FIG. 1. Log $[\sigma_f(Z, A)/\sigma_c(Z, A)]$ vs the binding energy of a neutron to (Z, A).

where Γ_f is the probability per unit time that the compound nucleus lose its excitation by fission, and Γ_c is the probability per unit time that the compound nucleus lose its excitation by gamma-ray emission. If Γ_c is a very slowly changing function of the nuclear excitation energy in the region under consideration (4.8 to 6.6 Mev), and if Γ_f is a highly dependent function of nuclear excitation energy in the above energy range, one would expect a correlation of the ratio σ_f/σ_c with neutron binding energies. Implicit in the above reasoning is that the critical deformation energies E_f are roughly constant for the nuclides considered. The difference between the neutron binding energy and E_f would be a better quantity to plot on the abscissa.

The plot of $\log\sigma_f(Z, A)/\sigma_c(Z, A)$ vs the binding energy of a neutron to (Z, A) for several nuclides is shown in Fig. 1. In Table I are listed the cross-section and binding energy data.

From Fig. 1, it is readily seen that the logarithm of the ratio σ_f/σ_c decreases sharply with decreasing neutron binding energy. With two of the three variables (σ_f, σ_c, neutron binding energy) known, the third can be predicted. Using our calculated neutron binding energies,[3] several interesting σ_f/σ_c ratios can be predicted. For example, σ_f/σ_c should be about 0.1 or less for U^{239}. The measured thermal neutron capture cross section of U^{239} is 22 barns.[4] Therefore, the predicted thermal neutron fission cross section of U^{239} is $\leqslant 2$ barns. Likewise, it would be of interest to

TABLE I. Binding energy of a neutron to (Z, A) and the thermal neutron capture and fission cross-sections of several nuclides.

Nuclide (Z, A)	$\sigma_f(Z, A)$[a] barns	$\sigma_c(Z, A)$[a] barns	$\dfrac{\sigma_f(Z, A)}{\sigma_c(Z, A)}$	Neutron binding energy to (Z, A)[b] Mev
Th^{230}	<0.001	45	$<2.2\times10^{-5}$	5.25
Th^{232}	<0.0002	7	$<2.8\times10^{-5}$	4.80
Pa^{231}	0.01	290	3.4×10^{-5}	5.41
U^{235}	545	100	5.45	6.43
Np^{237}	0.019	170	1.1×10^{-4}	5.36
Pu^{238}	20	455	4.4×10^{-2}	5.77
Pu^{239}	664	361	1.84	6.43
Pu^{241}	1060	400	2.65	6.27
Am^{241}	3	884	$\sim5\times10^{-3}$	5.57
Am^{242}	~6000	~2000	3	6.58
Cm^{242}	<5	~25	$<2\times10^{-1}$	5.93

[a] See reference 4.
[b] See reference 3.

measure the thermal neutron capture cross sections of such nuclides as Th^{229} and U^{232} since values[4] are reported for their thermal neutron fission cross sections.

It is also interesting to note that Cm^{242} and Am^{242} show no irregularity[5] in Fig. 1 when the variable σ_f/σ_c is plotted vs neutron binding energy. When the neutron binding energies decrease to values of about 5.4 Mev, the plot of $\log(\sigma_f/\sigma_c)$ vs neutron binding energy shows an irregularity (see Fig. 1). The fission mechanism may be considerably different at these energies, since the neutron binding energy may not be sufficient to bring the excited nucleus above the fission barrier.

Refinements, of course, can be made in Fig. 1 for the different type nuclei. However, the present data do not justify separating the odd compound nuclei from the even type.

It is a pleasure to acknowledge helpful discussion with P. R. Fields, L. B. Magnusson, and M. H. Studier.

