An Experimental Investigation of the Nucleon Cascade in Water

C. B. A. McCusker, H. Messel,* D. D. Millar† and N. A. Porter
Dublin Institute for Advanced Studies, Dublin, Ireland
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The nucleon cascade in water produced by primaries of energy greater than 20×10^9 ev has been examined using an array of Geiger-Müller counters and a 31-channel hodoscope. Over 15 000 showers have been studied. Transition curves for various primary energies, barometric coefficients for various multiplicities and at different depths, root mean square lateral spreads, and intensities at different depths are given. The experimental results are compared with the theory of the nucleon cascade developed by Messel and his co-workers. In particular, the variation of lateral spread with depth substantiates their conclusion that the differential cross section for nucleon-nucleon collision at high energies has a very peaked form in the forward direction.

INTRODUCTION

The apparatus described in this paper was originally set up to investigate the production of mesons in high energy nuclear encounters. The method used was to study local penetrating showers from water, paraffin, carbon, and lead using an array of Geiger-Müller counters operating a 31-channel hodoscope. While the experiment was in progress the theory of the nuclear cascade was considerably developed by Messel, Green, and their co-workers.[1,2] In this paper the results obtained from an examination of 15 000 penetrating showers from water are compared with this theory.

I. THE EXPERIMENTAL ARRANGEMENT

A sketch of the apparatus is given in Fig. 1. Trays A, B, C, D, and E each consisted of 12 Geiger-Müller counters 50 cm long by 3.8 cm wide. The axes of counters in tray A were at 90° to those of tray B. To provide a master pulse, at least 2 counters in each of trays B, C, and D had to be discharged, i.e., the master pulse was at least a sixfold coincidence. Each counter in trays A and B operated its own neon lamp. For the purpose of the hodoscope, the counters in tray D were connected in adjacent pairs. Each pair then operated a neon lamp. Finally the extended tray E, working with all its counters in parallel, operated a single neon lamp.

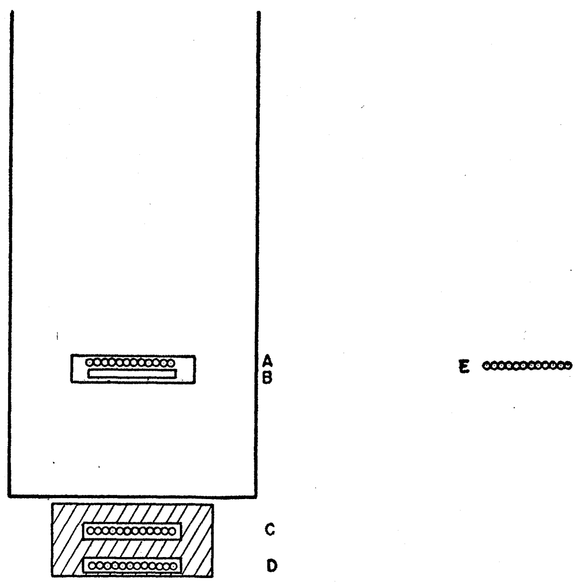

FIG. 1. The experimental arrangement.

Trays A and B were contained in a water tight sheet steel box. This box was itself inside an iron tank of height 8 ft and base 4 ft×4 ft. At the start of the experiment the water in the tank was just level with the top of the box (zero position). The water level was then raised in steps until a maximum depth of 180 cm over the box was reached, then reduced in similar steps and finally a second reading at the zero level was taken. This took 6 months.

Trays C and D were contained in a lead pile beneath the tank. Each counter in these trays was wrapped in lead sheet $\frac{1}{10}$ in. thick to reduce the effect of μ meson knock-ons. Above C the lead was 7.5 cm thick, between C and D 5 cm thick, and between D and the concrete floor 2.5 cm thick. On all sides there were 15 cm of lead. The extensive tray E was 2 m from the center line of the tank. The whole apparatus was housed in a wooden hut with a light roof.

