Symmetric Pseudoscalar Theory of Nuclear Forces

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IN his recent work on the problem of nuclear forces, Lévy[1] has employed a form of three-dimensional perturbation theory to deduce an adiabatic nuclear potential from the symmetric pseudoscalar theory with pseudoscalar coupling. We have re-examined the entire field-theoretic problem and will submit a full account shortly. The purpose of this letter is to report a number of errors in the Lévy potential.

Consider first the leading two-pair terms of the fourth-order potential ($V_4^{(a)}$ of L2). Contrary to reference 13 of L2, one must calculate beyond the zeroth approximation to the energy denominators if one wishes to avoid an error of relative order μ/M compared to the leading term. In the next approximation, the contribution from intermediate states with pairs precisely cancels $V_4^{(b)}$ of L2, whereas the contribution from the intermediate state without pairs yields a force which is velocity-dependent.[2] We have been able to show, however, that the velocity-dependent potential need not be included in application to the low energy two-nucleon problem.[3]

Consider next the one-pair terms. The basic matrix elements are represented in Fig. 2 of L2. It is easy to show that in the adiabatic limit the sets a_1, b_1, c_1, and a_2, b_2, c_2 yield precisely the same contribution except for charge dependence. The sum of the contributions is an ordinary force and just three times the value for the neutral theory.[4] The correct fourth-order adiabatic potential, not including terms of relative order $(\mu/M)^2$ compared to the leading term, has the form

$$V_4(r)=-3\left(\frac{G^2}{4\pi}\right)^2\left(\frac{\mu}{2M}\right)^2\frac{1}{\mu r^2}\left[\frac{2}{\pi}K_1(2\mu r)-\frac{\mu}{M}\left(1+\frac{1}{\mu r}\right)^2e^{-2\mu r}\right]. \quad (1)$$

The no-pair potential of L2, Eq. (20) is also inaccurate. This term should agree precisely with the leading contribution to the fourth-order potential of the pseudoscalar theory with derivative coupling if one multiplies the latter by $(\mu/M)^4$. The agreement fails in two respects. The first term of Eq. (20) cancels to first order in the coupling constant[3] against velocity-dependent corrections to the second-order potential. Secondly, there is a charge-dependent term missing of the form

$$-\left(\frac{G^2}{4\pi}\right)^2\frac{\boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^2}{(2M)^4}\int\frac{d\mathbf{k}_1d\mathbf{k}_2\exp[i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{r}]}{\omega_1\omega_2(\omega_1+\omega_2)}(\mathbf{k}_1\cdot\mathbf{k}_2)^2\left(\frac{2}{\omega_1^2}+\frac{1}{\omega_1\omega_2}\right). \quad (2)$$

Finally we have re-examined the leading terms of the sixth and eighth-order adiabatic potentials. If we include contributions only of the type represented explicitly in Fig. 3 of L2, we obtain essential agreement with Eqs. (23) and (24) of the latter. However, a large class of matrix elements of the same order of magnitude has been omitted in each case.[5] Moreover, we can delete from the results those parts which "cancel" the leading velocity-dependent contribution of the fourth-order result.[3] The remaining potentials are given by the expressions

$$V_6(r)=\left(\frac{G^2}{4\pi}\right)^3\left(\frac{\mu}{2M}\right)^4\frac{4}{3}\boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^2\{\boldsymbol{\sigma}^1\cdot\boldsymbol{\sigma}^2+S_{12}\}\frac{e^{-3\mu r}}{\mu^2r^3}\left[1+\frac{1}{\mu r}\right]^2, \quad (3)$$

$$V_8(r)=-3\left(\frac{G^2}{2M}\right)^4(2\pi)^{-12}\int\frac{\exp[i(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)\cdot\mathbf{r}]}{\omega_1\omega_2\omega_3\omega_4(\omega_1+\omega_2)}\times\left[\frac{3}{(\omega_1+\omega_3)(\omega_2+\omega_4)}+\frac{2}{(\omega_1+\omega_3)(\omega_1+\omega_4)}\right]d\mathbf{k}_1\cdots d\mathbf{k}_4. \quad (4)$$

The result of applying the above potentials to the low energy two-nucleon problem will be included in the full report.

