We wish to thank Professor L. Rosenfeld and Professor E. C. G. Stueckelberg for valuable discussions on this matter.

* Assisted by the Swiss Atomic Energy Commission (C. S. A.).
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New Equation in the Affine Field Laws

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(Received November 10, 1952)

In his recent affine field theory based on nonsymmetric affinities, Schrödinger has deduced his field equations from a variation principle.[1] He has taken as Lagrangian the square root of the determinant (with changed sign) of the Einstein tensor $R_{\mu\nu}$, defined by

$$R_{\mu\nu}=-\frac{\partial\Gamma_{\mu\nu}{}^{\sigma}}{\partial x_{\sigma}}+\frac{\partial\Gamma_{\mu\sigma}{}^{\sigma}}{\partial x_{\nu}}+\Gamma_{\mu\tau}{}^{\rho}\Gamma_{\rho\nu}{}^{\tau}-\Gamma_{\rho\sigma}{}^{\sigma}\Gamma_{\mu\nu}{}^{\rho}. \tag{1}$$

There is no reason why $\tilde{R}_{\mu\nu}$, defined by

$$\tilde{R}_{\mu\nu}=-\frac{\partial\Gamma_{\mu\nu}{}^{\sigma}}{\partial x_{\sigma}}+\frac{\partial\Gamma_{\sigma\nu}{}^{\sigma}}{\partial x_{\mu}}+\Gamma_{\sigma\nu}{}^{\rho}\Gamma_{\mu\rho}{}^{\sigma}-\Gamma_{\sigma\rho}{}^{\sigma}\Gamma_{\mu\nu}{}^{\rho}, \tag{2}$$

will not play an analogous role in the theory. It is therefore suggested here that the field equations should be obtained from two variation principles instead of one. The Lagrangians in them will be the square roots of the determinants of $R_{\mu\nu}$ and $\tilde{R}_{\mu\nu}$. (Use of two variation principles has already been made by Einstein.[2]) Manipulating the equations obtained from the variation principles, it is not difficult to show that

$$R_{\mu\nu;\sigma}\equiv\partial R_{\mu\nu}/\partial x_{\sigma}-R_{\mu s}\Gamma_{\sigma\nu}{}^{s}-R_{s\nu}\Gamma_{\mu\sigma}{}^{s}=0, \tag{3}$$

and

$$\Gamma_{i}\equiv\tfrac{1}{2}(\Gamma_{is}{}^{s}-\Gamma_{si}{}^{s})=0. \tag{4}$$

Let us now introduce the nonsymmetric $g_{\mu\nu}$'s. Manipulating in the same way as Schrödinger did to obtain his equations in *para* form from the *genuine* form,[1] we get

$$R_{\mu\nu}=\lambda g_{\mu\nu}, \tag{5}$$

$$g_{\mu\nu;\sigma}=0. \tag{6}$$

Equations (4)–(6) now constitute the complete field equations here suggested. These equations reduce to Einstein's strong field equations (described by him as I) if $\lambda=0$ and have the same position as regards over-determinacy.[3]

It may further be noted that, by virtue of (3) and (4),

$$R_{\mu\nu}=\tilde{R}_{\mu\nu}.$$

An interesting, particularly rigorous solution of these field equations has been obtained which (with suitable interpretation of the antisymmetric part of $g_{\mu\nu}$) leads to the conclusion that an isolated magnetic pole cannot exist.[4] In contrast to this the results of a similar solution[5] of Einstein's strong field equations may be mentioned. The remark about it by Schrödinger leads to the possibility of the existence of an isolated magnetic pole, though only in the absence of mass.[6]

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A Focusing Method for Large Accelerators

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For the guiding field of synchrotrons, fields providing continuous focusing, $1>n>0$, have usually been used. Recently, Courant, Livingston, and Snyder[1] showed that fields which consist of periodic focusing and defocusing regions, $n_1=-n_2\gg1$, have strong focusing properties. We have tried another application of the periodic field. The magnet is divided into guiding magnets and focusing magnets, and the latter are placed in the linear portion of the orbit. Quadrupole or solenoid magnets may be used as focusing magnets.