II. ANALYSIS OF THE DATA

(1) Method

Complete details of each individual shower were punched on Hollerith cards. This was essential since it would be quite impossible to sort by hand the 15 000 showers already obtained in the various ways necessary to get all the information possible from the crude data. There are, of course, various ways of specifying the showers. One particularly useful and simple description is that of the "multiplicity" in terms of counters discharged in trays A and B. If, in these two trays, there are m and n counters discharged, $m>n$, the shower is called an m fold event; m is obviously the minimum number of particles necessary to produce this event.

* Now at the University of Sydney, Sydney, Australia.
† Now at the University of Manchester, Manchester, England.
[1] H. Messel, Commun. of the Dublin Institute for Advanced Studies (1951), Series A, No. 7.
[2] H. Messel, *Progress in Cosmic Ray Physics*, Vol. II (1953) (to be published).

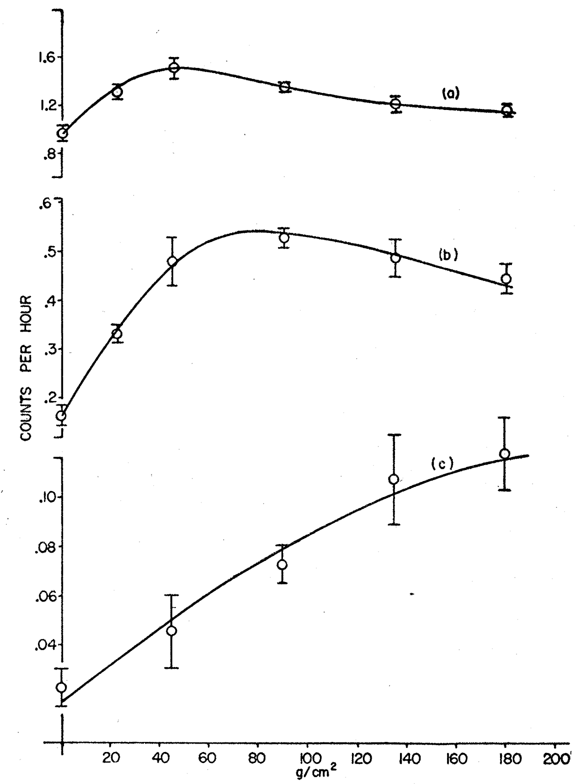

FIG. 2. The transition curves for (a) 2-fold, (b) 4-fold, and (c) 12-fold local showers.

Generally, the actual number of particles in the shower will be greater.

(2) The Extensive Showers

Of all the showers recorded, a fraction between $\frac{1}{3}$ and $\frac{1}{5}$, depending on the depth, discharged the extensive tray. The multiplicity distribution of these showers for the zero position is given in Table I. Multiplicity distributions for other thicknesses were not much different. In order to see if these showers were of the normal type (with $\gamma=1.5$, $k=1/40$) or not, the expected multiplicity distribution was calculated for such showers falling on an array consisting of two unshielded trays, one of 12 separate counters of which at least two must be discharged, the other of 12 counters in parallel, and a shielded tray of 12 counters of which at least two must be discharged. This array is an approximation to the real arrangement. Probably the greatest error is introduced by specifying that two penetrating particles are necessary. In extensive showers about $\frac{1}{3}$ of the

TABLE I. Multiplicity distribution for extensive showers.

Multiplicity		2	3	4	5	6	7	8	9	10	11	12
No. of showers	Expt.	130	109	116	86	84	66	81	76	70	98	342
	Calc.	72	70	68	68	68	71	75	84	101	141	442

penetrating particles are nucleons, and one nucleon if it interacts in the lead is sufficient to discharge the real shielded arrangement.

