Magnetic Lens Spectrometer Measurements of the Radiations from Light Nuclear Reactions*

R. G. THOMAS† AND T. LAURITSEN
California Institute of Technology, Pasadena, California
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The radiations from a number of light nuclear reactions are studied with a magnetic lens spectrometer. The γ-ray energies and intensities are determined from the photoelectric and Compton conversion processes; a new method for intensity measurements is developed in which "thick" Compton converters are used. The complicating effects of Doppler shift and broadening on energy determinations are discussed. The following transition energies, obtained from thick targets at bombarding energies from one to two Mev, are reported: $C^{13}(d,n)N^{14}$: 725±4 (assignment uncertain), 1638±8, 2310±12, 3381±13, 5052±25, and 5690±50 kev; $C^{13}(d,p)C^{14}$: 6110±30 kev; $C^{12}(d,p)C^{13}$: 3082±7 kev; $N^{15}(p,\alpha)C^{12}$: 4443±20 kev; $B^{10}(p,\alpha)Be^{7}$: 428.5±1.8 kev; $Li^{6}(d,n)Be^{7}$: 428.9±2 kev; $Li^{6}(d,p)Li^{7}$: 477.4±2 kev; $O^{16}(d,p)O^{17}$: 870.5±2 kev. The internally formed positron distribution from the 3.08-Mev transition in C^{13} is found to be in agreement with the theoretical distribution for an electric dipole transition; the internally formed positron distributions from $C^{13}+d$ and $Be^{9}+d$ are also observed but because of the uncertainty of the background, it is not possible to make unambiguous multipole assignments. The observed internal conversion line spectrum from the 870-kev transition of O^{17} indicates that the transition is electric quadrupole or a mixture of this and magnetic dipole. Semi-empirical formulas are given in the appendix for the most probable and effective energy losses of fast electrons traversing light materials.

I. INTRODUCTION

THE study of gamma-ray transitions in nuclear reactions is an important complement to the identification of energy levels by particle group observations. Not only is it sometimes possible to obtain from gamma-ray measurements more accurate values for the level energies than can be obtained by particle measurement techniques, but often, by observation of cascade transitions, one can derive important information about the character of the levels involved. With quantitative determinations of the relative probabilities of various transitions, one might hope to construct detailed decay schemes which may then be of assistance in establishing spins, parities, and possibly other quantities of fundamental significance pertaining to the levels involved.

The measurement of even relative intensities of gamma-rays poses rather formidable problems. Particularly when the spectrum is complex and extends over a large range of energies, recourse to a variety of techniques may be required. A major part of the present investigation has been devoted to the question of the precise determination of γ-ray intensities from the secondary electron spectra, as observed in a magnetic lens spectrometer. It appears to be possible, using the present techniques, to obtain both relative and absolute γ-ray intensities to an accuracy of five to ten percent, in not-too-unfavorable cases.

Further information about the levels involved in γ-ray transitions can be obtained from the multipole order of the radiation, as determined from the coefficients of internal conversion and internal pair formation. The latter process, which is essentially independent of the nuclear charge and for which the coefficient increases with decreasing multipole order, is particularly useful in the light nuclei, for γ-ray energies above $\sim$2 Mev. For the higher energy γ-rays, the conversion electrons and positrons are quite easily detectable, and a comparison of their spectrum with that of the secondary electrons produced by the γ-rays in a suitable converter yields the conversion coefficient directly, without knowledge of either the absolute intensity of the radiation or the solid angle of the spectrometer.

When a "prompt" gamma-ray produced in a nuclear reaction is observed in the direction of the bombarding beam, its energy may be subject to a Doppler shift, depending upon whether the radiation occurs before or after the emitting nucleus comes to rest in the target material. If the nuclei emitting the γ-radiation are traveling in various directions, the line may be broadened. The determination of whether or not Doppler effects occur may enable an estimate of the lifetime of the radiating nucleus, permitting again some inference as to the character of the states involved.

II. APPARATUS

The magnetic lens spectrometer used in the present investigation has been described in a previous communication.[1] Targets located at the focal point of the spectrometer were bombarded by deuterons or protons ranging in energy from 0.9 to 1.6 Mev, entering along the axis. Electrons and positrons produced in the target or in converters attached to the target were focused on an annular slit and counted by means of a Geiger counter with a 2–3 mg/cm² mica window. A helical baffle permitted observation of electrons and positrons

* This work was assisted by the joint program of the ONR and AEC.
† Now at Los Alamos Scientific Laboratory, Los Alamos, New Mexico.

[1] Hornyak, Lauritsen, and Rasmussen, Phys. Rev. **76**, 731 (1949).

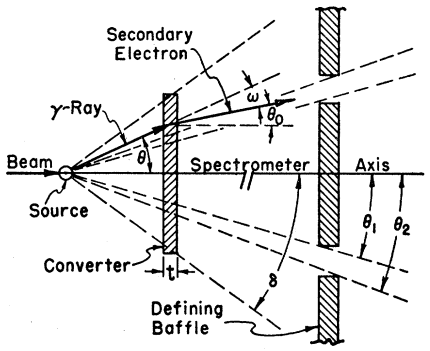

FIG. 1. The spectrometer source-converter-baffle geometry, illustrating a typical gamma-ray conversion process.

separately. Energy calibrations were generally based on the Th I and X lines, at 1754 and 9988 gauss cm, respectively.[2] The instrumental resolution curve was approximately Gaussian, with a width at half-maximum of 2.0 percent, and the effective solid angle, including transmission factor, was about two percent of a sphere.

III. DETERMINATION OF γ-RAY YIELDS AND CONVERSION COEFFICIENTS

The determination of the absolute number of gamma-rays produced in the target depends upon observation of the spectrum of the secondary electrons from a suitable converter. In general one may use the photo-electrons, Compton electrons, or pairs, depending upon the energy range of interest and upon the complexity of the primary spectrum. In each case the interpretation of the results requires consideration of the character of the converter and of the geometry of the instrument.

The source-converter geometry applicable to the present work is schematically illustrated in Fig. 1. The converter is a circular disk centered on the spectrometer axis and perpendicular to it. The axial distance of the converter from the source is ordinarily less than 3 mm and its diameter is 12 mm or less; it subtends an angle 2δ. The limiting acceptance angles of the spectrometer, $\theta_1=13°$ and $\theta_2=27°$, are determined by a circular baffle 16 cm from the source. If the source is produced in a nuclear reaction, the path of the incident beam is coincident with the spectrometer axis, and axial symmetry is maintained throughout. In the conversion process illustrated, θ is the angle between the direction of the gamma-quantum and the axis, ω is the angle between the directions of the gamma-quantum and the ejected electron, and θ_0 is the angle between the direction of the secondary electron and the axis. The dihedral angle between the planes defined by the quantum-electron paths and the quantum-axis directions will be referred to as ψ.

The following quantities are involved in the determination of the converter efficiencies: $\sigma(E)dE$, the total atomic conversion cross section for production of a secondary electron with an energy between E and $E+dE$; $S(\cos\omega)d(\cos\omega)d\psi$, the probability that the secondary electron is ejected into the solid angle $d(\cos\omega)d\psi$; and Y, the yield for a particular radiation from the source, integrated over the sphere. The angular distribution of the primary radiation is assumed to be isotropic so that the differential yield per unit solid angle is $Y/4\pi$ in any direction.

With a flat converter as shown in Fig. 1, the total number of gamma-rays traveling in the solid angle $2\pi d(\cos\theta)$ which eject electrons from a thickness dt into the solid angle $d(\cos\omega)d\psi$ and with energies between E and $E+dE$ is given by

$$(Y/4\pi)2\pi d(\cos\theta)\sigma(E)dENdt\sec\theta\, S(\cos\omega)d(\cos\omega)d\psi \quad (1)$$

if $\theta<\delta$, and is zero if $\theta>\delta$,

where N is the number of atoms per unit volume. (The attenuation of the gamma-radiation has been ignored.) Using the relation $\cos\theta_0=\cos\theta\cos\omega+\sin\theta\sin\omega\cos\psi$, it is possible to integrate (1) over all values of the angles θ and ω, obtaining a result in the form[3]

$$(Y/4\pi)d\Omega_0\sigma(E)dENdt\langle\sec\theta(\theta_0)\rangle, \quad (2a)$$

where

$$\langle\sec\theta(\theta_0)\rangle=\int_{4\pi}S(\cos\omega)\langle\sec\theta(\omega,\theta_0)\rangle d(2\pi\cos\omega) \quad (2b)$$

for the yield of secondary electrons traveling in the solid angle $d\Omega_0=2\pi d(\cos\theta_0)$ and with energies between E and $E+dE$. The kernel $\langle\sec\theta(\theta_0)\rangle$ is interpreted as the average value of $\sec\theta$ for secondary electrons ejected at an angle θ_0 with the axis, and $\langle\sec\theta(\omega,\theta_0)\rangle$ as the average value of $\sec\theta$ for those electrons ejected in the (two) directions characterized by the angles ω and θ_0. The form of the latter kernel depends upon the limits of integration; here we are concerned with the case where $\theta_0+\omega<\delta$ for which

$$\langle\sec\theta(\omega,\theta_0)\rangle=(\cos^2\theta_0-\sin^2\omega)^{-\frac{1}{2}}. \quad (3)$$

Compton Conversion

The treatment of the energy distribution of the ejected Compton electrons is readily carried out in terms of the quantities $x\equiv(E_\gamma-E_e)/E_\gamma$ and $K\equiv E_\gamma/m_0c^2$, where E_γ is the energy of the incident quantum and E_e is the kinetic energy of the ejected electron. The most energetic secondary electron corresponds to a value $x_{\min}=(1+2K)^{-1}$. The Klein-Nishina cross section for the production of a secondary electron with an energy between x and $x+dx$ can be written as

$$\sigma(x)dx=\pi Zr_0^2K^{-1}f(x)dx, \quad (4)$$

where

$$f(x)=(Kx)^{-2}+x^{-1}(1-2K^{-2}-2K^{-1})+x+K^{-2}+2K^{-1}, \quad (4a)$$

[2] G. Lindstrom, Phys. Rev. 83, 465 (1951); W. Brown, Phys. Rev. 83, 271 (1951).

[3] Rasmussen, Hornyak, Lauritsen, and Lauritsen, Phys. Rev. 77, 617 (1950).

$r_0=e^2/m_0c^2$, and Z is the atomic charge. The angle ω_0 between the initial path of the secondary electron and the incident quantum is given by

$$\sin^2\omega_0=[x(1+2K)-1]/[K^2(1-x)+2K], \quad (5)$$

the angular distribution function of Eq. (1) being the delta-function[4]

$$S(\cos\omega)=(2\pi)^{-1}\delta(\cos\omega-\cos\omega_0). \quad (6)$$

By integrating over the emergence angle θ_0 between the limits θ_1 and θ_2 set by the baffles, we obtain from (2), after substituting (4) and (3),

$$(Y/4\pi)\pi ZNdt\, r_0^2 f(x)K^{-1}dx\, q\langle\sec\theta\rangle\Omega_0, \quad (7)$$

where

$$\langle\sec\theta\rangle\Omega_0=2\pi\log\left\{\frac{\cos\theta_1+(\cos^2\theta_1-\sin^2\omega_0)^{\frac{1}{2}}}{\cos\theta_2+(\cos^2\theta_2-\sin^2\omega_0)^{\frac{1}{2}}}\right\}, \quad (7a)$$

for the number of secondary electrons emerging in the solid angle Ω_0 with energies corresponding to the interval between x and $x+dx$, $q(\leqslant 1)$ being a factor introduced to take into account the spectrometer transmission and counter efficiency. Since the angle ω_0 is small in the interesting high energy portion of the Compton spectrum and since the angles θ_1 and θ_2 are also small, it is evidently permissible to make the approximation[5]

$$q\langle\sec\theta\rangle\Omega_0=\Omega/\langle\cos\theta_0\rangle, \quad (7b)$$

where Ω is an effective solid angle which includes the q factor. If the spectrometer field setting is measured in units of gauss-cm ($B\rho$),

$$dx=-3.00\times10^{-4}\beta d(B\rho)/E_\gamma, \quad (8)$$

E_γ being measured in Mev and $\beta=v/c$, where v is the velocity of the ejected electron.