(A) The stability condition formulated by Courant *et al.* now involves the factor ξ, the ratio of the length of the focusing magnet to π/M, where $M=$ the number of pairs in 2π. Let $p_1{}^2$ and $p_2{}^2$ be the coefficients of the focusing field in Fig. 1(a). The stability conditionis as follows:

$$|\eta|<1,$$

$$\begin{aligned}\eta={}&\cos\frac{\pi}{M}\xi p_1\cos\frac{\pi}{M}\xi p_2\\&+\sin\frac{\pi}{M}\xi p_1\sin\frac{\pi}{M}\xi p_2\left[\left\{\frac{\pi}{M}(1-\xi)\right\}^2p_1p_2-\frac{p_1{}^2+p_2{}^2}{p_1p_2}\right]\frac{1}{2}\\&-\frac{\pi}{M}(1-\xi)\left\{p_1\sin\frac{\pi}{M}\xi p_1\cos\frac{\pi}{M}\xi p_2+p_2\cos\frac{\pi}{M}\xi p_1\sin\frac{\pi}{M}\xi p_2\right\}.\end{aligned} \tag{1}$$

Figure 1(b) shows the first stable region.

The quadrupole field shown in Fig. 2 increases linearly with x near the center. This field provides focusing and defocusing force in the x and z directions, respectively, and the forces are given by the equivalent n_{eq} and $-n_{eq}$, respectively, where,

$$n_{eq}=(H'/H_0)R. \tag{2}$$

$R=$ the radius of the orbit in the guiding field, $H_0=$ the guiding field strength, and $H'=$ the gradient of the quadrupole field.

The central part of this field is used and is placed in the linear part of the orbit, changing the direction of the field alternately, and is excited by ac synchronizing with the guiding field, keeping the p^2 near the maximum value allowed by Eq. (1).

n_{eq} of this field can easily be 2 to 3 times greater than that of Courant, *et al.*, and ξ can be taken small. There seem to be some advantages. This method of focusing may also be applied to linear accelerators.

Some examples of design are as follows: (1) 500-Mev electron synchrotron. If we take the effective radius of the whole orbit $=380$ cm, $M=16$, $n_{eq}\lesssim460$ and $\xi=0.12$, the converging distance will be $\gtrsim150$ cm, the length of a guiding field sector $=56$ cm, and the length of a focusing magnet $=9$ cm. The guiding field index may be $1>n>0$ roughly. (2) 100-Bev proton synchrotron. If we take the radius of the whole orbit $=336$ m, $M=150$, $n_{eq}\lesssim40\,000$, and $\xi=0.12$, the converging distance will be $\gtrsim14$ m, the length of a guiding field sector $=5.6$ m, and the length of a focusing magnet $=0.85$ m. The guiding field index may be 0 roughly.

(B) For a periodic focusing field as shown in Fig. 3(a), the stability condition is

$$|\eta|<1,$$

$$\eta=\cos\frac{\pi}{M}\xi p_1\cos\frac{\pi}{M}(2-\xi)p_2-\frac{p_1{}^2+p_2{}^2}{2p_1p_2}\sin\frac{\pi}{M}\xi p_1\sin\frac{\pi}{M}(2-\xi)p_2, \tag{3}$$

This focusing region is shown in Fig. 3(b). For the limit $p_2\to0$, the condition is

$$|\eta|<1,\ \eta=\cos\frac{\pi}{M}\xi p_1-\frac{\pi}{M}(2-\xi)\frac{p_1}{2}\sin\frac{\pi}{M}\xi p_1,$$

or for the first region, $2(f+g)>\pi/M(2-\xi)$,

$$f=\frac{1}{p_1}\csc\left(\frac{\pi}{M}\xi p_1\right),\quad g=\frac{1}{p_1}\cot\left(\frac{\pi}{M}\xi p_1\right). \tag{4}$$