The calculated distribution is given in Table I. The agreement between theoretical and calculated distributions is sufficiently good to make it likely that all the extensive showers recorded were of the normal type—the disagreement being due to the approximations in the calculation. It is, at first sight, surprising to find that it is possible to select showers of quite normal type which are represented on the apparatus by only two π-mesons or nucleons.[3]

(3) Transition Curves

For the local showers, transition curves for all multiplicities, 2-fold to 12-fold, were plotted (in this as in all other cases, the rates were corrected to a barometric pressure of 1000 millibars). Curves for 2 folds, 4 folds, and 12 folds are given in Fig. 2. Figure 3 shows the variation of the position of the cascade maximum with the number of particles at the maximum. This number was determined by the procedure given in the next section.

(4) Intensities of Charged Particles

It is possible to estimate the intensity of charged particles at a given depth caused by the single nucleons incident on the top of the water. To do this we must determine the number of charged particles which, on the average, produces the discharge of m and n counters on trays A and B [an (m, n) event] where $m>n$.

Let $P(N, n, m)$ be the probability that N particles produce an (n, m) event. A method of calculating these probabilities for a single tray assuming random distribution of the particles on the tray has been given by Schrödinger.[4] The extension to two crossed trays is elementary. The effect of the lack of randomness amongst the shower particles has been examined by Sitte and his co-workers[3] and found to be small. Now if $Q(N)$ is the probability of N particles occurring, the

FIG. 3. The variation of the depth of the cascade maximum with the number of particles at the maximum for the complete cascade.

[3] Froehlich, Harth, and Sitte, private communication (1952).
[4] E. Schrödinger, Proc. Phys. Soc. (London) A64, 1040 (1951).

TABLE II. Intensities of all charged particles at different depths.

Depth in cm H_2O	22.5	45	90	135	180
No. of charged particles per hour	4.6	8.7	15.0	17.2	16.6

TABLE III. Corrected root mean square lateral spread of showers.

Depth (cm)	Spread corrected for background (cm)	Spread corrected for background, finite size of tray and angle of incidence (cm)
22.5	6.4	8.0
45	8.5	12.4
90	10.2	17.1
135	11.4	21.3
180	12.1	24.9

absolute probability of an (n, m) event is

$$\sum_{N=m}^{\infty} Q(N)P(N, n, m).$$

If we know the value of $Q(N)$ for all N, we can use the above formula to calculate the average number of particles $\bar{N}$, associated with any given n, m event. Then, if $R(n, m)$ is the rate of any given (n, m) event and $\bar{N}(n, m)$ is the associated average number of particles, the intensity for a particular depth is

$$\sum_{(n,m)} R(n, m)\bar{N}(n, m).$$

This must then be corrected for the background intensity.

Strictly it is necessary to know $Q(N)$ in order to calculate the averages and hence the intensity. However, the intensities are rather insensitive to the form of $Q(N)$ because, for small N, $P(N, n, m)$ approximates to a δ-function and the form of $Q(N)$ is then unimportant, and secondly, events of large N are comparatively rare.

This insensitivity was shown by first calculating the intensities assuming that $Q(N)\propto 1/N$, which is probably a reasonable approximation to reality. The result of this is given in Table II. Then the calculation was again performed assuming that $Q(1)=Q(2)=Q(3)$, etc., a physically impossible case. The difference in the final intensities was in all cases less than 6 percent.

(5) Lateral Spread of Showers

The total spread of each shower at the level A, B can be determined by measuring the distances between the extreme counters discharged on each tray. The product of the distances gives the square spread and the root mean square spread for all the showers at that depth can be obtained at once. A number of corrections must be applied before these results can be compared with theory.

First a correction must be made for the background showers which come from the air and the surroundings. To do this the mean square spread was found for the zero position. Then for all subsequent depths the contribution of these showers to the run in question was calculated (normalized both to time and the average barometric pressure over the runs) and subtracted from the total spread for the run. The remainder was taken to be the spread due to the genuine locally produced showers for that depth. The results corrected in this way are given in the second column of Table III.

A second correction is necessary for the angle of incidence of the primary. Because the apparatus itself constitutes a vertical telescope and because the high energy nucleon component at sea level is strongly peaked in the vertical direction[5] the correction is not large. If we take the variation of intensity with angle to be $\cos^6\theta$, then the average angle to the vertical of the showers is 17.5° and all spreads must be multiplied by 0.95 (=cos 17.5°).