[1] N. Bohr and J. A. Wheeler, Phys. Rev. 56, 426 (1939).
[2] S. Frankel and N. Metropolis, Phys. Rev. 72, 914 (1947).
[3] J. R. Huizenga and L. B. Magnusson (unpublished results).
[4] See the compilation of cross sections by Manning, Ghiorso, and Seaborg, The Transuranium Elements: Research Papers (McGraw-Hill Book Company, Inc., New York, 1949), Chapter 20, National Nuclear Energy Series, Plutonium Project Record, Vol. 14B, Div. IV.
[5] G. T. Seaborg, Phys. Rev. 85, 157 (1952).
[6] Koch, McElhinney, and Gasteiger, Phys. Rev. 77, 329 (1950).

The Motion of Slow Electrons in Polar Crystals

TSUNG-DAO LEE* AND DAVID PINES
Department of Physics, University of Illinois, Urbana, Illinois
(Received September 17, 1952)

WE have investigated the low-lying energy levels of a conduction electron in a polar crystal, using the "intermediate coupling" variational technique introduced by Tomonaga.[1] This problem is of considerable interest because of the strong interaction between the electron and the ionic polarization it produces in its motion through the crystal. The electron may be pictured as accompanied by a cloud of phonons (i.e., the associated waves of ionic polarization), and the combined system (electron plus associated phonon cloud) is known as a polaron. The strength of the electron-lattice interaction furnishes a measure of the average number of phonons in the cloud around the electron, and hence of the effective mass of the polaron.

The reduction in the electron energy as a consequence of its interaction with the lattice was first computed semi-classically by Pekar,[2] and by Markham and Seitz,[3] using an adiabatic approximation. Fröhlich, Pelzer, and Zienau[4] calculated this energy and the effective mass of the polaron in a quantum-mechanical treatment which is appropriate provided the electron-lattice interaction is sufficiently weak. Pekar[5] has also given a quantum treatment of these aspects, using the adiabatic approximation. However, in most cases of interest the interaction is so strong that the method of Fröhlich et al. breaks down, and yet the kinetic energy of the electron is only comparable to that of the vibrational quantum, so that the adiabatic approximation is not applicable. Under these circumstances, Tomonaga's variational technique furnishes a promising method of attack.

We adopt the Hamiltonian and notation of Fröhlich, Pelzer, and Zienau, and we work in the Fock representation,[6] in which the Schrödinger function ψ of our system is described by a set of Schrödinger functions corresponding respectively to states of electron and no phonons, electron and one phonon, electron and two phonons, etc. Let $\langle \mathbf{k}_1, \mathbf{k}_2, \cdots, \mathbf{k}_n, \mathbf{r}_e/\psi \rangle$ be the probability amplitude of finding the electron at a position $\mathbf{r}_e$ and n phonons of momenta $\mathbf{k}_1 \cdots \mathbf{k}_n$, respectively in the phonon field. Tomonaga's method consists of assuming a trial function of the form

$$\langle \mathbf{k}_1, \mathbf{k}_2, \cdots, \mathbf{k}_n, \mathbf{r}_e/\psi \rangle = c_n f(\mathbf{k}_1) f(\mathbf{k}_2) \cdots f(\mathbf{k}_n) \chi_n(\mathbf{r}_e), \qquad (1)$$

where $f(\mathbf{k})$ and $\chi_n(\mathbf{r}_e)$ are normalized functions in the $\mathbf{k}$ and $\mathbf{r}_e$ spaces, respectively; c_n^2 thus represents the probability of finding n phonons and one electron in the system. If we further require that ψ be an eigenfunction of the total momentum of the system

P, the form of χ_n is automatically fixed. It is

$$\chi_n(r_e) = \frac{1}{\sqrt{V}} \exp\left\{\frac{i}{\hbar}(P - \sum_{i=1}^{n} \hbar k_i) \cdot r_e\right\}. \quad (2)$$

On varying the energy with respect to $f(k)$ and c_n, the best form of $f(k)$ and a difference equation for c_n can be obtained.[7] Our choice (1) of $\langle k_1, k_2, \cdots, k_n, r_e/\psi \rangle$ reflects our assumption of no correlation between the emission of successive phonons, so that the form of f is the same for all the phonons. Yet, as in any application of Ritz's variational principle, the value of E thus determined will be better than the form of ψ chosen.