For a "thin" converter, that is, a converter in which the electron energy loss is small compared to the energy spread represented by the spectrometer resolution function, expression (7) may, after convolution with the resolution function and the energy-loss distribution [Appendix, Eq. (22)], be matched to the observed spectrum. Adjustment of the parameters K and Y for best fit gives the gamma-ray energy and the yield in terms of the spectrometer constants.[3]

The energy distribution of secondary electrons emerging from a thick Compton converter is obtained by introducing an effective stopping force μ_e, by which the differential electron path length in the direction of the spectrometer acceptance solid angle, $dt/\langle\cos\theta_0\rangle$, is to be multiplied in order to obtain the energy loss in the converter:

$$-dE_e=E_\gamma dx=\mu_e dt/\langle\cos\theta_0\rangle. \quad (9)$$

Substituting (9) into (7) for dt and integrating from $x_{\min}$ to x, we obtain

$$\tfrac{1}{4}Y\Omega ZNr_0^2m_0c^2\mu_e^{-1}F(x)dx, \quad (10)$$

where

$$F(x)=2K^{-1}-\tfrac{1}{2}(1+2K)^{-2}+(1+2K)xK^{-2}+\tfrac{1}{2}x^2 -K^{-2}x^{-1}+(1-2K^{-2}-2K^{-1})\ln x(1+2K) \quad (10a)$$

for the number of electrons emerging from the converter into the spectrometer solid angle Ω in the energy range from x to $x+dx$. The determination of μ_e, which requires rather detailed considerations of the energy loss and scattering mechanisms in the converter, is discussed in the Appendix.

After convolution with the spectrometer resolution function, the distribution given by expression (10) may be compared with the observed spectrum from a thick converter to determine the gamma-ray energy and intensity. Except in the immediate neighborhood of the end point, it is permissible to ignore the curvature of $F(x)$ in the convolution with a symmetric resolution function, and one obtains for the number of counts recorded at the spectrometer field setting $B\rho$:

$$C(B\rho)=\tfrac{1}{4}Y\Omega ZNr_0^2m_0c^2\mu_e^{-1}F(x)\epsilon pB\rho dx/d(B\rho). \quad (11)$$

The factor $\epsilon pB\rho$ is the area under the resolution function,

$$\int_0^\infty R(B'\rho-B\rho)d(B'\rho)=\epsilon pB\rho, \quad (12)$$

where R is normalized to unity at the maximum. The quantity p represents the fractional resolution, and ϵ is a shape factor. For a Gaussian, if p is taken as the full width at half-maximum, $\epsilon=1.064$.

The comparison of expression (11) with the experimental data must be restricted to a rather limited energy range, in view of the assumption that μ_e is constant. For energies above 1 Mev, in aluminum and beryllium converters, however, μ_e varies by only a few percent over the upper 5 percent of the electron spectrum and quite good fits are possible. As a check on the method, we have determined the relative intensity of the two cascade gamma-rays of Na^{24} at 1.37 and 2.76 Mev, obtaining for the ratio, 1.03 ± 0.07. Using a calibrated Co^{60} source to determine the effective solid angle Ω, we obtained agreement within 10 percent of the published[6] value of the absolute yield of 6–7 Mev gamma-rays from $F^{19}(p,\alpha)O^{16*}$ (correcting for the known anisotropy).

Photoelectric Conversion in Thin Foils

It has not been possible to account for the observed thin-converter photoelectric line shapes in a quantita-

[4] A small redistribution in angle due to multiple scattering of electrons in the converter is considered in the Appendix.

[5] To the extent that approximation (7b) applies, the γ-rays whose yield is measured are those emitted in the direction of the spectrometer acceptance cone. Thus if the γ-radiation is not isotropic, the yield obtained is to be regarded as 4π times the differential yield in this direction.

[6] Chao, Tollestrup, Fowler, and Lauritsen, Phys. Rev. **79**, 108 (1950).

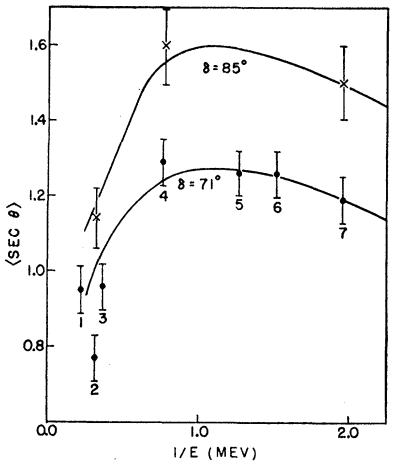

FIG. 2. Measured photoconversion geometrical efficiency factor $\langle\sec\theta\rangle$ plotted as a function of the reciprocal of the γ-ray energy in Mev. The sources used were: 1. $Be^9(p,\alpha)Li^6$, 2. $C^{12}(d,p)C^{13}$, 3. Na^{24}, 4. Co^{60}, 5. Cs^{134}, 6. Cs^{137}, 7. Na^{22}.

tive manner, the difficulty being the lack of precise knowledge concerning the angular distribution in the conversion process and the effects of elastic and inelastic scattering in the high atomic number materials required for this process. Therefore, the total area under the K-conversion line spectrum is used as a measure of the gamma-ray yield. We obtain from (2a) and (12)

$$(Y/4\pi)\Omega\sigma_K Nt\langle\sec\theta\rangle\epsilon p=\int C(B\rho)d\ \ln B\rho, \qquad (13)$$

for the relation between the yield and the area beneath the curve of counts divided by $B\rho$ for a converter of thickness t. The quantity $\langle\sec\theta\rangle$ now represents expression (2b) averaged over the solid angle Ω_0, and Ω includes the transmission factor q.

The total K-shell photoelectric cross section σ_K has been calculated by Hulme *et al.*,[7] for certain values of E_γ and Z. By means of a Z extrapolation of their results, one may obtain this cross section for thorium, the material used here for photoelectric converters, for values of E_γ equal to 1.13 and 0.354 Mev. Using Hall's asymptotic formula[8] it is possible to obtain a value of $\sigma_K^{\mathrm{Th}}\cdot E_\gamma$ for $E_\gamma=\infty$. A quadratic expression adjusted to these three values is

$$\sigma_K^{\mathrm{Th}}\cdot E_\gamma=0.371(1+0.448E_\gamma^{-1}+0.660E_\gamma^{-2})\times10^{-23}\ \mathrm{cm}^2, \qquad (14)$$

which is valid if $E_\gamma\geqslant0.3$ Mev. According to the considerations of Hulme *et al.*, such a formula should not be in error by more than about 5 percent, and it is in good agreement with Gray's formula,[9] extrapolated from lead, in the range 0.3 to 1.13 Mev.

[7] Hulme, McDougall, Buckingham, and Fowler, Proc. Roy. Soc. (London) A149 (1935).
[8] H. Hall, Phys. Rev. 84, 167 (1951).
[9] L. H. Gray, Proc. Cambridge Phil. Soc. 27, 103 (1931).

Unfortunately the quantity $S(\cos\omega)$ required for the determination of $\langle\sec\theta\rangle$ is only known through Sauter's equation, which is valid in the limit $Z=0$. Therefore it was necessary to determine $\langle\sec\theta\rangle$ of (13) experimentally, using sources of various energies whose strengths could be obtained in other ways, such as by means of a calibrated Geiger counter or by measurement of the Compton conversion electrons with the spectrometer. Some experiments were performed using radioactive sources such as Na^{22}, Cs^{137}, Co^{60}, Na^{24}, and gamma-radiation from nuclear reactions in order to determine the dependence of $\langle\sec\theta\rangle$ on the converter angle δ, the converter thickness, and E_γ. The details of these measurements will not be given here, but some of the qualitative features of the results are worth mentioning. As regards the dependence on δ, it was observed that for $E_\gamma=1.28$ Mev and a foil thickness of 22 mg/cm^2 (small compared to the scattering length) the geometrical factor could be represented as

$$\langle\sec\theta\rangle=4.2\sin^2(\delta/2), \quad \text{when} \quad 0\leqslant\delta\leqslant85°, \qquad (15)$$

the dependence on δ being even more critical in the case of annihilation radiation. The importance of controlling the angle δ in intensity measurements is thus apparent. Again for $E_\gamma=1.28$ Mev, $\langle\sec\theta\rangle$ was observed actually to increase with increasing foil thickness, roughly according to $\langle\sec\theta\rangle=1.6(1+0.010t)$ for t between 5 and 40 mg/cm^2, when $\delta=85°$. This critical dependence on t for large δ is presumably to be attributed to "scattering in" of electrons originally ejected at large angles to the spectrometer axis. Because of the geometry of the converter and the preference for forward ejection of the electron at high quantum energies, there will be an excess of electrons at large angles, and multiple scattering will increase the number accepted by the spectrometer at the expense of this excess. On the other hand, with a smaller converter angle of $\delta=71°$, $\langle\sec\theta\rangle$ was found to be independent of t, being equal to 1.3 for $t<50$ mg/cm^2. Also with this converter angle, $\langle\sec\theta\rangle$ was measured as a function of E_γ and found to be rather insensitive to energy; the results are shown in Fig. 2 together with several determinations of $\langle\sec\theta\rangle$ with $\delta=85°$. According to the theory, in the limit as $E_\gamma\to\infty$, the photoelectrons are ejected in the direction of the incident quantum and scattering is negligible, so that the geometrical factor becomes equal to the secant of the mean spectrometer acceptance angle, which was 1.06 in these experiments. There is some indication of this tendency in the results presented in Fig. 2.

Those experiments described in Sec. V which involve intensity determinations by means of photoelectric conversion were performed prior to the above-described investigation of the dependence of $\langle\sec\theta\rangle$ on the various factors. It was the practice to use a converter angle δ of about 85° in order to obtain maximum sensitivity, but as we have shown, the factor is rather critical to the exact value of δ and to the converter thickness;

consequently these photoelectric intensity determinations may be in error by 25 percent or more.

It was thought that the complications due to the uncertain angular distribution in the conversion process and the redistribution due to multiple scattering might be avoided by substituting for the flat converter a hemispherical one, centered at the source position. This did not turn out to be the case. The efficiency of such converters were observed to be less than 80 percent, indicating that a significant number of photoelectrons are ejected backwards with respect to the direction of the incident quantum. Furthermore, the line shapes were broad and the efficiency sensitive to the thickness, dropping rapidly with increasing thickness, presumably due to the fact that the electrons have to traverse greater distances in the foil before emerging.

Internal Conversion

As in the case of external photoelectric conversion, a convenient measure of the intensity of an internal conversion process is the total area under the line spectrum. The expression relating this area to the product of the internal conversion coefficient Γ and the yield Y of the associated γ-radiation is obtained from (13) by replacing the external conversion coefficient $\sigma_K Nt\langle\sec\theta\rangle$ with the internal conversion coefficient:

$$(Y/4\pi)\Omega\Gamma\epsilon p=\int C(B\rho)d(\ln B\rho). \tag{16}$$

Thus, if the gamma-ray yield can be measured by means of an external conversion process, Γ can be computed from (16), and multipolarities deduced by comparing Γ with the accurate theoretical values.[10]

Two complications arise in the application of (16) to the determination of Γ from the radiations from nuclear reactions. The first is that the angular distributions of the conversion electrons $\Gamma(\theta)$ and the associated γ-radiation $Y(\theta)$ may not be the same. That is, even ignoring the angular redistributions due to the external conversion process and to scattering, it may not be true that

$$\Gamma(\theta)/Y(\theta)=\int_{4\pi}\Gamma(\theta)d\Omega\Big/\int_{4\pi}Y(\theta)d\Omega, \tag{17}$$

which is implied by the combined use of (16) and (11), or (13) for the determination of Γ. It remains in each case to examine the extent of the validity of (17) by computing these angular distributions from the assumed spin-parity assignments of all the states involved and their occupation probabilities in the various angular momentum states along the beam-spectrometer axis. From the expressions for the internal conversion and gamma-ray angular distribution functions as obtained by Rose, Biedenharn, and Arfken,[11] it is possible to state that (17) is valid for electric multipole radiation if $E_\gamma\gg m_0c^2$, and for magnetic multipole radiation in the field of low-Z nuclei provided that $E_\gamma\gtrsim m_0c^2$; (17) is, of course, always valid if the radiations are isotropic.

The second complication lies in the determination of the average K-shell occupation probability during the time that the excited nuclei are radiating. These nuclei have initial velocities of the order of $e^2/\hbar$, so that the cross sections for capture and loss of orbital electrons are of the order of the Bohr area. Capture-and-loss equilibrium is established, then, in condensed materials, in about 10^{-16} sec, which is shorter than most gamma-ray lifetimes. From a study of existing experimental and theoretical information on capture and loss ratios as a function of velocity, we conclude that the K-shell occupation probabilities will usually be close to unity even before the nuclei are decelerated.