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[1] M. M. Lévy, Phys. Rev. **88**, 72, 725 (1952); hereafter referred to as L1 and L2. Our notation follows these papers.
[2] This much has recently been shown by G. Lüders, Institute for Theoretical Physics, Copenhagen, Denmark (unpublished).
[3] By using the variational principle, Eq. (49) of L1, one shows that the kernel consisting of velocity-dependent corrections to the fourth-order potential plus part of the sixth and eighth-order potential leads to no first-order change in the coupling constant.
[4] The result for the neutral theory given in reference 14 of L2 should be multiplied by two.
[5] As an example, there is a considerable class of eighth-order matrix elements which have at most two mesons in all intermediate states in which there are no pairs. These have apparently been omitted.

A Microwave Spectrum of the Free OH Radical*

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A STRONG microwave absorption spectrum resulting from the free radical $O^{16}H^1$ has been found.

The radicals are produced in a radiofrequency discharge through water vapor. The vapor flows continuously through the discharge and the absorption cell, at a pressure near 0.1 mm of mercury. The intensity of the lines is strongly dependent both on discharge current and on pressure.

The absorption cell is a one-meter length of low loss (Corning type 707) glass tubing approximately three centimeters in diameter and located within a split cylindrical wave guide. A solenoid wound on this assembly produces a magnetic field which is used for detection of lines by Zeeman modulation.

The microwave lines result from transitions between members of Λ-type doublets in the paramagnetic ground state of the molecule.[1] In these transitions none of the angular momenta $\Lambda=1$, $S=\frac{1}{2}$, K, $J=K+\frac{1}{2}$ changes. The energy levels are further split by magnetic hyperfine structure. Corresponding to the two values of F ($\mathbf{F}=\mathbf{J}+\mathbf{I}$) possible for each molecular level, one expects that each line will be split into four components. The transitions with $\Delta F=0$ are expected to be approximately fifty times as intense as the $\Delta F=\pm1$ lines. To date we have observed only the $\Delta F=0$ lines listed in Table I, although our signal-to-noise ratio is in excess of 300:1. The quantum numbers J, K are assigned on the basis of ultraviolet data.[2] The F values are determined from the observed relative intensity of the hyperfine components, which is approximately the ratio of their F values. The Zeeman patterns of these lines are markedly asymmetric even in small fields where $g_J\mu_0H/h\sim0.5$ Mc/sec. This suggests that for one member of the Λ-doublet,

$$|W(F=J+\tfrac{1}{2})-W(F=J-\tfrac{1}{2})|/h=|\Delta\nu|\sim0.5 \text{ Mc/sec},$$

whereas for the other $|\Delta\nu|\sim9$ Mc/sec. Under such circumstances the $\Delta F=\pm1$ lines would lie too near the main lines to be observed.

TABLE I. Observed frequencies of $O^{16}H^1$ absorption lines.

Transition	Frequency in Mc/sec
$K=4,\ J=4\frac{1}{2},\ F=5\rightarrow F=5$	23 826.90 ±0.05
$K=4,\ J=4\frac{1}{2},\ F=4\rightarrow F=4$	23 818.16 ±0.05
$K=5,\ J=5\frac{1}{2},\ F=6\rightarrow F=6$	36 994.43 ±0.15
$K=5,\ J=5\frac{1}{2},\ F=5\rightarrow F=5$	36 983.47 ±0.15

Three separate effects may enter the hyperfine structure. First, the interaction of the proton magnetic moment with electron spin magnetic moments; second, the interaction of the proton moment with the magnetic field arising from orbital motion; and third, the relativistic interaction which gives rise to the hyperfine structure of atomic S states. Of these, we expect only the first to produce different splittings in the two Λ-doubling states.[3] The observed splitting agrees reasonably well with an approximate calculation of the dipole-dipole interaction assuming the coupling to be pure Hund's case (b), and assuming the odd electron to have a probability near one-half of being found in a $2p$ state about the proton.

