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Coulomb Functions for Heavy Nuclear Particles

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Values of the regular and irregular solutions and the derivative of the regular solution of the wave equation for repulsive Coulomb potential have been computed by contour integrals at the classical turning point. Results are given for the range $\eta=10$ to $\eta=200$, corresponding to the Coulomb interaction of heavy particles.

COULOMB functions for certain useful ranges of parameters have been computed by Breit *et al.*[1,2] These do not extend above $\eta=30$. For large η's, the steepest descent approximation is very useful[3] but breaks down near the classical turning point. The results of calculations by contour integrals for $\rho=2\eta$ are tabulated in Table I. The quantities F_0, G_0, and F_0' were chosen because they allow quantities for higher L's to be calculated with a minimum of cancellation from recursion formulas.[1] G_0' can easily be obtained from the Wronskian relation, $F_L'G_L-G_L'F_L=1$. Recursion relation 11.5 of reference 1 and its derivative with respect to ρ are useful to obtain values at higher L's. Values of F, G, and F' were also computed by contour integration for the values of L up to $L=4$, and recursion formulas were used to check these values. None of these checks indicated an error greater than 0.1 percent, and many checked to better than 0.01 percent.

TABLE I. Coulomb functions calculated by contour integration for $\rho=2\eta$.

η	$F_0(2\eta)$	$G_0(2\eta)$	$F_0'(2\eta)$
10	1.034	1.797	0.2868
15	1.108	1.923	0.2668
20	1.163	2.016	0.2531
25	1.207	2.092	0.2431
30	1.244	2.157	0.2353
35	1.277	2.213	0.2290
40	1.306	2.262	0.2236
50	1.355	2.348	0.2150
60	1.397	2.420	0.2083
70	1.434	2.483	0.2028
80	1.466	2.539	0.1982
90	1.495	2.589	0.1942
100	1.522	2.635	0.1907
110	1.546	2.677	0.1876
120	1.569	2.716	0.1848
130	1.590	2.753	0.1823
140	1.609	2.787	0.1800
150	1.628	2.819	0.1779
160	1.646	2.849	0.1760
170	1.662	2.878	0.1742
180	1.678	2.906	0.1725
190	1.693	2.932	0.1709
200	1.708	2.957	0.1694

TABLE II. Number of terms, N, required in a series expansion around $\rho=2\eta$ to give a value of G_0 to within 1 percent accuracy at $2\eta-\rho=5$.

η	N	Error in steepest descent formula
20	6	8.1%
100	4	8.4%
200	3	6.2%

Since the Coulomb functions and their first derivatives are given, an expansion around the point $\rho=2\eta$ can be made. Table II lists the number of terms in this expansion required to give a value of G_0 to within 1 percent of the correct value at $2\eta-\rho=5$. It also gives the error in the steepest descent approximation at this point. The steepest descent approximation should improve in accuracy for ρ less than $2\eta-5$.[1] From the table we see that a combination of the steepest descent approximation and the expansion around $\rho=2\eta$ gives G_0 to better than 10 percent in this region of ρ for η's greater than 20.

The values listed in Table I may also be used to start a numerical integration to extend the range in ρ.

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[1] Bloch, Hull, Broyles, Bouricius, Freeman, and Breit, Phys. Rev. **80**, 553 (1950).
[2] Bloch, Hull, Broyles, Bouricius, Freeman, and Breit, Revs. Modern Phys. **23**, 147 (1951).
[3] A. A. Broyles and J. L. Powell, Phys. Rev. **72**, 155 (1947).