Internal Pair Formation

In the approximation of representing the positron and electron wave functions by plane waves, formulas have been obtained by Rose[12] for the differential internal pair formation coefficient $\Gamma(w)dw$, for the production of a positron (or electron) of energy w for various electric and magnetic multipoles. End point corrections for $E1$ (electric dipole) and $E2$ (electric quadrupole), obtained by representing the slow particle by means of spherical waves, have been given by Rose and Uhlenbeck.[13] Except near the end point, we can neglect the effect of the spectrometer resolution function on the shape of the distribution, and obtain for the relation between $\Gamma(w)$ and the observed number of counts, $C(B\rho)$, as a function of $B\rho$:

$$(Y/4\pi)\Omega\Gamma(w)\epsilon p dw/d(B\rho)=C(B\rho)/B\rho, \tag{18}$$

with

$$dw/d(B\rho)=5.86\beta\times10^{-4};$$

w is the total electron energy, in units m_0c^2; β is the velocity, in units of c.

When dealing with nuclear reactions where the radiations may not be isotropic, an assumption equivalent to (17) is also involved in the use of (18). It is not possible at present to estimate the validity of this assumption since the internal pair angular distribution functions have not been determined. If the transition energy is very large, however, the angular distribution of the positrons (and electrons) and the associated gamma-radiation will be the same.

From a theoretical point of view, the processes of internal conversion and internal pair formation are similar; in one case an orbital electron is ejected from the surroundings of the nucleus, and in the other case a negative-energy electron in the surroundings of the nucleus is raised to a positive-energy state, the resulting vacancy in the negative energy states being the positron. Therefore, we may expect the same order of magnitude of error to arise from the use of plane waves

[10] Rose, Goertzel, Spinrad, Harr, and Strong, Phys. Rev. **83**, 79 (1951).
[11] Rose, Biedenharn, and Arfken, Phys. Rev. **83**, 5 (1952).
[12] M. E. Rose, Phys. Rev. **76**, 678 (1949); **78**, 184 (1950).
[13] M. E. Rose and G. Uhlenbeck, Phys. Rev. **48**, 211 (1935).

in the internal pair formation process as does in the same approximation in the internal conversion process. When the internal conversion coefficients calculated by Dancoff and Morrison,[14] using relativistic plane waves and neglecting the binding of the orbital electrons, are compared with the calculations using exact Coulomb wave functions made by Rose and his collaborators,[10] considerable deviations are noted, of the order of 10 percent for Z as low as ten and at moderate energies. Thus we may expect similar deviations for the differential pair coefficient $\Gamma(w)$, although as noted by Rose and Uhlenbeck,[13] these deviations tend to cancel at each end point in the determination of the total pair formation coefficient. Unfortunately, in the present experiments it is possible to measure only the high energy portion of the positron distributions and consequently uncertainties of the order of 10 percent must be expected.‡

IV. DOPPLER EFFECTS

The Doppler shift due to the center-of-mass motion and the Doppler broadening due to the angular distribution of the radiating nuclei produced in a disintegration reaction can affect the spectrometer energy determinations by as much as one part in two hundred, which is usually more than the inherent uncertainty in the energy determination from other causes. From studies of such effects, it is sometimes possible to obtain information regarding the lifetimes of the excited nuclei involved, as was done by Elliott and Bell[15] and by Rasmussen, Lauritsen, and Lauritsen[16] in the case of the first excited state of Li^7. The existence or absence of a Doppler shift and broadening depends on whether the nuclei radiate before or after they are significantly decelerated in the target material. A measure of the time required for this deceleration is the ratio of their range to initial velocity; this ratio will be referred to as the stopping time, $T_0 \equiv R/v_0$.

With the geometry shown in Fig. 1, the center-of-mass motion will increase the energy of those quanta which produce secondary electrons in the direction of the spectrometer acceptance cone on the average by the Doppler shift,

$$\delta_s = (v_c/c)E_\gamma\langle\cos\theta\rangle,$$

where v_c is the velocity of the center of mass, E_γ is the quantum energy in the center-of-mass system, and the quantity $\langle\cos\theta\rangle$ will be approximately equal to $\langle\sec\theta\rangle^{-1}$ in the case of photoconversion or to the cosine of the spectrometer (mean) acceptance angle in the case of Compton conversion.

The full extent of the Doppler energy spread due to the angular distribution of the radiating nuclei in the center-of-mass system is given by

$$2\delta_b = 2(v_r/c)E_\gamma,$$

where v_r is their velocity in the center-of-mass system. If the angular distribution in the center-of-mass system is asymmetric about the plane perpendicular to the spectrometer axis, a shift in the observed photoconversion peak may occur. An energy determination by the Compton method, which is primarily based on the high energy edge of the spectrum, will be affected by this broadening even if the angular distribution is symmetric.

In order to determine the extent of the Doppler corrections, it is necessary to know the lifetime τ and the range-velocity relation of the radiating nuclei. For the low initial velocities which occur in the reactions to be studied, cloud-chamber measurements are available[17] on the range-velocity relations (reduced to standard air) of Li^7, C^{12}, N^{14}, O^{16}, Ne^{20}, and some heavier ions. The interpolation between these data, which is required to obtain the values for other light nuclei, can be accomplished with reasonable accuracy by means of the empirical expression,

$$R_{\text{air}}(\text{cm}) \sim 3\times10^{-10} M_1 Z_1^{-0.8} v - R_0,$$

for $v \gtrsim 1\times10^8$ and $Z_1 \lesssim 20$, where M_1 and Z_1 are the mass- and charge-numbers of the ion and v its velocity in cm/sec. For velocities $\sim 0.01c$ this expression appears to fit the experimental data with an accuracy of about 35 percent, which is adequate for the present purpose. The quantity R_0, which is rather small ($\sim\frac{1}{2}$ mm), and the restriction on v have to do with nuclear collision effects which are of negligible importance here.[18] Since there is no experimental information on the stopping powers relative to air of the stopping materials for these nuclei, we assume them to be the same as the stopping powers for protons or alpha-particles of the same velocity. The recent data of Warshaw[19] and others indicate that in the low velocity region the Bragg $Z^{\frac{1}{2}}$ dependence is probably adequate for interpolation.

Since the range is a nearly linear function of the

[14] S. M. Dancoff and P. Morrison, Phys. Rev. **55**, 122 (1939).

‡ *Note added in proof:* Dr. Rose has kindly pointed out to us that the error introduced by use of the Born approximation may be expected to be much less in the case of internal pair formation than in the internal conversion process, since no bound state is involved in the former.

[15] L. G. Elliott and R. E. Bell, Phys. Rev. **76**, 168 (1949); **74**, 1869 (1948).

[16] Rasmussen, Lauritsen, and Lauritsen, Phys. Rev. **75**, 199 (1949).

[17] C. W. Gilbert, Proc. Cambridge Phil. Soc. **44**, 447 (1948); J. K. Bøggild, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **23**, No. 4 (1945); N. Feather, Proc. Roy. Soc. (London) **141**, 194 (1933); G. A. Wrenshall, Phys. Rev. **57**, 1095 (1940); P. Blackett and D. Lees, Proc. Roy. Soc. (London) **A134**, 658 (1932); W. W. Eaton, Phys. Rev. **48**, 921 (1935); J. T. McCarthy, Phys. Rev. **53**, 30 (1938); R. L. Anthony, Phys. Rev. **50**, 726 (1936); W. Hansen and G. A. Wrenshall, Phys. Rev. **57**, 750 (1940).

[18] The linear dependence upon v reflects the predominant influence of capture and loss on the stopping phenomenon. Theoretical arguments for heavy ions in heavy substances suggest a $Z_1^{-\frac{2}{3}}$ dependence and, for stopping materials other than air, a stopping power proportional to $Z_2^{\frac{1}{2}}$. See N. Bohr, Kgl. Danske. Videnskab. Selskab, Mat.-fys. Medd XVIII, 8 (1948) and Phys. Rev. **57**, 275 (1941).

[19] S. D. Warshaw, Phys. Rev. **76**, 1759 (1951).

velocity at low velocities, the time consumed in reducing the velocity of the nucleus from v_0 to v is approximately

$$t = T_0 \ln(v_0/v).$$

Thus, T_0 is actually the time required to reduce the velocity to $1/e$ of its initial value. The distribution of Doppler shifts due to the center-of-mass motion is immediately given in terms of the lifetime by

$$P(\delta)d\delta = \xi x^{\xi-1}dx,$$

where $\xi = T_0/\tau$ and $x = v/v_0 = \delta/\delta_0$, δ and δ_0 being the Doppler shifts in the direction of motion corresponding to velocities v and v_0, respectively. If $\xi = 1$, the distribution of Doppler shifts is rectangular.

Unless the lifetime τ is known, it is not possible to state whether the Doppler shift correction should be applied. However, if an energy level is known with sufficient precision from particle-group measurements, it may be possible by comparison with the γ-ray measurement to determine whether or not the Doppler shift exists and thereby to obtain a limit on the radiative lifetime. This can be done in two of the reactions studied in Sec. V.

It is also necessary to consider the possible effects on energy determinations of the Doppler broadening. In some reactions the particle group angular distributions are known so that the correction can be applied. If no angular-distribution data are available, it seems reasonable to regard $\pm\delta_b/2$ as a measure of the additional uncertainty in the energy determination due to possible asymmetry. The Li^{7*} radiation from Be^9+d is an extreme example of a peak shift which may be due to such an asymmetry.[16] In this reaction the Doppler broadening was so pronounced that the asymmetry was readily detected. More often the Doppler broadening will be less than the combined spectrometer and converter energy-loss width so that it may escape detection.

In the 717-kev B^{10*} radiation from Be^9+d, the absence of Doppler effects has been established by using a thin converter and high spectrometer resolution. Hornyak *et al.*[1] observed the secondary electrons from a 7-mg/cm² thorium converter, for which the energy loss distribution is expected to be about $0.008B\rho$, the spectrometer resolution width (at half-maximum) being $0.015B\rho$; the combination of these two widths is $0.017B\rho$. If the B^{10*} nuclei radiate before being significantly decelerated, there will be a Doppler broadening contribution of $0.013B\rho$ and, consequently, an over-all line width of about $0.021B\rho$. The observed line width (at half-maximum), however, is $0.017B\rho$, indicating no Doppler broadening. It may be concluded either that the lifetime of this state or of the higher states producing cascades to this state is greater than the stopping time of about 3×10^{-13} sec. Therefore, the energy determination which is uncorrected for Doppler shift, 716.6 ± 1.0 kev, is preferred to the corrected value, 713 ± 1.5 kev. This conclusion is also arrived at by Craig *et al.*[20] who obtain 719 ± 1.6 kev by measurement of the energy of the protons inelastically scattered by B^{10}.

[20] Craig, Donahue, and Jones, Phys. Rev. **87**, 206 (1952).

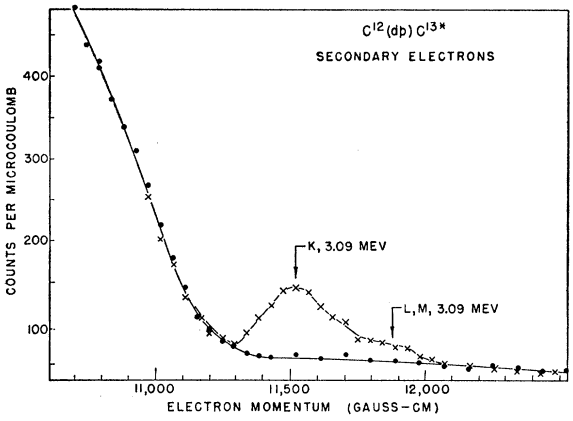

FIG. 3. Secondary photoelectric and Compton electron spectra from $C^{12}(d,p)C^{13*}$ at 1.46-Mev bombarding energy. Closed circles: 350-mg/cm² Al converter; crosses: same plus 22-mg/cm² Th. The target was graphite, 200 mg/cm² thick.