The Λ-doubling frequencies agree with ultraviolet data[2] to within 1000 Mc/sec, which is about the expected accuracy of the earlier measurements. Using the value $\lambda\equiv A/B=-7.547$ obtained by Dieke and Crosswhite, we arrive at the following tentative values for the two parameters describing the Λ-type doubling:[4]

$$4\Sigma\frac{(-1)^S}{\nu_{\Pi\Sigma}}(\Pi|AL_y+2BL_y|\Sigma)(\Sigma|BL_y|\Pi)=+3592\pm18 \text{ Mc/sec},$$

$$4\Sigma\frac{(-1)^S}{\nu_{\Pi\Sigma}}|(\Pi|BL_y|\Sigma)|^2=-524.4\pm0.8 \text{ Mc/sec}.$$

We are not now able to give an accurate value for the abundance of radicals, since the line intensity is proportional to the square

of the as yet unknown electric dipole moment of the molecule. On the basis of a rough estimate of this quantity, however, we believe the fraction of OH present to be a few tenths of one percent. This figure is in agreement with some earlier values,[5] but does not seem to be consistent with others.[6]

The very substantial intensity of the lines gives hope that one may be able to study by microwave methods OH radicals produced in a variety of chemical reactions, and that other free radicals may prove accessible to microwave techniques.

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[1] For a discussion of this type of spectrum see Hicks, Ossofsky, and Jones, Technical Note No. 130, Ballistics Research Laboratory, Aberdeen Proving Grounds, November, 1949 (unpublished).
[2] G. H. Dieke and H. M. Crosswhite, Bumblebee Report No. 87, The Johns Hopkins University, November, 1948 (unpublished).
[3] The different behavior of the two Λ-doubled states is a consequence of the interaction between a spin on the symmetry axis and off-axis spins. This type of effect has been observed in the inversion spectrum of ammonia, and will be discussed in a future paper by G. R. Gunther-Mohr *et al.*
[4] J. H. Van Vleck, Phys. Rev. **33**, 467 (1929); R. S. Mulliken and A. Christy, Phys. Rev. **38**, 87 (1931).
[5] A. A. Frost and O. Oldenberg, J. Chem. Phys. **4**, 642 (1936); O. Oldenberg and F. F. Rieke, J. Chem. Phys. **7**, 482 (1939).
[6] W. H. Rodebush and M. H. Wahl, J. Chem. Phys. **1**, 696 (1933). See also R. W. Campbell and W. H. Rodebush, J. Chem. Phys. **4**, 293 (1936) and K. H. Geib, J. Chem. Phys. **4**, 391 (1936).

Experimental K to $L+M$ Ratios for Internal Conversion Lines

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MEASUREMENTS have been made on the K to $L+M$ ratios of seven γ-rays, four of which have not been reported previously. These values were obtained by means of a solenoidal type β-ray spectrometer of the design described by DuMond,[1,2] using a value for the fundamental radius R of 20 cm.

Sources were prepared by deposition and evaporation of solutions onto a polyethelene backing $\sim$0.5 mg/cm^2 thick. Gold foil $\sim$100 μg/cm^2 was vacuum evaporated on the backing to render the source conducting. Sources varied in thickness between 1 mg/cm^2 and 10 mg/cm^2, yielding resolutions of the order of one percent.

The K conversion line was completely separated from the corresponding $L+M$ line in every case, although neighboring conversion lines of other γ-rays interfered in some measurements. In the case of Cs^{134}, the K line resulting from the 602-kev γ-ray masked the $L+M$ line of the 560-kev γ-ray, so that it was necessary to estimate the size of the latter. In the case of the equilibrium run of $Ba^{140}-La^{140}$, the K line resulting from the 540-kev γ-ray in Ba^{140} partially interfered with the $L+M$ line of the 488-kev γ-ray in La^{140}.

Uncertainties in the ratios were increased by the low intensity of the $L+M$ lines as a result of the relatively low transmission of the instrument. The counting rates at the peaks of the $L+M$ lines in Nb^{95}, Sb^{124}, and Ba^{140} were all less than 10 cpm above background, and in addition, the latter line was superimposed on top of a relatively strong continuous β-background $\sim$400 cpm.

TABLE I. Values of $K/(L+M)$ obtained.

Isotope	γ-ray energy, kev	$K/(L+M)$
Cs^{137}	663	4.52 ±0.07
Cs^{134}	602	6.6 ±0.2
Cs^{134}	799	7.8 ±0.4
Sb^{124}	607	15.5 ±2
Nb^{95}	774	6.6 ±0.4
Ba^{140}	540	6 ±2
La^{140}	488	3.7 ±0.2

The results obtained for the two Cs^{134} conversion lines compare favorably with those reported recently by LeBlanc *et al.*[3] The value for the 663-kev γ-ray in Cs^{137} also is in good agreement with the results of Kelly.[4]

The experimental values appear in Table I.

