of the as yet unknown electric dipole moment of the molecule. On the basis of a rough estimate of this quantity, however, we believe the fraction of OH present to be a few tenths of one percent. This figure is in agreement with some earlier values,[5] but does not seem to be consistent with others.[6]

The very substantial intensity of the lines gives hope that one may be able to study by microwave methods OH radicals produced in a variety of chemical reactions, and that other free radicals may prove accessible to microwave techniques.

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† Radio Corporation of America Fellow.
‡ Carbide and Carbon Chemical Corporation Postdoctoral Fellow. Now at Bell Telephone Laboratories, Murray Hill, New Jersey.

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Experimental K to $L+M$ Ratios for Internal Conversion Lines

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MEASUREMENTS have been made on the K to $L+M$ ratios of seven γ-rays, four of which have not been reported previously. These values were obtained by means of a solenoidal type β-ray spectrometer of the design described by DuMond,[1,2] using a value for the fundamental radius R of 20 cm.

Sources were prepared by deposition and evaporation of solutions onto a polyethelene backing $\sim$0.5 mg/cm^2 thick. Gold foil $\sim$100 μg/cm^2 was vacuum evaporated on the backing to render the source conducting. Sources varied in thickness between 1 mg/cm^2 and 10 mg/cm^2, yielding resolutions of the order of one percent.

The K conversion line was completely separated from the corresponding $L+M$ line in every case, although neighboring conversion lines of other γ-rays interfered in some measurements. In the case of Cs^{134}, the K line resulting from the 602-kev γ-ray masked the $L+M$ line of the 560-kev γ-ray, so that it was necessary to estimate the size of the latter. In the case of the equilibrium run of $Ba^{140}-La^{140}$, the K line resulting from the 540-kev γ-ray in Ba^{140} partially interfered with the $L+M$ line of the 488-kev γ-ray in La^{140}.

Uncertainties in the ratios were increased by the low intensity of the $L+M$ lines as a result of the relatively low transmission of the instrument. The counting rates at the peaks of the $L+M$ lines in Nb^{95}, Sb^{124}, and Ba^{140} were all less than 10 cpm above background, and in addition, the latter line was superimposed on top of a relatively strong continuous β-background $\sim$400 cpm.

TABLE I. Values of $K/(L+M)$ obtained.

Isotope	γ-ray energy, kev	$K/(L+M)$
Cs^{137}	663	4.52 ±0.07
Cs^{134}	602	6.6 ±0.2
Cs^{134}	799	7.8 ±0.4
Sb^{124}	607	15.5 ±2
Nb^{95}	774	6.6 ±0.4
Ba^{140}	540	6 ±2
La^{140}	488	3.7 ±0.2

The results obtained for the two Cs^{134} conversion lines compare favorably with those reported recently by LeBlanc *et al.*[3] The value for the 663-kev γ-ray in Cs^{137} also is in good agreement with the results of Kelly.[4]

The experimental values appear in Table I.

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The Charge Independence of Nuclear Forces*

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IN this note we should like to call attention to some consequences of the charge-independence hypothesis for high-energy neutron-proton and proton-proton scattering. Let us consider the general process of the elastic scattering of a nucleon by a nucleon,

$$N_1+N_2\rightarrow N_3+N_4, \tag{1}$$

where N_i denotes a nucleon of momentum $\mathbf{p}_i$ and spin s_i. If we agree to keep the incident and final momenta and spins fixed throughout our considerations, then, by varying the charge assignments of the individual nucleons, we can obtain a number of physically distinguishable reactions.[1] These include

$$p_1+p_2\rightarrow p_3+p_4, \tag{2a}$$
$$n_1+p_2\rightarrow n_3+p_4, \tag{2b}$$
$$n_1+p_2\rightarrow p_3+n_4, \tag{2c}$$

plus three additional reactions which are directly related to (2a), (2b), and (2c) by charge symmetry. The notations n and p indicate neutron and proton, respectively; the subscripts refer to the preassigned momenta and spins.