V. RADIATIONS FROM LIGHT NUCLEAR REACTIONS

A. $C^{12}+d$

The bombardment of C^{12} with deuterons of about 1.5 Mev energy leads to the following reactions:

$$C^{12}(d,n)N^{13} \qquad Q_0 = -0.280$$
$$C^{12}(d,p)C^{13} \qquad Q_0 = 2.723.$$

Aside from the annihilation radiation from the decay of N^{13}, only one γ-ray, from the 3.1-Mev state of C^{13}, is produced. This circumstance makes this reaction particularly appropriate for a study of the internal pair conversion coefficient. The pairs in question were first observed by Dougherty *et al.*,[21] who reported the conversion coefficient to be of the order of 10^{-3} pair per gamma-quantum. By utilizing the methods of intensity measurement described above, we have been able to obtain a more precise value for this coefficient, and thus to determine the multipole order of the transition. At the same time we have found it convenient to redetermine the γ-ray energy.

The energy of the γ-radiation was obtained from photoconversion in 15- and 22-mg/cm² foils of thorium, in which the photopeak shifts are 4 and 6 kev, respectively,[1] the spectrometer being calibrated by means of the nearby internal conversion X line of ThD. Both determinations gave 3094 ± 6 kev. By averaging this value with previous determinations from this laboratory, we obtain 3097 ± 5 kev, uncorrected for the Doppler shift, or 3082 ± 7 kev corrected for the Doppler shift. The angular distribution of the short-range protons from this reaction has not been reported, so that it is not possible to make a correction for a possible asymmetry in the Doppler broadening; therefore we consider the corrected determination to be uncertain by an additional amount $\delta_b/2 = 5$ kev. Our resolution

[21] Dougherty, Hornyak, Lauritsen, and Rasmussen, Phys. Rev. **74**, 712 (1948).

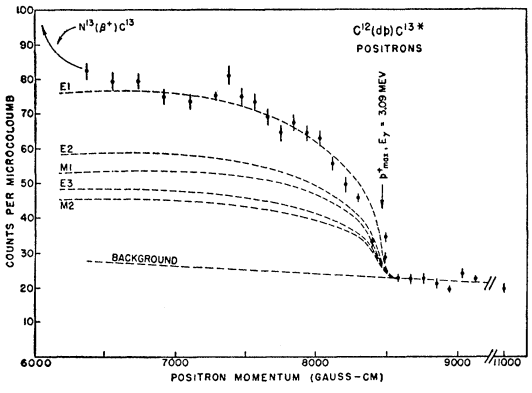

FIG. 4. Positron spectrum of internal conversion pairs from the 3.09-Mev gamma-ray of $C^{12}(d,p)C^{13*}$ at $E_d=1.46$ Mev. Target: 17-mg/cm² graphite. The dashed curves are theoretical distributions for various multipoles, normalized by the yield measurement of Fig. 3.

was not sufficient to detect this small broadening in the line spectrum should it exist. The photoconversion spectrum from the 22-mg/cm² foil is shown in Fig. 3, as is also the Compton electron spectrum from a thick aluminum converter. The absolute yield of the radiation obtained from the latter measurement, using Eq. (11), was 13.2×10^{-6} γ-quanta per deuteron at 1.46-Mev bombarding energy. An independent determination, using a 27.2-mg/cm², "thin" aluminum converter and Eq. (7) gave 13.3×10^{-6} γ-quanta per deuteron. Both measurements were made with graphite targets sufficiently thick to stop the deuterons.

The positron spectrum from a 17-mg/cm² graphite target, bombarded by 1.46-Mev deuterons, is shown in Fig. 4. The expected contribution of externally formed pairs from the target and support is of the order of one or two percent of the observed effect. Also indicated in the figure, by the dashed curves, are the theoretical distributions for several multipole orders, normalized in accordance with the measured γ-ray yield and added to the background. This background, determined from the beam-off counting rate in the spectrometer, increases slightly with decreasing positron momentum because of the presence of positrons from gaseous N^{13}. The positron spectrum was also observed through a 1.7-gram/cm² aluminum absorber; a gradual increase in the counting rate starting from about 8475 gauss-cm could be quantitatively ascribed to external pair formation in the "thick" converter, indicating that there were no extraneous sources of positrons from external pair formation in the baffles, spectrometer walls, etc., and that the background as shown in Fig. 4 is correct to ±2 counts at the lowest field setting.

Discussion

In comparing the theoretical and experimental distributions shown in Fig. 4, the uncertainty in the absolute γ-ray yield and possible error in the theoretical curves due to the use of plane wave functions, are estimated to be less than 10 percent, in addition to the background uncertainty of about 5 percent. Therefore, the observed distribution is consistent with only an $E1$ assignment. There is independent experimental and theoretical support[22] for such an assignment, based on the analysis of other data on the mirror levels of C^{13} and N^{13}. If the radiating state has a spin of $\frac{1}{2}$ as expected, both positron and γ-ray angular distributions must be isotropic so that assumption (17) is valid.

The most accurate values for the C^{13*} level position are 3083 ± 5 kev from the measurement[23] of the alpha-particle groups from $N^{15}(d,\alpha)C^{13}$ and 3086 ± 6 kev from the measurement of the proton groups from $C^{12}(d,p)C^{13}$; an average of these values is 3084 ± 4 kev. This average

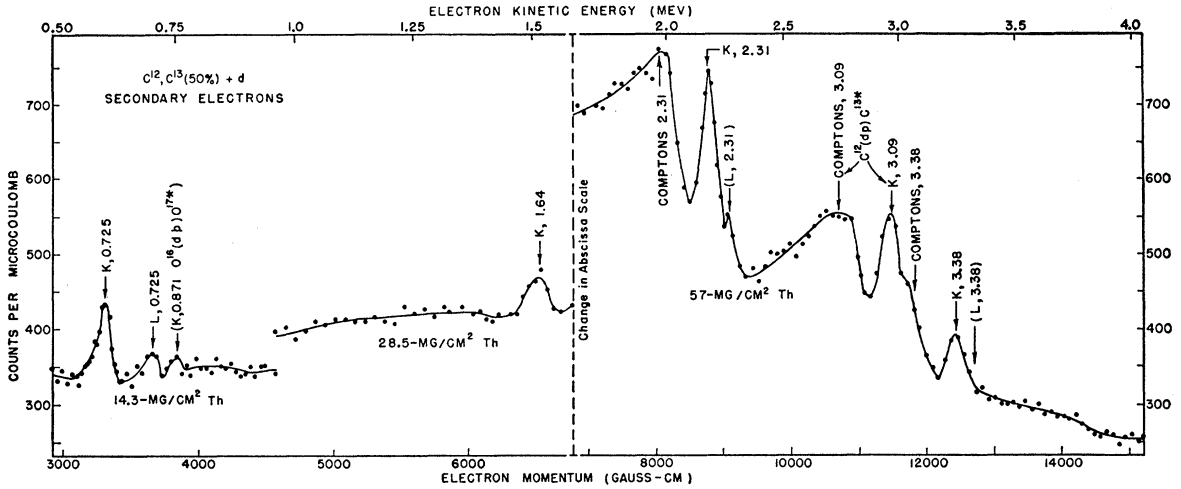

FIG. 5. Secondary photoelectric and Compton electron spectrum from an enriched C^{13} (50 percent) target bombarded by 1.58-Mev deuterons.

[22] R. G. Thomas (to be published).
[23] R. Malm and W. W. Buechner, Phys. Rev. 81, 519 (1951) and D. M. Van Patter (private communication).

agrees with our Doppler corrected value but not with the uncorrected value, indicating that the lifetime of C^{13*} is less than 3×10^{-13} sec, its stopping time in the target material. This observation is consistent with the fact that the corresponding level in N^{13} at 2.37 Mev has a resonance radiative width of 0.63 ev, corresponding to a radiative lifetime of 10^{-15} sec.

B. $C^{13}+d$

The following reactions occur when C^{13} is bombarded with deuterons:

$C^{13}(d,n)N^{14}$	$Q_0=5312$ kev
$C^{13}(d,p)C^{14}$	$Q_0=5940$ kev
$C^{13}(d,\alpha)B^{11}$	$Q_0=5164$ kev
$C^{13}(d,t)C^{12}$	$Q_0=1310$ kev.

In the first three reactions a deuteron energy of 1.5 Mev is sufficient to produce known excited levels of the residual nuclei. For a general survey of the γ-radiation a $C^{12}-C^{13}$ target was made by consolidating soot, enriched to 50 percent in C^{13}, to a thickness of about 30 mg/cm² in a 45-mg/cm² copper cup. To the back of the cup were attached photoconverters of thorium or Compton converters of beryllium or aluminum. Secondary electrons ranging from 0.4 to 4 Mev were observed from thorium foils of thickness 14.3, 28.5, and 57 mg/cm², the thicknesses having been selected so that the energy loss of the secondary electrons in passing through the converter would be comparable to the spectrometer resolution width, yielding optimum sensitivity without significant loss in resolution. The spectra observed at a bombarding energy of 1.58 Mev, are shown in Fig. 5 and the γ-ray energies determined therefrom are listed in Column 1 of Table I. The maximum possible Doppler shifts are given in Column 2. With the exception of the 3.1-Mev γ-ray from C^{13*} (see $C^{12}+d$), it is not known whether or not the nuclei radiate while in motion, so that it is likewise not known whether or not to include the Doppler correction. The uncertainties given in Column 1 for the uncorrected energies are based on the uncertainties in peak locations, and the peak-shift corrections, the additional uncertainties due to the possibilities of Doppler shift, complete or partial, being ignored. The yields obtained from (13), assuming the angular distributions to be isotropic, are listed in Column 3; these yields may be in error by 25 percent or more since a rather large converter angle, $\delta\sim85°$, was used. The relative excitation functions for deuterons ranging from 1.0 to 1.6 Mev are given in Fig. 6.

TABLE I. Energies and thick-target yields of C^{12}, C^{13} (50 percent) $+d$ γ-rays, determined from photoelectric conversion in thorium, at 1.58-Mev bombarding energy. The yields are uncorrected for isotopic abundance, and it is assumed that the angular distributions are isotropic. The energies are not corrected for the possible Doppler shifts δ, which are given in Column 2; these shifts, if they apply, should be subtracted from the energies given in Column 1. Energies are given in kev.

Energy	δ	Yield $(\gamma/d)\times10^6$	Radiating nucleus
725±4	3	1.6	?
1638±8	6	1.5	N^{14}
2310±12	10	5.3	N^{14}
3092±15	15	5.5	C^{13}
3377±15	15	1.8	N^{14}

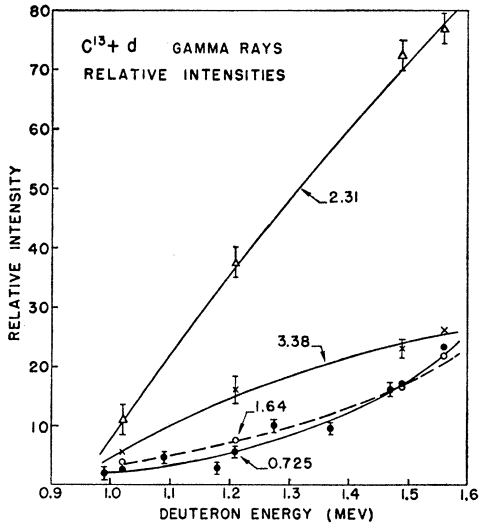

FIG. 6. Excitation functions for thick-target yields of the 0.725, 1.64, 2.31, and 3.38-Mev γ-rays from $C^{13}+d$. The general trend of the individual curves is believed to be correct within the probable errors indicated (shown only for a few typical points) but the relative scales may be subject to a considerably larger systematic error.

Compton conversion is a more sensitive process for detecting γ-radiation above about 3 Mev. Figure 7 shows the Compton spectrum obtained at a deuteron energy of 1.42 Mev from a 94-mg/cm² beryllium converter. The energies obtained by fitting "thick" converter distributions of the form given by Eqs. (10) and (11) to the front edges are given in Column 1 of Table II, and the maximum possible Doppler shifts due to the

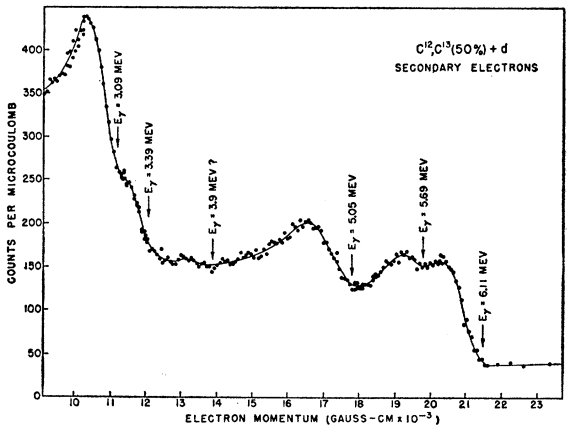

FIG. 7. Compton conversion spectrum from a $C^{12}+C^{13}$ (50 percent) target bombarded by 1.42-Mev deuterons. Converter: 94-mg/cm² Be.