Thirdly, a correction must be made for the finite size of the detecting trays. This has been done assuming that (a) if a shower has a spread of n counters, all counters within the spread are discharged; and (b) showers produced near the edge of apparatus have an equal chance of discharging the apparatus as centrally produced showers. The correction was then made by finding the recorded spread of every possible shower of a given real spread, including those striking the edges (remembering that at least 2 counters in tray B must be discharged). The recorded spreads were averaged and compared with the assumed spread, and the resulting correction table was used (together with the correction for angle of incidence) to give the results in the third column of Table III.

It is worth considering the effects produced by the breakdown of assumptions (a) and (b). If either of these assumptions is not fulfilled, the correction to be applied will be smaller, since in both cases it will be showers hitting the edge of trays which will be affected.

As a check on our method of measuring the root mean square spread we have measured the spread for the extensive showers for the zero position. The result was 18.2 cm which is to be compared with the 21.0 cm which would be produced if all the showers were of infinite extent and density. For such large spreads the correction table is necessarily inaccurate but the value given is >70 cm.

The further corrections which may be applied are: (a) A correction for the finite width of individual counters (3.8 cm). Since at least 2 counters in B must be discharged, showers whose spread is less than 3.8 cm will tend to be missed and hence the recorded average will be greater than the real average. (b) The complete cascade is measured, and it is obvious from the method of measurement that the effect of this inclusion of the nonnucleonic component can only be to increase the

[5] M. G. Mylroi and J. G. Wilson, Proc. Phys. Soc. (London) **A64**, 404 (1951).

measured spread. The true spread of the nucleon cascade will, if anything, be less than the measured spread.

(6) Barometric Effects

The barometric coefficient was determined for various multiplicities at various depths for both extensive and nonextensive showers. All gave values close to -10 percent per cm Hg with the exception of the nonextensive 2 folds at 0 and 90 cm. For these the values were -0.57 ± 1.0 percent per cm Hg and -4.2 ± 0.6 percent per cm Hg.

III. DISCUSSION OF RESULTS

Throughout the discussion which follows a number of points should be kept in mind; the intensities and results obtained in our experiment are necessarily for *all* ionizing particles capable of penetrating the sheet steel ($\frac{1}{16}$-in.) box. That is to say the intensity due to the complete cascade of nucleons, mesons, and electrons is measured. This cascade is probably started by the energetic sea-level nucleon component of the cosmic radiation. In order, therefore, to carry out a completely satisfactory comparison between theory and experiment, it would be necessary to measure the various effects of the different components separately, and secondly to compare the results obtained with those which would be given by a mixed-cascade theory. It is obvious that both of these tasks are formidable, and not likely to be achieved immediately.[6] The comparisons which follow should therefore be treated as qualitative, bringing out the main features of the various phenomena involved in the cascade process.

(1) Transition Curves and Intensities of Charged Particles

The lower limit to the energy of the particles measured must lie somewhere in the range 10^7–10^8 ev since they need only have sufficient energy to penetrate the steel box.

The minimum triggering energy of the apparatus, on the other hand is probably about 20.10^9 ev. This may be estimated in two ways. Firstly, Salant *et al.*[7] and the Bristol group[8] have shown that stars with two shower particles are produced on average by primaries of about 5×10^9 ev energy. A π-meson is classified as a shower particle if its energy is greater than 80×10^6 ev. Our apparatus, on the other hand, requires at least two penetrating particles of energy greater than 300×10^6 ev. This suggests that the minimum triggering energy is greater than 10×10^9 ev. Secondly, we can estimate the flux of energetic protons and neutrons through the apparatus using the intensity of protons and the exponent of the energy spectrum given by Mylroi and Wilson. Assuming the number of protons and neutrons to be equal, the calculated number of nucleons greater than 10×10^9 ev is 8 per hour. But at the maximum of our total transition curve we got 2.3 showers per hour over background. Thus, it seems reasonable to estimate that the minimum triggering energy is of the order of 20×10^9 ev.