It may now be readily shown that, if f and g denote the isotopic triplet and singlet scattering amplitudes, respectively, for the two-nucleon system (where f and g are functions of the initial and final momenta as well as the spins), the scattering amplitudes for the three processes (2a), (2b), (2c) are f, $\frac{1}{2}(f+g)$, $\frac{1}{2}(f-g)$, so that with proper normalization of the wave functions we have for the corresponding differential cross sections

$$\sigma_{pp}=|f|^2, \tag{3a}$$
$$\sigma_{np}=\tfrac{1}{4}|f+g|^2, \tag{3b}$$
$$\sigma_{pn}=\tfrac{1}{4}|f-g|^2. \tag{3c}$$

The three cross sections are here expressed in terms of three unknowns (the magnitudes of the triplet and singlet scattering amplitudes as well as their relative phase) so that one cannot derive *equalities* relating the cross sections, but one can deduce *inequalities*. For example, it can be proved that σ_{pp}, σ_{np}, and σ_{pn} will be compatible with the charge-independence hypothesis if and only if

$$(\sigma_{np})^{\frac{1}{2}}+(\sigma_{pn})^{\frac{1}{2}}\geq(\sigma_{pp})^{\frac{1}{2}}, \tag{4a}$$
$$(\sigma_{pn})^{\frac{1}{2}}+(\sigma_{pp})^{\frac{1}{2}}\geq(\sigma_{np})^{\frac{1}{2}}, \tag{4b}$$
$$(\sigma_{pp})^{\frac{1}{2}}+(\sigma_{np})^{\frac{1}{2}}\geq(\sigma_{pn})^{\frac{1}{2}}. \tag{4c}$$

Now, when the incident nucleons are unpolarized, the corresponding differential scattering cross sections σ_{pp}, σ_{np}, and σ_{pn} (which involve an averaging over the initial and a summation over the final spins) are given by expressions similar to (3a)–(3c), so that the inequalities (4a)–(4c) remain valid. In the center-of-momentum system we have $\sigma_{pn}(\theta)=\sigma_{np}(\pi-\theta)$, whence the restrictive relations become

$$[\sigma_{np}(\theta)]^{\frac{1}{2}}+[\sigma_{np}(\pi-\theta)]^{\frac{1}{2}}\geq[\sigma_{pp}(\theta)]^{\frac{1}{2}}, \tag{5a}$$
$$[\sigma_{np}(\pi-\theta)]^{\frac{1}{2}}+[\sigma_{pp}(\theta)]^{\frac{1}{2}}\geq[\sigma_{np}(\theta)]^{\frac{1}{2}}, \tag{5b}$$
$$[\sigma_{pp}(\theta)]^{\frac{1}{2}}+[\sigma_{np}(\theta)]^{\frac{1}{2}}\geq[\sigma_{np}(\pi-\theta)]^{\frac{1}{2}}. \tag{5c}$$

At high energies the Coulomb interaction in the proton-proton system plays a small role in the scattering except in the region of very small angles. Hence equations (5a)–(5c) may be directly applied as a test of the charge-independence hypothesis. For the neutron-proton scattering cross section one has[2] $\sigma_{np}(\theta)\cong\sigma_{np}(\pi-\theta)$ so that relations (5b) and (5c) are essentially satisfied identically. Furthermore, in view of the known angular dependences of $\sigma_{np}(\theta)$[2,3] and $\sigma_{pp}(\theta)$,[4–6] the most critical test of (5a) will be obtained for $\theta=90°$, whence one is led to the inequality

$$R\leq 4, \tag{6}$$

where $R=\sigma_{pp}(90°)/\sigma_{np}(90°)$.

Equation (6) has already been quoted in the literature[4] as holding in the absence of tensor forces; however, it is clearly subject to no limitations beyond those contained in the assumption of charge independence. From the data which are currently available,[2–6] R is largest for energies of the order of 220–260 Mev. In view of the experimental discrepancies and uncertainties (particularly with respect to σ_{pp}), it is unfortunately impossible at the present time to say whether (6) is or is not violated. The ratio R is 2.8 ± 0.5 or 3.8 ± 0.6 according to whether one uses the Berkeley[4] or the Rochester-Harwell[5,6] determination of σ_{pp}. It is clear that a more precise determination of R, both at the energies already referred to and particularly at higher energies, may serve as a useful check of the charge-independence hypothesis.