TABLE II. Energies and yields of C^{13} (50 percent), $C^{12}+d$ γ-rays determined from Compton conversion. The energies were measured at a bombarding energy of 1.42 Mev, while the thick target yields were obtained at 1.21 Mev. The yields are uncorrected for isotopic abundance, and it is assumed that the angular distributions are isotropic. The energies are not corrected for the possible Doppler shifts δ_s; these shifts, if they apply, are to be subtracted from the energies given in Column 1. Energies are given in kev.

Energy	δ_s	Yield $(\gamma/d)\times10^6$	Radiating nucleus
3086±20	15	3.81	C^{13}
3390±20	15	1.85	N^{14}
5052±25	26	1.56	N^{14}
5690±50	29	0.69	N^{14}
6110±30	30	2.30	C^{14}

center-of-mass motion are given in Column 2. The yields were obtained from another run at a bombarding energy of 1.21 Mev, in which a 350-mg/cm² aluminum converter was used; the values obtained by the "thick" converter method of analysis are given in Column 3. The uncertainty in these yields is estimated to be about 15 percent, except for the 3.1-Mev line which is less certain because of the sloping background, and the 5.69-Mev line which is less certain because of its low intensity. These energies and yields appear to be in satisfactory agreement with the recent determinations of Baggett and Bame,[24] who used a magnetic pair spectrometer and 1.65-Mev deuterons.

An additional uncertainty in the Compton and photoconversion energy determinations is the possibility of Doppler broadening effects. However, in the (d,n) and (d,p) reactions the broadening will in any case be rather small, the quantity $\delta_b/2$ ranging from about 4 kev for the low energy radiations to about 10 kev for the high energy radiations.

In order to permit the observation of internally-formed positrons, a flake of C^{13} enriched soot, about 30 mg/cm² thick, was mounted in a light frame fabricated from 3.5-mg/cm² aluminum foil. The spectrum obtained from this target at a bombarding energy of 1.21 Mev is shown in Fig. 8. In this experiment there was a continuous production of gaseous N^{13} from the $C^{12}(d,n)N^{13}$ reaction, which gave rise to a field insensitive but somewhat time-dependent background. On this account alternate "beam-off" count readings were taken at each field setting and the background thus obtained subtracted from the "beam-on" count reading for the same time interval. Shown dashed in the figure are the theoretical internally formed positron distributions for $E1$ and $E2$ multipoles predicted on the basis of the γ-ray intensities listed in Column 3 of Table II, these distributions were added to an assumed background from the higher energy distributions. The distributions for all other multipoles follow in close order below that for $E2$. The total contribution from external positron formation in the target and the aluminum frame is about 5 percent of the $E1$ theoretical distribution in each case; no correction was made for this contribution. The positron spectrum was not studied below 5500 gauss-cm because of the intense β^+ activity from N^{13}.

Discussion

In the last columns of Tables I and II we have listed the probable sources of the radiation. The 6110-kev γ-ray is assigned to C^{14} because there was a small but definite yield observed with a deuteron energy of 650 kev, which is below the threshold for such a level in either B^{11} or N^{14}. This assignment has been confirmed by Sperduto *et al.*,[25] who find a level at 6096±9 kev in C^{14} from magnetic analysis of the protons; this value agrees with our determination, with or without the Doppler correction due to the center-of-mass motion. They observed no other proton groups which might correspond to levels in C^{14} from 5.2 to 6.1 Mev with an intensity of greater than 20 percent of that of the group associated with the 6.1-Mev level. This observation may exclude a level at 5.24 Mev in C^{14} reported by Humphreys and Watson[26] from range measurements; it may also exclude a level reported at 5.59±0.04 Mev by Curling and Newton,[27] also from range measurements. The 5.69-Mev γ-ray which was formerly thought to be associated with this level can easily be accommodated in the N^{14} level scheme.

Aside from the 3.1-Mev γ-ray from $C^{12}(d,p)C^{13*}$, the 6.1-Mev γ-ray from C^{14}, and the 725-kev γ-ray which is unassigned, all of the radiations listed in Tables I and II are believed to be from transitions in N^{14}. Three cascades are possible within the uncertainty of the measurements: (1) 1638+3381=5019, (2) 3381+2310 =5691, and (3) 1638+2310=3948, the last being considered since a weak γ-ray of about 3.9-Mev energy

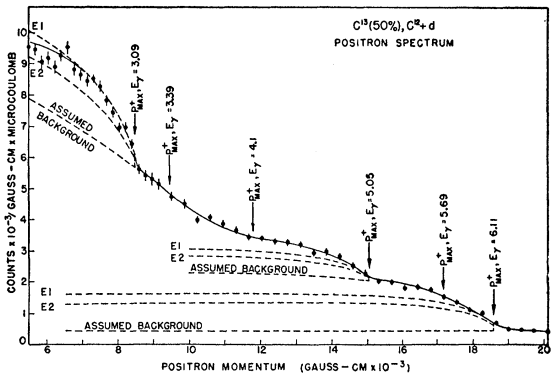

FIG. 8. Internally-formed positron spectrum from C^{12}, C^{13} (50 percent)$+d$. E_d=1.21 Mev; target, 30-mg/cm². Dashed curves labeled $E1$ and $E2$, are theoretical distributions for internal pair conversion, normalized according to the yields indicated in Table II.

[24] L. M. Baggett and S. J. Bame, Phys. Rev. **84**, 154 (1951).

[25] Sperduto, Holland, Van Patter, and Buechner, Phys. Rev. **80**, 769 (1950); Strait, Van Patter, Buechner, and Sperduto, Phys. Rev. **81**, 747 (1951); and Van Patter (private communication).

[26] R. F. Humphreys and W. W. Watson, Phys. Rev. **60**, 542 (1941).

[27] C. D. Curling and J. O. Newton, Nature **165**, 609 (1950).

cannot be excluded and since such a level is indicated by observed particle groups. From magnetic spectrometer analysis of α-particles from the bombardment of O^{16} with 6.8-Mev deuterons, Ashmore and Raffle[28] find levels in N^{14} at 5.70, 5.01, and 3.95-Mev, all uncertain by 0.08 Mev. From inelastic scattering of protons, Heydenburg, Phillips, and Cowie[29] report levels at 2.35 and 3.95 Mev, while Arthur *et al.*[30] report a possible additional level at 3.80 Mev. Mandeville and Swann[31] have observed the neutron groups from $C^{13}+d$ at 1.43-Mev bombarding energy: they find levels in N^{14} at 2.19, 3.47, 3.87, and 4.90 Mev, with probable errors of 0.07 Mev. Benenson,[32] who used a bombarding energy of 3.9 Mev, has reported neutron groups corresponding to levels at 2.23, 3.85, 4.80, 4.97, 5.5 (uncertain), 5.78 Mev, and several others at higher energies. A level scheme for N^{14} incorporating these data is exhibited in Fig. 9 together with the most plausible assignment of the γ-transitions. Cascade (1) has been eliminated because of lack of definite evidence for a 3.4-Mev state and because it seems unlikely that it can occur together with (2) and (3). The existence of cascades (2) and (3) requires that $Y(3.38)+Y(1.64) \leqslant Y(2.31)$: from Table I we find $Y(3.38)+Y(1.64)=3.3$ and $Y(2.31)=5.3\times10^{-6}\gamma/d$, so that the inequality is satisfied. Li and Whaling[33] have observed alpha-particles from $C^{13}+d$ which they attributed to the reaction $C^{13}(d,\alpha)B^{11*}$ leaving B^{11} in an excited state at 2107 ± 17 kev; such a level has also been observed in other reactions. A rough estimate indicates that the corresponding γ-ray would have only about one-tenth of the intensity of the neighboring 2.3-Mev line. The present data do not permit a definite statement of its presence or absence.

We have been unable to make an assignment for the 725-kev γ-ray. The possibility that it is a level at $6.11+0.73=6.84$ Mev in C^{14}, say of spin 0^- for which one-quantum radiation to the ground state would be forbidden, is excluded by the indication in Fig. 6 that its threshold lies below a deuteron bombarding energy of $(15/13)(6.84-5.94)=1.04$ Mev. It would appear not improbable that the 725-kev radiation is a part of a cascade transition involving some of the levels observed in the neutron spectrum which do not otherwise appear in the present work.[34]

In view of the evidence supporting the assumption that nuclear forces are charge-independent, it is of interest to look in N^{14} for the level corresponding to the 6.1-Mev level of C^{14}. On the basis of the beta-decay of

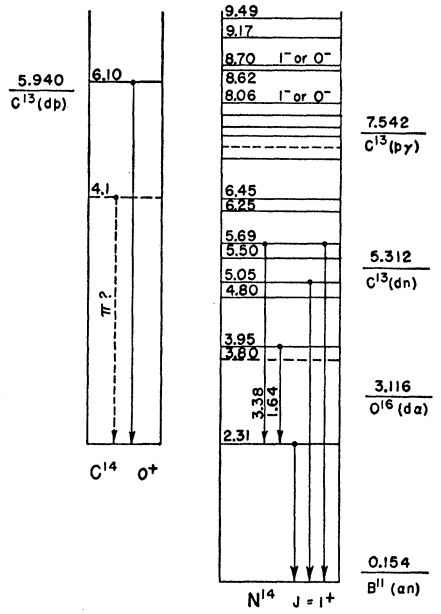

FIG. 9. The energy levels of C^{14} and N^{14} and the gamma-rays from $C^{13}+d$.

O^{14} and consideration of the Coulomb energies, Sherr, Muether, and White[35] conclude that the 2.31-Mev level of N^{14} is the analog of the ground state of C^{14}. Neglecting for the moment the contribution to level displacements from the differences of the boundary conditions on the nuclear surfaces, the mirror level in N^{14} is expected to appear at an excitation energy of $6.110+2.310=8.420$ Mev, which would correspond to a $C^{13}+p$ resonance at a proton bombarding energy 0.94 Mev. The nearest reported level is a narrow resonance at 1.16 Mev.[36] However, the well-known 554-kev resonance has a rather large reduced width which could give rise to an appreciable level displacement, as is observed in the case of the first excited states[22] of the mirror nuclei, N^{13} and C^{13}. If the variation of the level shift is taken into account, an s-wave assignment for the protons forming this state is the only one which yields a non-negative value for the reduced width, which is then 0.48 in units of $\hbar^2/2Ma$ for a channel radius $a=4.32\times10^{-13}$ cm. An s-wave assignment has also been made by Devons and Hine[37] on the basis of the isotropy of the capture γ-radiation. Using this reduced width in the one-channel, one-level approximation, the position of the mirror level in C^{14} is predicted to be 6.05 Mev. The small discrepancy of 60 kev may be due to the possible differences of the internal Coulomb energies of the various levels involved or to the approximation. Since the 554-kev resonance in $C^{13}+p$ is due to s-wave protons, it can have either spin 1 or 0 and odd

[28] A. Ashmore and J. F. Raffle, Proc. Phys. Soc. (London) **A64**, 754 (1951).
[29] Heydenburg, Phillips, and Cowie, Phys. Rev. **85**, 742A (1952).
[30] Arthur, Allen, Bender, Hausman, McDole, Diana, Rhodes, and Bajon, Phys. Rev. **87**, 237 (1952).
[31] C. E. Mandeville and C. P. Swann, Phys. Rev. **79**, 787 (1950).
[32] R. E. Benenson, Phys. Rev. **87**, 207 (1952) and private communication.
[33] C. W. Li and W. Whaling, Phys. Rev. **82**, 122 (1951) and private communication.
[34] H. T. Richards (private communication).
[35] Sherr, Muether, and White, Phys. Rev. **75**, 282 (1949).
[36] J. D. Seagrave, Phys. Rev. **85**, 197 (1952).
[37] S. Devons and M. G. N. Hine, Proc. Roy. Soc. (London) **A199**, 56 (1949).