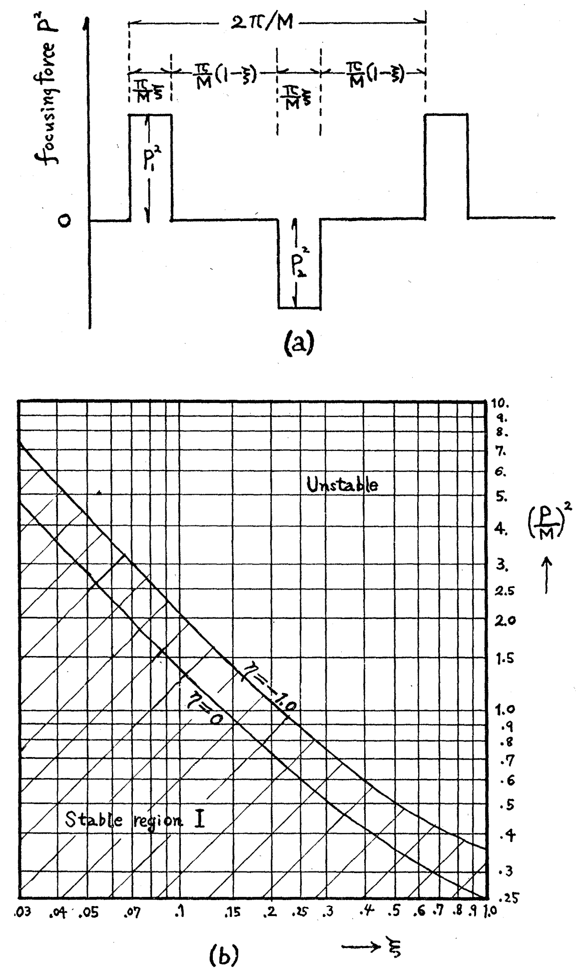

FIG. 1. (a) A periodic focusing field. (b) Stable region I.

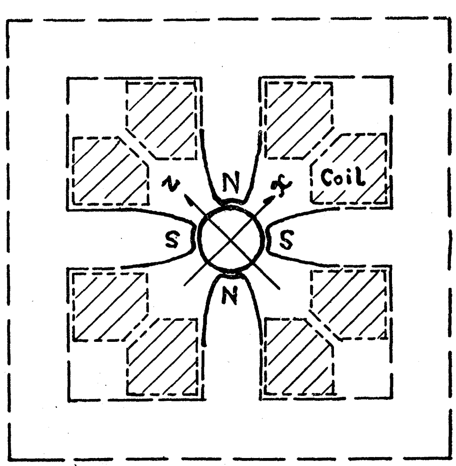

FIG. 2. Quadrupole focusing magnet.

This is the condition that a converging distance should be longer than $2\pi/M$, and has been reported by DePackh[2] for thin solenoid fields.

The focusing index is as follows:

$$n_{eq}=\{H_s/2H_0\}^2, \tag{5}$$

where H_s = solenoid field. It can be used in a way similar to that as in (A) with ac excitation in the linear part of the orbit, though it is not suitable near the maximum beam energy. This may be enough for small accelerators, since the scattering occurs predominantly in the earlier stages of acceleration. For the low energy period, from injection until the synchrotron phase starts, a conventional type of machine with solenoids on the donut may be enough.

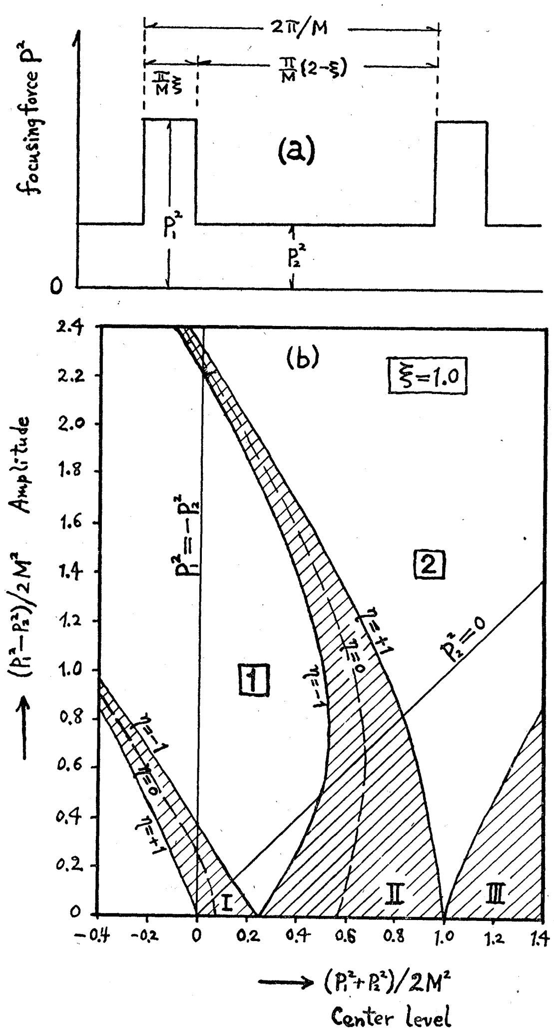

FIG. 3. (a) A general periodic focusing field. (b) Stable regions are shaded.

These focusing principles are now being tested. The author wishes to express his thanks to Professor M. Kimura for his valuable suggestions and encouragement.

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