The general considerations which we have applied to nucleon-nucleon scattering can also be extended to other cases involving the elastic and inelastic scattering of light nuclei by light nuclei at sufficiently high energies. For example, in the inelastic scattering of nucleons by deuterons, we have three physically distinguishable reactions,

$$p+d\rightarrow n_1+p_2+p_3, \tag{7a}$$

$$p+d\rightarrow p_1+n_2+p_3, \tag{7b}$$

$$p+d\rightarrow p_1+p_2+n_3, \tag{7c}$$

plus the three reactions corresponding to neutron-deuteron scattering. If we denote the differential scattering cross sections for (7a), (7b), and (7c) by σ_{npp}, σ_{pnp}, and σ_{ppn}, respectively, these cross sections will satisfy inequalities of the type given by (4a)–(4c), *viz.*,

$$(\sigma_{npp})^{\frac{1}{2}}+(\sigma_{pnp})^{\frac{1}{2}}\geq(\sigma_{ppn})^{\frac{1}{2}},\quad \text{etc.}$$

Finally, we should like to note that restrictive inequalities of the sort we have been dealing with always appear when one compares reactions which differ from one another only as to charge assignments, but that these inequalities *may* already be included in whatever restrictive equalities one may have. Thus, in the single-meson production in nucleon-nucleon scattering,[1] there are two equalities which relate the cross sections, one of which is Watson's relation[7] and the other is a phase relationship [see Eqs. (1) and (2) of reference 1]. Actually, the phase relationship already contains implicitly all the restrictive inequalities for this case.[8] On the other hand, in the elastic scattering of mesons by nucleons, there is only one restrictive equality, *viz.*, Heitler's relation,[9]

$$\tfrac{1}{2}\sigma(\pi^-+p\rightarrow\pi^0+n)+\sigma(\pi^0+p\rightarrow\pi^0+p)$$
$$=\tfrac{1}{2}\sigma(\pi^++p\rightarrow\pi^++p)+\tfrac{1}{2}\sigma(\pi^-+p\rightarrow\pi^-+p). \tag{8}$$

The absence of a restrictive equality in the form of a phase relationship means that the experimental meson-nucleon scattering cross sections will be compatible with charge independence when and only when Eq. (8) and the following are satisfied:

$$[\sigma(\pi^++p\rightarrow\pi^++p)]^{\frac{1}{2}}+[\sigma(\pi^-+p\rightarrow\pi^-+p)]^{\frac{1}{2}}$$
$$\geq[2\sigma(\pi^-+p\rightarrow\pi^0+n)]^{\frac{1}{2}}, \tag{9}$$

plus the two additional inequalities which are obtained from (9) by permuting the bracketed quantities. Since $\sigma(\pi^0+p\rightarrow\pi^0+p)$ is not a readily measurable quantity, it is the inequalities (9) which alone have any practical value.

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Divergence of Perturbation Expansion*

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THE difficulties encountered in quantum field theory in the ultraviolet region have been successfully resolved by the renormalization technique. Other difficulties, discussed by many authors[1,2] and resulting from the failure of the Born approximation, can be removed by a procedure developed by Dyson.[3] But the problem of the convergence after renormalization of the power-series expansions encountered in calculating physical quantities has not yet been resolved, although some attempts have been recently made in this direction.[4–6] A direct proof of divergence is the subject of the present letter.

We consider the calculation of a Green's function in electrodynamics. It is first shown that the total number K_{2n} of Feynman graphs belonging to the nth approximation is given by an exact but very complicated formula, the asymptotic form of which is

$$K_{2n}=N_{2n}=(2n-1)N_{2n-2}. \tag{1}$$

As an example, we give the exact number K_{2n} of graphs for the four first approximations:

$$K_4=4;\quad K_6=27;\quad K_8=248;\quad K_{10}=2830. \tag{2}$$

Retaining now only a subset of the K_{2n} graphs belonging to the nth approximation, namely the "pattern" graphs as defined by Neuman,[7] one can show that the number N_{2n}' of these self-energy graphs is

$$N_{2n}'=(2n-3)N_{2n-2}'. \tag{3}$$

These N_{2n}' graphs can be described in the following ways. Starting with the simplest self-energy graph, from the source line we draw a photon line crossing the one already existing and joining the source line again. This diagram contains now three source lines (crossed self-energy graph); from each of them we iterate the preceding operation, and so on $(n-1)$ times.

Limiting ourselves to the simple case of a scalar theory,[8] we can show by intricate calculations, exhibiting the structure of the n fold integrals, that the contribution of the self-energy nth approximation is asymptotically greater than

$$\alpha^n(4c)^{-n}n^{-1}(n/2)!\equiv a_n \tag{4}$$

in the limit $(-p^2)\rightarrow 0$, and the series

$$\sum_n^\infty a_n \tag{5}$$

is consequently divergent.

A generalization of the present method to other field theories must be possible without extensive modifications. A detailed account of the contents of this letter will shortly be published.