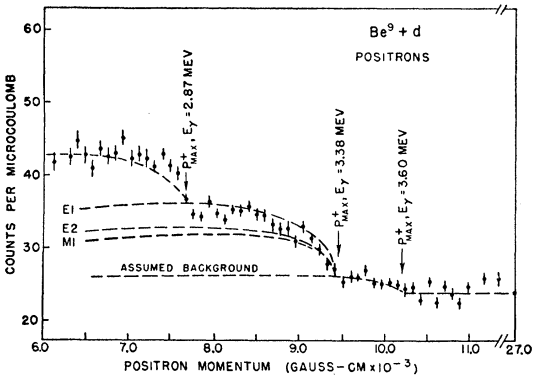

FIG. 10. Internally formed positron spectrum from Be^9 bombarded by 1.19-Mev deuterons. Target: 15-mg/cm² Be. The dashed curves marked $E1$, $E2$, and $M1$ are theoretical distributions for the corresponding multipoles, normalized according to the yields determined from Fig. 11.

parity. However, a zero-spin assignment cannot be given to the 6.11-Mev level of C^{14} because one-quantum emission to the spin-zero ground state would be strictly forbidden, contrary to observation. Therefore, on the basis of this reasoning, these two mirror levels would have a 1^- assignment, and the radiation from the C^{14} level would be $E1$. The internally formed positron distribution from this level as shown in Fig. 8 is in closest agreement with this assignment, but other assignments, such as $E2$ and $M1$, cannot be excluded because of the uncertainty in the γ-ray intensity and in our estimate of the background.

There are indications in Fig. 8 of internally formed positrons from the 5.69- and 5.05-Mev transitions, as well as the 3.1-Mev transition of C^{13} from $C^{12}+d$; the low intensity and uncertainty of the background and γ-ray intensity again preclude the determination of multipolarities for these transitions. There is a marked rise in the positron intensity which is associated with a transition energy of about 4.1 Mev. It is not entirely clear how this rise is to be accounted for, since only a very weak, if any, γ-radiation was observed at this energy. There is the possibility of a nuclear pair emitting state in C^{14} similar to the one in O^{16}; the observed slow rise of the distribution is characteristic of nuclear pairs as distinct from the abrupt rise which characterizes the more common internal or external pairs (see Fig. 1 of reference 3). Although no level has been reported in C^{14} below 5.2 Mev from particle-group measurements, it does not appear that any experiment has been performed which would have revealed a group associated with such a level; in the work of Humphreys and Watson[26] as well as that of Bennett *et al.*[38] such a group would have been obscured by the intense proton group from $C^{12}(d,p)C^{13}$, and Sperduto *et al.*[25] did not search for levels below 5.2 Mev.

[38] Bennett, Bonner, Richards, and Watt, Phys. Rev. **59**, 904 (1941).

C. Be^9+d

The γ-radiation from the bombardment of Be^9 by 1 to 1.5-Mev deuterons has been observed by Rasmussen *et al.*[39] Three of the high energy transitions at 2.87, 3.38, and 3.60 Mev arising from levels in B^{10}, Be^{10}, and B^{10}, respectively, are sufficiently intense to permit the detection of the internally formed positrons. The reactions involved are

$$Be^9(d,n)B^{10} \qquad Q_0=4.360$$
$$Be^9(d,p)Be^{10} \qquad Q_0=4.585.$$

The positron spectrum from the bombardment of a 15-mg/cm² Be target by 1.19-Mev deuterons is shown in Fig. 10, and the thick-converter Compton spectra from 350 mg/cm² of Al for the 3.60- and 3.38-Mev γ-rays are shown in Fig. 11. The dashed curves in the figure are calculated from Eq. (11). Table III lists the intensities determined from this calculation together with the values obtained by Rasmussen at the same bombarding energy using the thin converter method. We did not measure the intensity of the 2.87-Mev γ-ray since the background from the two higher-energy lines was rather intense for the thick converter method. In Fig. 10 we have indicated the calculated distributions for internally-formed positrons from the 3.38-Mev transition for $E1$, $E2$, and $M1$ multipoles, with a normalization based on the gamma-ray yield obtained from Fig. 11. The theoretical distributions for higher multipoles lie in close sequence below $M1$. Since the normalization does not depend upon the absolute value of Ω, it is somewhat more accurate than the yield. Unfortunately the uncertainty in the background in Fig. 10 is such that we cannot say definitely whether the transition is $E1$ or $E2$; $M1$ seems, however, less likely, and higher multipoles would appear to be excluded.

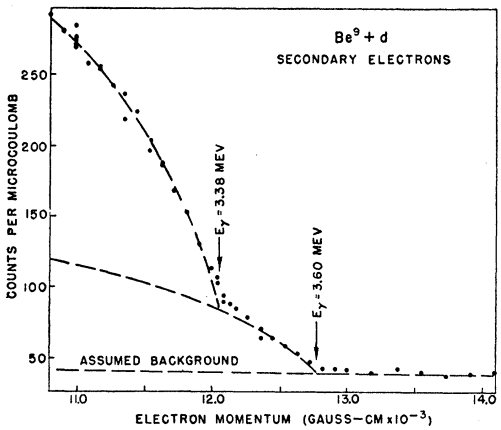

FIG. 11. Thick-converter Compton electron spectrum from Be^9+d. $E_d=1.19$ Mev: converter, 350-mg/cm² Al. Dashed curves are theoretical fits, using Eq. (11).

[39] Rasmussen, Hornyak, and Lauritsen, Phys. Rev. **76**, 581 (1949); and V. K. Rasmussen, Ph.D. thesis, California Institute of Technology (1950) (unpublished).

Discussion

The multipole order of the 3.38-Mev transition in Be^{10} is of particular interest since it bears on the determination of the spin and parity of the first excited state of Be^{10}. If the transition is $E1$, the 3.38-Mev state has total angular momentum 1 and odd parity, assuming the ground state to be 0, (even): $E2$ radiation implies $J2$, (even). Indications from the angular distributions of the protons[40] are that both states have even parity, which is consistent with $E2$ but not with $E1$. Cohen *et al.*[41] find that the angular correlation of short-range protons and γ-rays is consistent with $J=2$ for the excited state.

D. $N^{15}+p$

Bombardment of N^{15} with protons leads to production of the ground state and first excited state of C^{12} through the reaction

$$N^{15}(p,\alpha)C^{12} \qquad Q_0=4.961 \text{ Mev.}$$

The target in the present experiment consisted of a disk of titanium, nitrided on one surface with nitrogen enriched to 32 percent in N^{15}; the TiN layer was thick

TABLE III. Be^9+d thick target γ-ray yields for $E_d=1.19$ Mev.

Reaction	Energy	Yield[a]	Yield[b]
$Be^9(d,n)B^{10}$	2.87	2.4±0.6	...
$Be^9(d,p)Be^{10}$	3.38	2.1±0.5	2.5 ±0.3
$Be^9(d,n)B^{10}$	3.60	0.9±0.2	0.96±0.15

[a] Rasmussen (see reference 39) thin converter. [b] This work, thick converter. Yields are in units of 10^{-6} γ/d. The radiations are assumed to be isotropic.

enough to constitute a thick target for the incident proton beam and the disk was a thick converter for the secondary Compton electrons. The Compton spectrum from the bombardment with 1.63-Mev protons is shown in Fig. 12. By fitting this spectrum and a similar one taken at $E_p=1.20$ Mev with a thick-converter Compton distribution, appropriately convoluted with the spectrometer resolution curve, we find the γ-ray energy to be 4463±20 kev, uncorrected for Doppler effects. If the radiation is emitted before the C^{12*} nuclei are significantly decelerated, a Doppler correction of 14 kev due to the center-of-mass motion is to be subtracted. In addition, the front edge of the observed Compton distribution will be modified by the Doppler broadening ($\delta_b/2=20$ kev), requiring a further correction of 6 kev. The corrected energy of the first excited state of C^{12} is then 4443±20 kev. The thick-target yields computed from the Compton spectra were $6.3\times10^{-8}\gamma/p$ at $E_p=1.20$ Mev and $8.7\times10^{-8}\gamma/p$ at $E_p=1.63$ Mev, assuming the angular distribution of the radiation to be isotropic. Analysis of the elastically scattered proton

[40] F. A. El Bedewi, Proc. Phys. Soc. (London) **A65**, 64 (1952); C. F. Black, Phys. Rev. **87**, 205 (1952).
[41] Cohen, Shafroth, Class, and Hanna, Phys. Rev. **87**, 206 (1952).

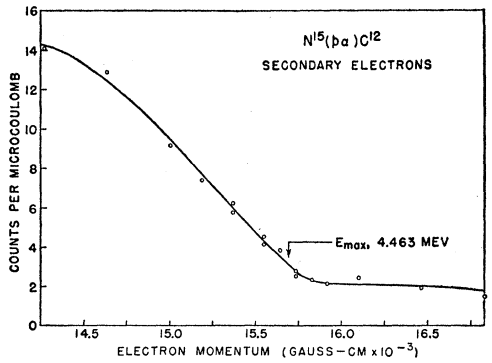

FIG. 12. Thick-converter Compton electron spectrum from N^{15}-$(p,\alpha)C^{12}$. $E_p=1.63$ Mev; converter, ~300-mg/cm² Ti.

groups from the target[42] gave the composition near the surface as 1 nitrogen atom per 1.45 titanium atoms; taking the stopping power of Ti to be 2.1 times that of nitrogen, the above yields would be 13 times larger in pure N^{15}.

Discussion

Measurements have been made in this laboratory by Schardt[42] of the Q of the low energy alpha-particle group from $N^{15}+p$ and by Li and Whaling[43] of the Q of the ground-state group. The values obtained were 529±8 and 4961±6 kev, respectively, giving 4432±10 kev for the energy of the C^{12} level. The level energy can also be obtained by using the Q of the reaction $N^{14}(d,\alpha)C^{12*}$ as measured by Malm and Buechner[23] together with other accurately known Q's: thus

$$Q[N^{14}(d,p)N^{15}]+Q[N^{15}(p,\alpha)C^{12}]-Q[N^{14}(d,\alpha)C^{12*}]$$
$$=(8608\pm9)+(4960\pm4)-(9137\pm6)$$
$$=(4431\pm12) \text{ kev.}$$

These particle group values are in agreement with our Doppler corrected value but not with the uncorrected value, suggesting that the C^{12*} lifetime is shorter than the stopping time of about 3×10^{-13} sec. There is some evidence that the first excited state of C^{12} has a spin of two with even parity[44]; since the ground state is spinless and presumably of even parity, the transition should be $E2$. For such a transition the lifetime estimates of Bethe[45] and Goldhaber-Sunyar[46] are $\sim10^{-15}$ and $\sim10^{-14}$ sec, respectively, both of which are consistent with the evidence for the Doppler correction.

E. $B^{10}+p$

The bombardment of B^{10} with protons was first shown by Brown *et al.*[47] to yield an α-particle group

[42] Schardt, Fowler, and Lauritsen, Phys. Rev. **86**, 527 (1952).
[43] Li, Whaling, Fowler, and Lauritsen, Phys. Rev. **83**, 512 (1951).
[44] R. Haefner, Revs. Modern Phys. **23**, 228 (1951); A. Kraus and A. P. French (unpublished).
[45] H. A. Bethe, Phys. Rev. **55**, 436 (1939).
[46] M. Goldhaber and A. W. Sunyar, Phys. Rev. **83**, 1073 (1951).
[47] Brown, Snyder, Fowler, and Lauritsen, Phys. Rev. **82**, 159 (1951).

corresponding to an excited state of Be^7 at 434.4±4 kev. We have measured the accompanying radiation with the spectrometer using a thick target of amorphous B^{10}, pressed into a shallow cup of 14-mg/cm² thorium foil. The resulting momentum spectrum of photoelectrons, together with a calibration line from annihilation radiation is shown in Fig. 13. The mean of several such determinations yields an energy of 429.8±1.5 kev for the $(B^{10}+p)$ γ-ray. The thick target yield, as determined from the area beneath the conversion spectrum, is $Y=1.7\times10^{-6}\gamma/p$ at a bombarding energy of 1.41 Mev. If the Be^{7*} radiates before it is significantly decelerated in the target material, the Doppler shift due to the center-of-mass motion will be 1.3 kev and the transition energy 428.5±1.8 kev. According to the measurement of Elliott and Bell,[15] the lifetime of the mirror level of Li^{7*} is 0.75×10^{-13} sec; since only a slightly longer lifetime is to be expected for Be^{7*} and since the stopping time is about 5×10^{-13} sec, the Doppler correction is required.

