Fig. 4) *versus* $|\Theta|$, the deviation from the angular correlation curve for $\pi+$H scattering. $\Theta=(\Delta\theta^2+\Delta\phi^2)^{\frac{1}{2}}$, where $\Delta\theta$ and $\Delta\phi$ represent the respective angular deviations of each event from the calculated $\pi+$H scattering curve as obtained from Fig. 2.

The distribution of the $\pi^+ +$H scattering between the forward and backward direction in the center-of-mass system is

$$d\sigma_{\text{c.m.}}=2.7\pm1.3 \text{ mb/steradian for } \phi_{\text{c.m.}}=35-90°,$$

and

$$d\sigma_{\text{c.m.}}=4.4\pm1.6 \text{ mb/steradian for } \phi_{\text{c.m.}}=90-180°.$$

As the scanning efficiency drops off for recoil prongs shorter than $\sim 50\mu$, a cutoff was taken at a recoil proton length of 100μ, which corresponds to a meson scattering angle of $\phi_{\text{c.m.}}=35°$. The asymmetry of the differential cross section with a peak in the backward direction found by Anderson and co-workers[9] at π^+ energies of 135 Mev and 110 Mev appears to be still evident in the present work at 75 Mev. From the work of Shutt and co-workers,[10] there is very little asymmetry at 53 Mev. The total cross section for $\pi^+ +$H scattering obtained is 41 ± 15 mb for $\phi_{\text{c.m.}}>35°$. This is in agreement with other measurements.[11]

The author would like to express his appreciation to Professor G. Bernardini and Dr. D. Bodansky, who participated in the initial stages of this work, to Mrs. E. Arase and Mrs. C. Major for their efforts in scanning the plates, and to Dr. S. Goldhaber and Dr. T. A. Green for discussions in connection with this work. Thanks are also due Mrs. J. Bielk for help in the development of the emulsions, and Mr. J. Spiro and the Nevis Cyclotron staff for their invaluable assistance.

[8] This relation holds for either the original dip angles or the final dip angles in the shrunk emulsion. In Fig. 4, $|F|$ is plotted in the form using the final dip angles.

[9] Anderson, Fermi, Nagle, and Yodh, Phys. Rev. **86**, 793 (1952).

[10] Fowler, Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **86**, 1053 (1952).

[11] Anderson, Fermi, Long, and Nagle, Phys. Rev. **85**, 936 (1952).

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The Occurrence of Singularities in the Elastic Frequency Distribution of a Crystal

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It is shown that for a crystal, under the assumption of harmonicity for the interatomic forces and as a consequence of the periodic structure, the frequency distribution function of elastic vibrations has analytic singularities. In the general case, the nature of the singularities depends only on the number of dimensions of the crystal. For a two-dimensional crystal, the distribution function has logarithmically infinite peaks. In the three-dimensional case, the distribution function itself is continuous whereas its first derivative exhibits infinite discontinuities. These results are elementary consequences of a theorem of Morse on the existence of saddle points for functions defined on a torus.

I. INTRODUCTION

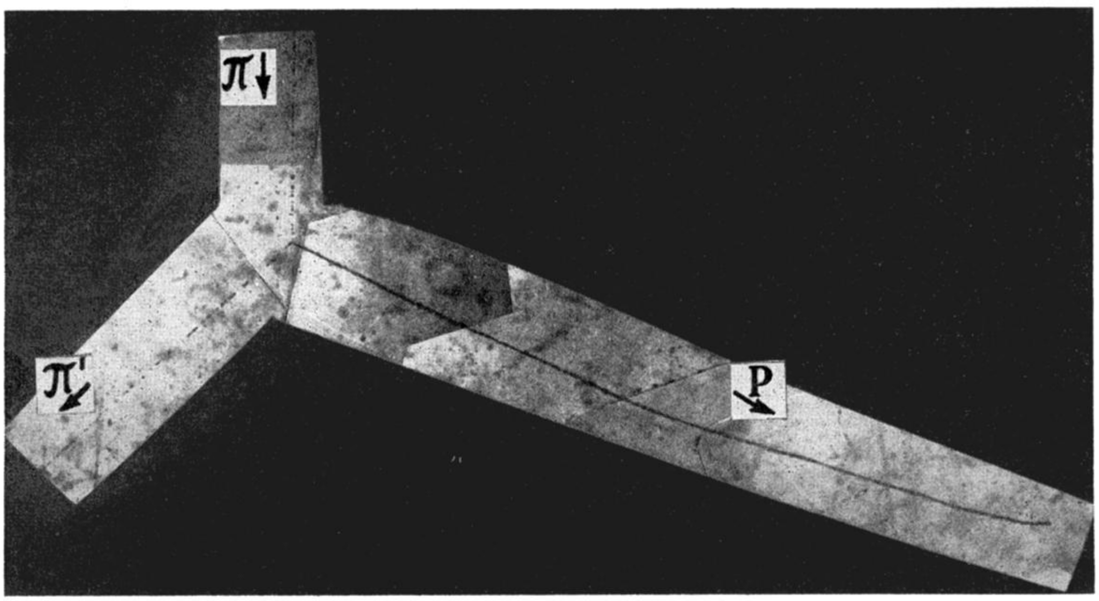
It is well known that the shape of the frequency distribution function $g(\nu)$ of a crystal (defined as the number of elastic frequencies per unit frequency interval, divided by the total number of frequencies) determines an important part of the thermodynamical properties of the crystal. The frequency distribution of two- and three-dimensional crystals, as predicted by the Born-von Karman theory of crystal dynamics, has been extensively studied by approximate methods based on the use of a finite sample of elastic vibrations.[1] A smooth distribution function was obtained in all cases, showing for small frequencies a behavior in full agreement with the Debye continuum theory, tending continuously to zero at the maximum frequency and displaying for intermediate frequencies a few finite maxima.[2]

The analytical nature of the frequency distribution function, or rather of its asymptotic form for a crystal of infinite extension, has recently attracted attention as a consequence of an exact calculation by Montroll for a two-dimensional square lattice.[3] As found by

[1] M. Blackman, Repts. Prog. Phys. **8**, 11 (1941) and references there quoted; E. Montroll, J. Chem. Phys. **10**, 219 (1942); E. Montroll and D. Peaslee, J. Chem. Phys. **12**, 98 (1944); H. M. J. Smith, Trans. Phil. Soc. **A241**, 105 (1948); W. V. Houston, Revs. Modern Phys. **20**, 161 (1948); R. B. Leighton, Revs. Modern Phys. **20**, 165 (1948).

[2] The frequency distribution obtained by Leighton (reference **1**), has at the maximum frequency of each branch a singularity very similar to some of the singularities we shall show to exist in three dimensions. The singularities obtained by Houston (reference 1) are entirely spurious.

[3] E. Montroll, J. Chem. Phys. **15**, 575 (1947).



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