[1] J. W. M. DuMond, Rev. Sci. Instr. **20**, 160 (1949).
[2] J. W. M. DuMond, Rev. Sci. Instr. **20**, 616 (1949).
[3] LeBlanc, Nester, Martin, Brice, and Cork, Bull. Am. Phys. Soc. **27**, No. 5, 22 (1952).
[4] W. C. Kelly, Phys. Rev. **85**, 101 (1952).

The Charge Independence of Nuclear Forces*

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IN this note we should like to call attention to some consequences of the charge-independence hypothesis for high-energy neutron-proton and proton-proton scattering. Let us consider the general process of the elastic scattering of a nucleon by a nucleon,

$$N_1+N_2\rightarrow N_3+N_4, \tag{1}$$

where N_i denotes a nucleon of momentum $\mathbf{p}_i$ and spin s_i. If we agree to keep the incident and final momenta and spins fixed throughout our considerations, then, by varying the charge assignments of the individual nucleons, we can obtain a number of physically distinguishable reactions.[1] These include

$$p_1+p_2\rightarrow p_3+p_4, \tag{2a}$$
$$n_1+p_2\rightarrow n_3+p_4, \tag{2b}$$
$$n_1+p_2\rightarrow p_3+n_4, \tag{2c}$$

plus three additional reactions which are directly related to (2a), (2b), and (2c) by charge symmetry. The notations n and p indicate neutron and proton, respectively; the subscripts refer to the preassigned momenta and spins.

It may now be readily shown that, if f and g denote the isotopic triplet and singlet scattering amplitudes, respectively, for the two-nucleon system (where f and g are functions of the initial and final momenta as well as the spins), the scattering amplitudes for the three processes (2a), (2b), (2c) are f, $\frac{1}{2}(f+g)$, $\frac{1}{2}(f-g)$, so that with proper normalization of the wave functions we have for the corresponding differential cross sections

$$\sigma_{pp}=|f|^2, \tag{3a}$$
$$\sigma_{np}=\tfrac{1}{4}|f+g|^2, \tag{3b}$$
$$\sigma_{pn}=\tfrac{1}{4}|f-g|^2. \tag{3c}$$

The three cross sections are here expressed in terms of three unknowns (the magnitudes of the triplet and singlet scattering amplitudes as well as their relative phase) so that one cannot derive *equalities* relating the cross sections, but one can deduce *inequalities*. For example, it can be proved that σ_{pp}, σ_{np}, and σ_{pn} will be compatible with the charge-independence hypothesis if and only if

$$(\sigma_{np})^{\frac{1}{2}}+(\sigma_{pn})^{\frac{1}{2}}\geq(\sigma_{pp})^{\frac{1}{2}}, \tag{4a}$$
$$(\sigma_{pn})^{\frac{1}{2}}+(\sigma_{pp})^{\frac{1}{2}}\geq(\sigma_{np})^{\frac{1}{2}}, \tag{4b}$$
$$(\sigma_{pp})^{\frac{1}{2}}+(\sigma_{np})^{\frac{1}{2}}\geq(\sigma_{pn})^{\frac{1}{2}}. \tag{4c}$$

Now, when the incident nucleons are unpolarized, the corresponding differential scattering cross sections σ_{pp}, σ_{np}, and σ_{pn} (which involve an averaging over the initial and a summation over the final spins) are given by expressions similar to (3a)–(3c), so that the inequalities (4a)–(4c) remain valid. In the center-of-momentum system we have $\sigma_{pn}(\theta)=\sigma_{np}(\pi-\theta)$, whence the restrictive relations become

$$[\sigma_{np}(\theta)]^{\frac{1}{2}}+[\sigma_{np}(\pi-\theta)]^{\frac{1}{2}}\geq[\sigma_{pp}(\theta)]^{\frac{1}{2}}, \tag{5a}$$
$$[\sigma_{np}(\pi-\theta)]^{\frac{1}{2}}+[\sigma_{pp}(\theta)]^{\frac{1}{2}}\geq[\sigma_{np}(\theta)]^{\frac{1}{2}}, \tag{5b}$$
$$[\sigma_{pp}(\theta)]^{\frac{1}{2}}+[\sigma_{np}(\theta)]^{\frac{1}{2}}\geq[\sigma_{np}(\pi-\theta)]^{\frac{1}{2}}. \tag{5c}$$