Discussion

In an earlier communication[48] the gamma-radiation from $B^{10}+p$ was ascribed to the reaction $B^{10}(p,p')B^{10*}$ and was considered to establish that a line of energy 413 kev observed in the reaction $Be^9(d,n)B^{10*}$ represented the transition from the lowest excited state of B^{10}. Reference to the dashed curve of Fig. 13, where the K-conversion spectrum from this line is directly compared with the $(B^{10}+p)$ K-line, reveals that the two energies differ by 17 kev and that hence the earlier assignment was incorrect.

The measurements of the excitation energies of Be^{7*} have been reviewed in the paper by Brown *et al.* Subsequent accurate determinations, from $Li^7(p,n)Be^{7*}$ reaction are 431±5 kev[49] and 434±1 kev[50] and from $B^{10}(p,\alpha)Be^{7*}$, 430±3 kev.[20] Our determination appears to be slightly low, especially with the required Doppler correction. An extreme asymmetry in the Doppler broadening ($\delta_b=6$ kev) could account for the discrepancy but this seems unlikely for a (p,α) reaction. There appear to be no angular distribution measurements of the α-particles from this reaction, although it is noted that the yield of α-particles at 140° is nearly the same as the yield of γ-radiation at 90°,[47] the latter distribution being presumably isotropic, since Be^{7*} very likely has a spin of $\frac{1}{2}$.

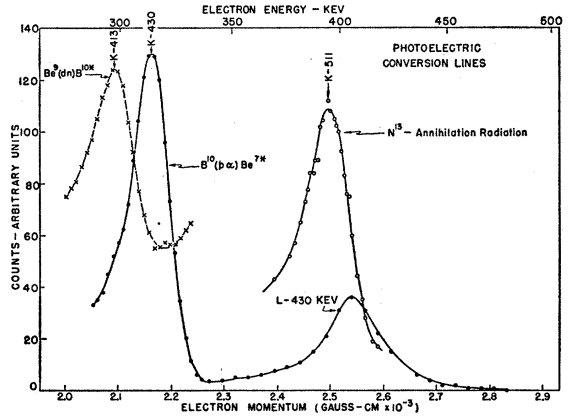

FIG. 13. Photoelectric conversion lines from $Be^9(d,n)B^{10}$, $B^{10}(p,\alpha)$-Be^7 and annihilation radiation. Converter, 14-mg/cm² Th.

[48] Rasmussen, Hornyak, and Lauritsen, Phys. Rev. **76**, 581 (1949).

[49] C. H. Johnson and H. H. Barschall, Phys. Rev. **81**, 317 (1951).

[50] H. B. Willard and W. M. Preston, Phys. Rev. **81**, 480 (1951) (only the statistical and reading error is quoted).

F. Li^6+d

In the hope of obtaining a precise value for the displacement of the first excited states of the mirror nuclei, Be^7 and Li^7, we investigated the γ-ray spectrum from the bombardment by 1.5-Mev deuterons of a thick target of LiCl, the Li being enriched[51] to 96 percent in Li^6. Figure 14 displays the two prominent K-conversion lines observed with an 8.1-mg/cm² thorium converter. From calibrations using annihilation radiation as well as the thorium I line, the energies are found to be 430.0±1.5 kev for $Li^6(d,n)Be^{7*}$ and 478.5±1.5 kev for $Li^6(d,p)Li^{7*}$, uncorrected for possible Doppler effects. The latter determination agrees with previous spectrometer measurements in this laboratory from other reactions.

A striking feature of these mirror reactions is the near-equality of the intensities of the two radiations; correction of the peak heights for the difference of the photoconversion cross section shows that the radiations are of equal intensity to within 10 percent. The absolute yields were determined from a thick target of $(Li^6)_2SO_4$ bombarded with 1.49-Mev deuterons and was found to be $1.0\times10^{-6}\gamma/d$ for each line. Interpolating between available stopping-power data, the absolute yields for pure Li^6 would be about 14 times greater.

Referring again to Fig. 14, it appears that an upper limit of about 5 percent can be placed on the relative intensity of any other line below 420 kev, down to 100 kev, since the photoconversion efficiency is increasing so rapidly with decreasing γ-ray energy that even below the K-binding energy of 110 kev, the L conversion lines would give a sensitive indication of any gamma-radiation. In the region above the 479-kev line up to 1 Mev, an upper limit of about 25 percent can be set. The background in this region is partly due to Li^8 electrons from residual Li^7 in the target and partly from the strong 871-kev line of $O^{16}+d$, ascribed to oxygen present in the water of crystallization. The identification of this line was checked with a $(Li^6)_2SO_4$ target where it is about five times as strong as the lithium lines. From 1 to 6 Mev no γ-radiation was found but, because the Li^8 spectrum was quite intense in this region, we could not have detected any γ-radiation whose yield is less than $10^{-6}\gamma/d$.

[51] We are indebted to the Isotopes Division, AEC for the loan of this material.

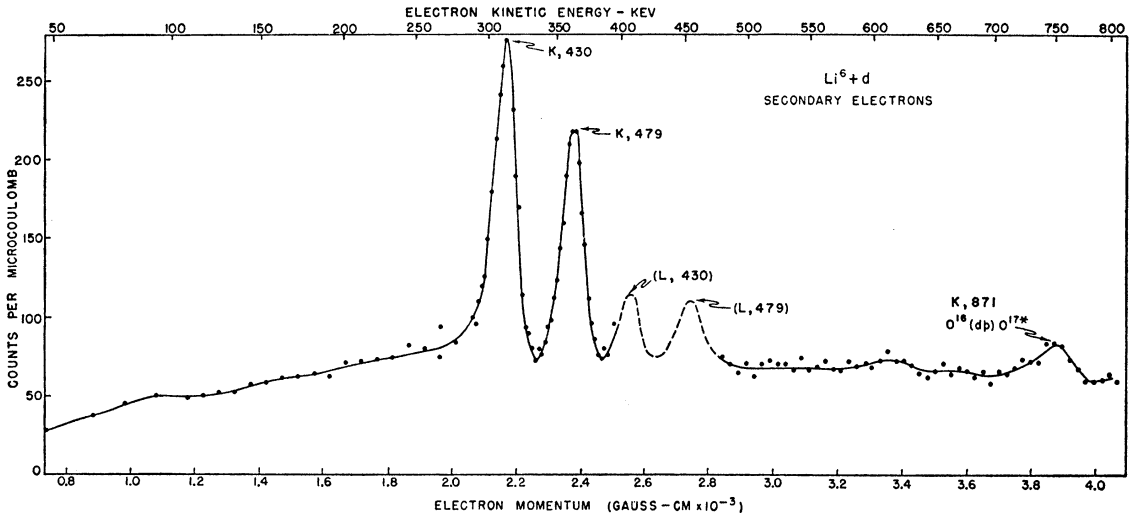

FIG. 14. Photoelectric conversion lines from $Li^6(d,p)Li^7$ and $Li^6(d,n)Be^7$. $E_d=1.49$ Mev; converter, 8.1-mg/cm² Th.

Discussion

As in the case of the $B^{10}(p,\alpha)Be^{7*}$ γ-ray, Doppler corrections are required: the corrections to be subtracted for the center-of-mass motion are 2.6 kev for Be^{7*} and 2.9 kev for Li^{7*}. Whaling and Bonner[52] have measured the angular distribution of the short range protons from Li^6+d for bombarding energies below 1.4 Mev and observe considerable asymmetry. By using their angular distribution at 1.4 Mev to determine the Doppler broadening distribution ($\delta_b=7$ kev) and convoluting this distribution with the spectrometer resolution curve, we find an effective "red" shift of 1.8 kev. The angular distribution of the short-range neutrons from the mirror reaction have not been reported but assuming it to be the same as that of the protons, we obtain for Be^{7*} ($\delta_b=6$ kev) an effective "red" shift of 1.5 kev. The broadening shifts thus partially cancel the shifts due to center-of-mass motion, and the corrected transition energies are 428.9 and 477.4 kev, with a possible error of ± 2 kev in each case, considering the uncertainty in the Doppler corrections. The Be^{7*} energy agrees with our value from the $B^{10}+p$ reaction so that the same comments in this connection given in the discussion of that reaction also apply here. A mean of the Li^{7*} energy determinations from particle group measurements and radiation determinations from other laboratories (involving no Doppler effects) is 478.5 ± 0.5 kev; our value, though low, is in agreement within the combined stated errors. The level displacement, which is less subject to errors in the Doppler corrections and photopeak shifts, is 48.5 kev with an uncertainty of about ± 1.0 kev.

G. $O^{16}+d$

An intense, low energy γ-radiation is observed in the deuteron bombardment of O^{16} from the reaction

$$O^{16}(d,p)O^{17}, \qquad Q_0=1.918.$$

The energy of this radiation was obtained from the photoconversion spectrum from 8.1-mg/cm² thorium observed when $(Li^6)_2SO_4$ was bombarded with 1.5-Mev deuterons. By calibrating the spectrometer with the thorium I line as well as with the photoconversion line of annihilation radiation in the same thorium foil, we obtained an energy of 870.5 ± 2.0 kev, uncorrected for the Doppler shift, or 867.0 ± 2.5 when corrected for the motion of the center-of-mass as well as the asymmetry of the angular distribution of the short-range protons[53] ($\delta_b/2=2$ kev).

Because of the high intensity of this transition, it was possible to detect the internal conversion electrons. Figure 15 shows the "no-converter" spectrum in the vicinity of the internal conversion line from the bombardment of an 8-mg/cm² SiO_2 target with 1.35-Mev deuterons; also shown is the thick converter Compton spectrum from 140-mg/cm² Al under otherwise identical experimental conditions. The arrow above the "no-converter line" indicates the position predicted for the peak on the basis of the spectrometer calibration; the observed line has the proper position, shifted slightly due to energy loss within the target, and the proper

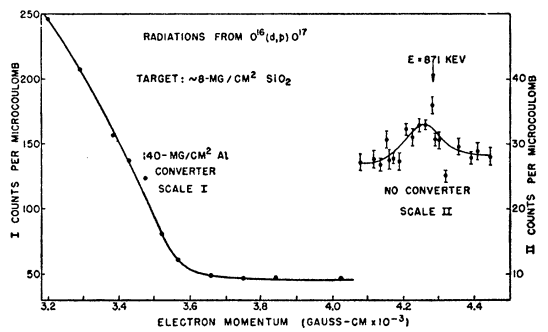

FIG. 15. Thick-converter Compton electron spectrum and internal conversion line from $O^{16}(d,p)O^{17}$. $E_d=1.35$ Mev.

[52] W. Whaling and T. W. Bonner, Phys. Rev. **79**, 258 (1950).

[53] N. P. Heydenburg and D. R. Inglis, Phys. Rev. **73**, 230 (1948).

width for a spectrometer resolution of 2 percent in momentum. From the Compton spectrum the thick target yield from SiO_2 is found to be $Y \approx 6\times10^{-6}\gamma/d$ and by comparing the two spectra, using relations (11) and (16), the internal conversion coefficient from all atomic shells is found to be $(7.1\pm2.6)\times10^{-6}$. The thick target yield is somewhat uncertain because of the difficulty of charge collection on a quartz target; the determination of Γ is not subject to this uncertainty since the two spectra shown in Fig. 15 were obtained for an equal number of counts on a Geiger counter monitor. Since the spin of O^{17*} is very likely $\frac{1}{2}$, assumption (17) in the internal conversion coefficient determination is valid.

Discussion

The theoretical values of the K-shell conversion coefficient as given by Rose *et al.*[10] are (in units of 10^{-6}): $E1=3.71$, $E2=9.15$, $E3=21.2$, $M1=4.87$, $M2=11.90$, $M3=27.2$. In view of the large experimental uncertainty, we need not be concerned with the contribution from the L-shell or with screening corrections. The observed coefficient is consistent with pure $E2$ or a mixture of $E2$ and $M1$. On the basis of angular distribution measurements, the 0.87-Mev level of O^{17} is assigned a spin of $\frac{1}{2}$ (even),[54] and on the basis of spin, magnetic moment, and angular distribution measurements, the ground state is assigned a spin of 5/2 (even).[54,55] Therefore, there is little doubt that the radiation between these two levels should be predominently $E2$, as observed.

If the radiation is indeed $E2$, its lifetime can be estimated as $\sim10^{-11}$ sec from the formula given by Goldhaber and Sunyar[46] or 2×10^{-12} sec from Bethe's formula.[45] These times are longer than the stopping time of about 3×10^{-13} sec so we can be rather certain that the effective K-shell occupation probability for internal conversion is nearly unity. Moreover, the Doppler shift correction should not be included in the energy determination. Our uncorrected energy determination of 870.5 ± 2.0 kev is not, however, in agreement with the most recent value of 880 ± 5 kev for the first excited state of O^{17} as obtained by the MIT group[23] from magnetic analysis of the energies of the protons from this reaction.