The transition curves for local penetrating showers produced in water have been determined by Wataghin and his co-workers.[9] The arrangement used was a simple fourfold coincidence, and the single curve had a cascade maximum at 75 g/cm² (see Fig. 2). An examination of our results given in Fig. 2 reveals that as one passes from twofold to twelvefold events the depth at which the cascade maximum is attained increases. In other words there is a shift in the position of the maximum towards greater depths, with increasing primary energy (fixed secondary energy) or with decreasing secondary energy and a fixed primary energy. This behavior is in accordance with that predicted by cascade theory. From Messel's[2] work on the nucleon cascade it may easily be shown that the position of the cascade maximum for particles with energies $>E$ due to a primary nucleon of energy E_0 is given by $\theta_{\max}\approx0.72\{\log_{10}(E_0/E)-0.79\}$ interaction lengths, whereas for a primary nucleon power law spectrum with exponent γ and particle energies $>E<E_c$ (E_c is the cut-off energy), $\theta_{\max}\approx0.72\{\log_{10}(E_c/E)-0.79+0.43/(\gamma-0.55)\}$ interaction lengths. The corresponding expressions for electrons produced by a photon or spectrum of photons are given by

$$x_{\max}=2.3\{\log_{10}(E_0/E)-0.32\}\ \text{cascade lengths},$$
$$x_{\max}=2.3\{\log_{10}(E_c/E)-0.32+0.43/(\gamma-1)\}\ \text{cascade lengths},$$

respectively.[10] For energies $E>E_c$, in the case of a primary power law spectrum, no maximum occurs, since the intensity now decreases monotonically with increasing depth. The interpretation of Fig. 2 by means of the above theoretical results is now obvious, and qualitatively at least agreement between theory and experiment on this point is good.

It is now interesting to use the expression for $\theta_{\max}$ in the case of nucleons and the result in Fig. 2 for the twofolds to obtain an estimate for the minimum triggering energy required by the apparatus. Taking $\gamma=1.8$ we find $E_c=68E$. The minimum energy E of the 2 folds is about 3×10^8 ev, since 12.5 cm of lead must be traversed

[6] Messel, Potts, and McCusker, Phil. Mag. **43**, 889 (1952) have recently set up and solved completely the equations for a mixed cascade forming within a nucleus, for various schemes of meson production. Master equations have also been formulated by Messel for a mixed-cascade developing in a finite absorber; numerical results for this last case are, however, unlikely to appear for some time because of the heavy computations involved.

[7] Salant, Hornbostel, Fisk, and Smith, Phys. Rev. **79**, 184 (1950).

[8] Camerini, Davies, Fowler, Franzinetti, Muirhead, Lock, Perkins, and Yekutieli, Phil. Mag. **42**, 1241 (1951).

[9] Meyer, Schwachheim, Wataghin, and Wataghin, Phys. Rev. **76**, 598 (1949).

[10] See, for instance, L. Janossy and H. Messel, Proc. Roy. Irish Acad. **A54**, 217 (1951).

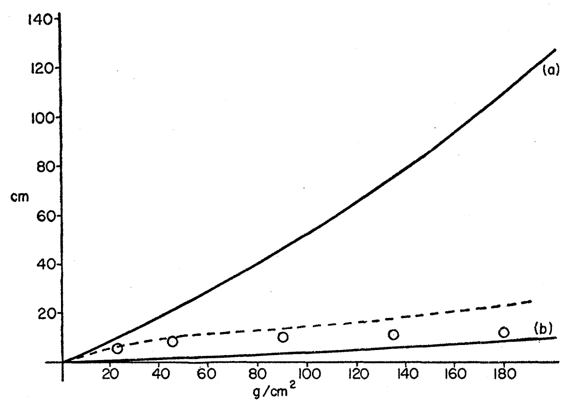

FIG. 4. The lateral spread of the complete cascade at different depths. Curve (a) gives the calculated curve for the nucleon cascade from the Messel-Green theory assuming a quasi-isotropic differential cross section; curve (b) that assuming a cross section of exponential form. The circles give the experimental points corrected for background, and the dashed curve is the estimated experimental upper limit.

in order to obtain a sixfold coincidence. The minimum triggering energy is therefore 2×10^{10} ev.