We confine our attention here to the low-lying energy levels of the system, such that the total *system* momentum P satisfies $(P^2/2m) < \hbar\omega$, where m is the effective mass of the electron (as a consequence of its interaction with the *periodic* lattice field), and ω the frequency of the optical vibrational mode of the lattice oscillation. Under these circumstances, the difference equation can be solved analytically, and the best form of $f(k)$ turns out to be

$$f(k) = N\{k(k^2 + 2m\omega/\hbar) - [12kP \cos\vartheta/(\alpha+6)\hbar]\}^{-1}, \quad (3)$$

where N is a normalization constant, ϑ is the angle between k and P, and α is the coupling constant of the lattice-electron interaction.

$$\alpha = \left(\frac{e^2}{\hbar c}\right)\left(\frac{mc^2}{2\hbar\omega}\right)^{\frac{1}{2}}\left(\frac{1}{n^2} - \frac{1}{\epsilon}\right),$$

where n is the index of refraction and ϵ the dielectric constant of the crystal. The average number of phonons in the cloud around the electron turns out to be approximately $\alpha/2$. The corresponding energy of the ground state of the system with a total momentum **P** is

$$E = -\alpha\hbar\omega + (P^2/2m)(1+\alpha/6)^{-1} + \cdots + O(P^2/2m\hbar\omega)^2 + \cdots, \quad (4)$$

relative to the energy of a free electron at the bottom of the crystal conduction band.

Thus, for the slow electrons we here considered, the interaction introduces two effects: (1) all electronic energy levels are reduced by $\alpha\hbar\omega$; and (2) the motion of the polaron is that of a free particle with an effective mass $m_{eff} = m(1+\alpha/6)$. In the limit of weak coupling $\alpha \ll 1$, our result reduces to that of Fröhlich *et al.*, and for larger values of α, our result always corresponds to a lower energy. For a typical polar crystal, NaCl, ($\alpha = 5.2$, $\omega = 4.8 \times 10^{13}$ sec^{-1}), we find $E_0 = -\alpha\hbar\omega \cong -0.16$ ev, and $m_{eff} = 1.9$ m. In this case Fröhlich *et al.*, and also Pekar, obtain $E_0 \cong -0.09$ ev. Thus, the Tomonaga scheme provides a better determination of the lowest energy of the system. For NaCl, Pekar[5] and Landau and Pekar[8] found $m_{eff} = 432$. However, as might be inferred from Pekar's much higher value of E_0, this high effective mass is due to the lack of applicability of the adiabatic approximation. A detailed calculation confirms this point.

The interaction of a fast electron with both the acoustic and polarization waves is now under investigation. A detailed account of the above work will be submitted for publication soon.

We thank Professor John Bardeen for many stimulating discussions. One of us (D.P.) would like to acknowledge the support of the Office of Ordnance Research during this work.

* Now at the Institute for Advanced Study, Princeton, New Jersey.
[1] S. Tomonaga, Prog. Theor. Phys. 2, 6 (1947).
[2] S. Pekar, J. Phys. (U.S.S.R.) 10, 341 (1946).
[3] J. Markham and F. Seitz, Phys. Rev. 74, 1014 (1948).
[4] Fröhlich, Pelzer, and Zienau, Phil. Mag. 41, 221 (1950).
[5] S. Pekar, J. Exptl. Theor. Phys. (U.S.S.R.) 19, 796 (1949).
[6] V. Fock, Z. Physik 75, 622 (1932).
[7] Our approach differs from Tomanaga's in two respects. First, we have determined the best form of f by a variation calculation, rather than assuming a definite form for f. (Actually, in the case considered by him, Tomonaga's assumed form for f is also the best one.) Second, we have extended his technique to include the case of a moving particle (here, the electron).
[8] L. Landau and S. Pekar, J. Exptl. Theor. Phys. (U.S.S.R.) 18, 419 (1948).