It is a pleasure to record our indebtedness to Professors C. C. Lauritsen, R. F. Christy, and W. A. Fowler for many stimulating discussions and much active assistance. We are also grateful to Mr. R. J. Mackin for his assistance in the experiments and in many of the calculations. One of us (RGT) held an AEC predoctoral fellowship during part of the period of this work.

[54] Burge, Burrows, Gibson, and Rotblat, Proc. Roy. Soc. (London) **210**, 534 (1951).
[55] F. Alder and F. C. Yu, Phys. Rev. **81**, 1067 (1951).

APPENDIX: THE EFFECTIVE STOPPING FORCE

The extent to which the distribution in energy and angle of the secondary Compton electrons emerging from the converter is affected by elastic and inelastic collisions in the converter depends, of course, on its thickness. If the thickness of the converter in energy loss units is large compared with the spectrometer resolution width in energy units, it is possible to combine these effects into a single energy dependent quantity μ_e, referred to as the effective stopping force, which enters simply as a direct factor in the yield determination as indicated by Eq. (10). The determination of the effective stopping force involves consideration of the distribution of energy losses for a given path length (straggling) as well as the distribution of path lengths for a given converter thickness; furthermore, it will be convenient to include in the effective stopping force the effect on the yield determination of the angular redistribution of the secondary electrons due to the elastic collisions in the converter. These effects will be regarded as independent of one another and then combined in the final result; this procedure, though approximate, is sufficiently accurate for the present applications.

The most important consideration concerns the distribution of energy losses for electrons which have traveled a certain path length in the material; this distribution for fast electrons has been calculated approximately by Landau[56] and more recently in greater detail by Schultz,[57] and Blunck and Leisegang.[58] The characteristic feature of the distribution is the most-probable energy loss, and except for a minor correction, it is completely specified in terms of this quantity. Thus, the detailed consideration of the properties of the stopping material, such as the distribution of atomic oscillators, can be confined to the determination of the most probable loss.

The Most Probable Energy Loss

The most probable energy loss for electrons of velocity v traveling a path length l in a condensed medium is given by[59]

$$\Delta E_{\rm mp}=\mu_{\rm mp}l=Bl\left[\log\frac{2Blm_0v^2\gamma^2}{I(\gamma)^2}-(1-F_p)\beta^2+0.37\right], \tag{19}$$

where $B=2\pi NZe^4/m_0c^2\beta^2$, $\beta=v/c$, $\gamma=(1-\beta^2)^{-\frac{1}{2}}$, and N is the number of atoms of atomic number Z per unit volume. The quantity $I(\gamma)$ is the average excitation

[56] L. Landau, J. Phys. (U.S.S.R.) **8**, 201 (1944).
[57] W. Schultz, Z. Physik **129**, 530 (1951).
[58] O. Blunck and S. Leisegang, Z. Physik **128**, 500 (1950).
[59] This equation is derived from expressions given by N. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. XVIII, No. 8 (1948), with polarization effects included in accordance with a prescription by A. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. XXIV, No. 19 (1948), and using the limiting energy loss, $Ble^{0\ 37}$ as given by Landau (see reference 56).

potential, defined by

$$\log I(\gamma)=F_p \log(\gamma\hbar\omega_p)+\sum_{\omega_i>\gamma\omega_p} f_i \log I_i,$$

where I_i is the excitation energy of the ith atomic transition and

$$F_p=\sum_{\omega_i<\gamma\omega_p} f_i$$

is the sum of the fractional oscillator strengths for all oscillators with frequencies less than γ times the polarization frequency,

$$\omega_p=(4\pi NZe^2F_p/m_0)^{\frac{1}{2}}.$$

In principle, the evaluation of (19) requires complete knowledge of the frequency distribution of the quantum mechanical oscillators of the stopping material, which can be obtained approximately from x-ray absorption data. Christy[60] has found that, within the accuracy of the somewhat limited experimental data, a reasonably good fit for platinum is given by

$$F(\omega)=\sum_{\omega_i<\omega} f_i=\frac{\omega}{\omega_0+\omega}, \tag{20}$$

the parameter ω_0 being adjusted to obtain the measured value $I(1)$. With the distribution represented by (20), it follows from a straightforward calculation that

$$I(\gamma)=\hbar\omega_0+\gamma\hbar\omega_p=\gamma\hbar\omega_p F_p^{-1},$$
$$F_p=[(1+\xi\gamma^2)^{\frac{1}{2}}-1][(1+\xi\gamma^2)^{\frac{1}{2}}+1]^{-1},$$

and

$$\mu_{\mathrm{mp}}=B[\log(lF_p/a_0)-(1-F_p)\beta^2+0.37], \tag{21}$$

where

$$\xi\equiv 16\pi NZe^2/m_0\omega_0^2, \quad a_0\equiv\hbar^2/m_0e^2.$$

Of course the details of the absorption spectrum are not reproduced by (20), but it was observed that the deviations of (21) from the determination for aluminum using the x-ray data[61] were less than 1 percent for all $\gamma\gtrsim 1$. A mean of the recent determinations[62] of $I(1)$ for aluminum is 155 ev, which yields a value of $\hbar\omega_0=148$ ev for the parameter in (20) and a value of $\xi=0.195$. The values of μ_{mp} given by (21) with this value of ξ are in good agreement with the recent measurements of Chen and Warshaw.[63]

Since the effects of scattering in a thick aluminum Compton converter become important for electrons whose energies are less than about 1 Mev, it is sometimes desirable to use a lighter material such as beryllium. Although no x-ray absorption data is available for Be, the theoretical expression, corrected in the usual manner for inner and outer screening, should provide an accurate description of the actual K-shell absorption for such a simple element. Moreover, a knowledge of the L-shell absorption spectrum is unnecessary in view of A. Bohr's[59] observation that the two valence electrons will be effectively bound by the polarization forces for all $\gamma\gtrsim 1$. The most probable energy loss was evaluated in this way and compared with (21) in which $\xi=0.95$ as obtained from the measured value $I(1)=64$ ev[64]; the differences were less than 2 percent for all $\gamma\gtrsim 1$. The energy losses given by (21) are in agreement with the measurements by Chen and Warshaw[63] of the most probable loss of 620-kev electrons in Be foils of various thicknesses.

The Distribution of Energy Losses

According to Landau[56] the probability that an electron whose initial energy is E suffers an energy loss between Δ and $\Delta+d\Delta$ after traveling a distance l in a material can be written as $W(l,\Delta)d\Delta$, where

$$W(l,\Delta)=\phi(\lambda)/lB,$$

$\phi(\lambda)$ being a universal function which is derived and plotted in his paper and whose argument is

$$\lambda=(\Delta/lB)+\lambda'; \quad -\lambda'=(\mu_{\mathrm{mp}}/B)+0.05.$$

For the moment the effects of elastic scattering are neglected so that the path length l is equal to the component distance z along the spectrometer axis (for simplicity, we assume here that $\cos\theta_0=1$). If the number of electrons originating in the conversion process between z and $z+dz$ with initial energies between E and $E+dE$ is $Q(E)dEdz$, then the number of these electrons suffering losses between Δ and $\Delta+d\Delta$ in traveling to the converter surface is given by

$$Q(E)dEdzW(z,\Delta)d\Delta,$$

and the total number generated in a material of thickness t with these initial energies and energy losses is

$$Q(E)dEd\Delta B^{-1}\int_0^t \phi(\lambda)dz/z. \tag{22}$$

If the slowly varying logarithmic dependence of μ_{mp} on $l(=z)$ is neglected, then

$$dz/z=-d\lambda/(\lambda-\lambda'),$$

and the distribution (22) may then be written

$$-Q(E)dE\mu_{\mathrm{mp}}^{-1}d\Delta(\lambda'+0.05)\int_{\lambda(t)}^{\infty}\phi(\lambda)d\lambda/(\lambda-\lambda'),$$

where

$$\lambda(t)=(\Delta/Bt)+\lambda'.$$

For a "thick" converter $t\to\infty$, and the emerging distribution can be written

$$Q(E)dEd\Delta G(\lambda')\mu_{\mathrm{mp}}^{-1},$$

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where

$$G(\lambda')=-(\lambda'+0.05)\int_{\lambda'}^{\infty}\phi(\lambda)d\lambda/(\lambda-\lambda').$$

The number of electrons emerging with energies between E_0 and E_0+dE_0, where $E_0=E-\Delta$, is therefore

$$\int_{E_0}^{E_{\max}} Q(E)dEdE_0\mu_e^{-1}, \tag{23}$$

where

$$\mu_e=\mu_{\mathrm{mp}}/G(\lambda') \tag{23a}$$

is the effective stopping force. By numerical integration, using the curve and asymptotic expression given by Landau for $\phi(\lambda)$, we find that

$$G(\lambda')=0.765-0.0058\lambda'$$

when $9<-\lambda'<17$, which is the range of λ' occurring in our applications. Thus the effective energy loss $\mu_e l$ is greater than the most probable energy loss but considerably less than the average energy loss. For the analysis of Compton thick converter spectra, μ_e should be evaluated for a thickness corresponding to the middle of the range of energies where the curve of Eq. (11) is fitted. The approximation of neglecting the variation of the logarithmic terms in μ_{mp} leads to an incorrect treatment of the electrons arising from very shallow and very deep laminae in the converter; however, the greater bulk of the electrons is accurately treated and the proper normalization is maintained for all laminae. For very small energy loss, of the order of a few times the atomic excitation energies, Landau's derivation no longer applies, and the somewhat better approximation of Blunck and Leisegang[58] should be used. Under the conditions of the present experiments, for $E_\gamma>1$ Mev in Al or >0.5 Mev in Be, the correction to $G(\lambda')$ is less than one percent.

A derivation of an effective stopping force which is similar to the one given above but uses the experimental results of White and Millington for $W(l, \Delta)$ has been given by Ellis and Aston.[65]

Corrections for Multiple Scattering

As a consequence of multiple scattering in the converter, an electron originating at a distance z within the converter will have traveled a path length $l\geqslant z$ on emergence. Since the energy loss is proportional to l rather than z, there will thus be fewer electrons emerging with an energy E_0 when multiple scattering is taken into account. In the small angle approximation, the distribution function giving the probability that an electron has a path length l after traveling a component distance z along a given direction has been obtained by Yang.[66] According to him (Case I, which corresponds approximately to the present situation) the average path length is given by

$$\langle l\rangle=z[1+(z/\Lambda)],$$

where Λ is the scattering length.[67] If we neglect straggling of energy losses by considering all losses to be given by the effective value $\mu_e l$ and in addition, if we neglect the distribution in path lengths by considering all path lengths to be given by $\langle l\rangle$, then to first order in z/Λ expression (23) is to be multiplied by

$$1-(2\Delta/\Lambda\mu_e). \tag{24}$$

A detailed derivation using Yang's distribution function for the path lengths also gives this result to first order in z/Λ.

Since a flat converter is used, the angular distribution of the most energetic Compton electrons will not be isotropic but will be very nearly $(\cos\theta_0)^{-1}$ for not-too-large angles, and the number of electrons emerging from the converter in the forward direction of the spectrometer acceptance solid angle ($\cos\theta_{1,2}\approx 1$) will therefore be increased as a result of angular redistribu- in the multiple scattering. The redistribution process can be treated in the same manner as the angular redistribution in the conversion process. By carrying out a computation which is similar to (2b) but with $S(\cos\omega)$ as the multiple scattering law, given in the Gaussian small angle approximation by

$$S(\omega, z)=(\Lambda/4\pi z)\exp(-\omega^2\Lambda/4z),$$

we obtain to first order in z/Λ a factor

$$1+(2\Delta/\Lambda\mu_e), \tag{25}$$

by which (23) is to be multiplied to include this effect.

Thus, to first order, the multiple scattering effects leading to (24) and (25) compensate, and the effective stopping force can be taken directly from (23a). Because the factors (24) and (25) are approximate, the compensation cannot be expected to occur in general and it is therefore desirable to minimize both by using a light material as converter. For a 3-percent energy loss in aluminum, the correction term, $2\Delta/\Lambda\mu_e\approx 0.17\gamma^{-1}$ which may be quite significant for low energy electrons. In beryllium the term is only $0.052\gamma^{-1}$ and may be ignored for energies greater than 1 Mev.

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