In Fig. 3 we have given the depth of the transition curve maximum plotted against the total number of charged particles at the shower maximum. This curve was obtained from our experimental results by the method discussed in the previous section of this paper. We note that as the depth at which the cascade maximum is attained increases, the number of charged particles at the maximum point of shower development increases. For showers which attain their maximum development at small depths, the number of particles at the maximum is small; this remains so until depths of the order of 160 g/cm² are attained. For showers which attain their maximum development beyond this depth, the number of particles at the maximum increases very rapidly. Agreement between these experimental facts and predictions by cascade theory is again very good. For instance, the average number of protons at the cascade maximum for the two cases discussed previously is given by

$$N_p(E_0, E; \theta_{\max}) \approx 0.36(E_0/E)^{0.55}\{9.36\log(E_0/E)-4.08\}^{-\frac{1}{2}},$$

and

$$N_p(E_c, E; \theta_{\max}) \approx \frac{0.36}{\gamma-0.55}\left(\frac{E_c}{E}\right)^{0.55}\left\{9.36\log(E_c/E) -4.08+\frac{4.05}{\gamma-0.55}+\frac{1}{(\gamma-0.55)^2}\right\}^{-\frac{1}{2}}.$$

In case of the electrons we have

$$N_e(E_0, E; x_{\max}) \approx 0.09(E_0/E)\{\log(E_0/E)-0.25\}^{-\frac{1}{2}},$$

and

$$N_e(E_c, E; x_{\max}) \approx \frac{0.09}{\gamma-1}(E_c/E)\left\{\log(E_c/E)-0.25+\frac{0.43}{\gamma-1}+\frac{0.28}{(\gamma-1)^2}\right\}^{-\frac{1}{2}}.$$

Hence, with increasing ratio (E_0/E) or (E_c/E), both the depth of the cascade maximum and number of particles at this maximum increases. This phenomenon occurs for both the electrons and the nucleons separately. The experimental curve given in Fig. 3 must, therefore, consist of a *weighted* average of these two results. A further difficulty in making a quantitative comparison in this instance arises because of the low energies of the particles considered. The expressions for N_p and N_e are only valid for high energies where energy loss by ionization is included. If these losses are taken into account, the average number of particles at the cascade maximum given by N_p and N_e may be decreased by a factor of 2 or 3 depending upon the secondary energy considered. A rough calculation has been carried out taking ionization losses into account. The results obtained agree quantitatively with those found experimentally, provided a sufficiently low secondary energy E and a reasonable value of the weights to be applied to the average numbers are chosen.

(3) The Lateral Spread

The solid curves in Fig. 4 give the calculated lateral spread of the nucleon cascade in water for secondaries of energy 500 Mev and an incident power law of exponent 1.1 according to the theory of Messel and Green.[11] The upper curve is for a quasi-isotropic cross section, the lower for a cross-section exponential in the laboratory system. Also displayed on the graph are the experimental points corrected for the background showers and the experimental curve (shown dashed) corrected also for the angle of incidence of the primary and the finite size of the detecting tray. No correction for the finite size of the counters or for the nonnucleonic component has been made. Since the further corrections to be applied all tend to reduce the spread, it seems likely that the dashed curve gives an upper limit for the spread of the cascade in water. It will be seen that the experimental curve strongly favors a differential cross section much more strongly peaked than the quasi-isotropic form.

There are two further effects to consider. It seems likely that the exponent of the proton spectrum at sea level is higher than 1.1,[5] and it also seems likely that particles of less than 500 Mev would be detected by

[11] H. S. Green and H. Messel, Phys. Rev. 85, 679 (1952); Proc. Phys. Soc. (London) A65, 245 (1952); Phys. Rev. 88, 331 (1952).

our apparatus. Both of these effects would increase the calculated spread for a given depth thus increasing the agreement between the experimental points and the curve calculated for the exponential form of the differential cross section. Evidence in favor of this form of differential cross section has already been given by Messel and Green,[11] and Hazen *et al.*[12] has shown that the Fermi distribution tends to give lateral spreads of extensive air showers considerably greater than those reported experimentally.

CONCLUSION

We have been able to determine experimentally the variation of the intensity of charged particles with depth, the variation of the lateral spread with depth, and the variation of the number of charged particles at the maximum of the transition curve with the depth of that maximum for the nucleon induced cascade in water. In all cases there is qualitative agreement with theory. Whereas in some cases the theory of the complete cascade is not sufficiently developed for a quantitative comparison to be possible, in the case of the lateral spread our results make it seem very likely that the differential cross section in high energy nucleon-nucleon collisions cannot have a quasi-isotropic form. On the other hand the exponential form suggested by Messel and Green gives good agreement.

We wish to thank Professor E. Schrödinger and Dr. R. C. Geary for their help and advice, Professor L. W. Pollak for allowing us the use of his meteorological records, and Messrs. A. Guinness, Son and Co. Ltd. for the loan of a large iron tank.

[12] Hazen, Heineman, and Lennox, Phys. Rev. 86, 198 (1952).

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Relativistic Corrections to the Magnetic Moments of Nuclear Particles*

G. Breit and R. M. Thaler
Yale University, New Haven, Connecticut
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A re-examination of the problem is reported. The results for the vector and scalar cases in the case of the deuteron are explained in terms of known correction factors for the one-body problem. For the vector equation part of the result is caused by an induction effect which is the meson theoretic generalization of Faraday's law of induction. In the scalar case the relation to the one-body result is made in a form employing an effective change in mass caused by the presence of the scalar. These interpretations are substantiated by an analysis in terms of plane waves. Simple forms are obtained for one particle in a pseudoscalar field and a tentative application to the deuteron is made and criticized.

1. INTRODUCTION AND NOTATION

RELATIVISTIC corrections to the magnetic moment of a single Dirac particle in a central potential field have been discussed by Breit[1] and by Margenau.[2] The latter has pointed out that an application of the formula applicable to the one-electron case to the deuteron problem gives effects comparable with the change of the magnetic moment expected on account of the admixture of a D-wave to the ground state of H^2. Caldirola[3] was the first to consider the relativistic correction for the case of a particle having an intrinsic magnetic moment in Pauli's sense. Caldirola's signs are either inconsistently used or applied with a misunderstanding regarding the correction factor for the proton, which differs from unity by an amount which is too small in absolute value under his postulated assumptions having been obtained as $-0.667+0.596 = -0.071$ rather than $-0.667-0.596$. Since there is an almost compensating error for the neutron, the result for the deuteron is practically unaffected by this ambiguity. In view of this situation and the fact that Caldirola's work considered the particle with intrinsic Pauli moment to be in a central field, a condition which is not satisfied in the deuteron, the problem was again briefly treated by Breit.[4] In this discussion the correction for the intrinsic moment has the form

$$1-\langle T_\sigma\rangle/Mc^2,$$

where T_σ is the part of the kinetic energy owing to motion along the direction of the particle spin. The character of the field enters the result only through T_σ, and this part of the correction may therefore be used directly for the deuteron. The relativistic factor for a single charged particle in a central scalar field was also contained in this work. It was pointed out in the same note that a single particle treatment does not suffice for the calculation of effects stemming from the Dirac current of the particle's charge; the contribution to the relativistic correction arising from the charge of the

* Assisted by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.
[1] G. Breit, Nature 122, 649 (1928).
[2] H. Margenau, Phys. Rev. 57, 383 (1940).
[3] P. Caldirola, Phys. Rev. 69, 608 (1946).
[4] G. Breit, Phys. Rev. 71, 400 (1947).