Meson-Nucleon Scattering and the Classical Strong-Coupling Theory*

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The ordinary elastic and charge-exchange scattering of mesons by nucleons is considered within the framework of a classical strong-coupling treatment of the charge-symmetric pseudoscalar field.

It has been recently observed[1,2] that the qualitative features of the neutral photomeson production cross section can be understood if one assumes the existence of metastable nucleon isobars[3] of the sort which appear in strong-coupling meson theory.[4] If such excited nucleon states should actually occur, they would manifest themselves in other processes, and, indeed, the much larger cross sections for the scattering of π^+ mesons in hydrogen as compared with π^- mesons[5] have been interpreted by Brueckner[6] in terms of a broad resonance level corresponding to a nucleon isobar of total angular momentum $J=\frac{3}{2}$ and total isotopic angular momentum $I=\frac{3}{2}$.

The current evidence in favor of metastable nucleon states is, of course, hardly conclusive. One must bear in mind that the presence of a resonant meson-nucleon interaction does not of itself necessarily imply the existence of isobars. There is also a good deal of uncertainty at the present time that meson-nucleon scattering does in fact take place predominantly in the $J=\frac{3}{2}$, $I=\frac{3}{2}$ state.[7] Nevertheless, much interest has been focused recently on the strong-coupling theory, and we therefore feel it worthwhile to indicate the qualitative features of meson-nucleon scattering which result from a classical treatment of the charge-symmetric pseudoscalar field.[8]

It must be emphasized from the very outset that, in addition to the usual approximations of strong-coupling theory,[8] the classical method treats the meson field as an unquantized field and the spin $\boldsymbol{\sigma}$ and isotopic spin $\boldsymbol{\tau}$ of the nucleon as classical unit vectors.[9] It is this last restriction particularly which has led to differences in the quantitative comparison of the results of the classical and quantum-mechanical methods, even in the extreme strong-coupling limit.[10] Actually, we are interested in moderately strong or intermediate coupling in which case the classical theory cannot be expected to serve other than as a qualitative guide.

In consequence of the classical treatment of $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$, it follows that one cannot separate the two processes of ordinary elastic scattering of a charged meson by a nucleon

$$\pi^+ + P \rightarrow \pi^+ + P, \tag{1}$$

$$\pi^+ + N \rightarrow \pi^+ + N, \tag{2}$$

from one another, but one is led, instead, to a classical average of the differential cross sections for (1) and (2) which we denote by $(d\sigma/d\Omega)_{\text{ord}}$. The exchange scattering of charged mesons by nucleons, corresponding to

$$\pi^+ + N \rightarrow \pi^0 + P, \tag{3}$$

is separable and leads to $(d\sigma/d\Omega)_{\text{exch}}$. Using natural units with $\hbar = c = 1$, one finds for the differential scattering cross sections the following expressions:[11]

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{ord}} = \frac{4}{225}\left(\frac{f}{\mu}\right)^4 \frac{k^4}{\nu^2|1-R^2|^2}[(25-10\cos^2\theta) + |R|^2(11+52\cos^2\theta)], \tag{4}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exch}} = \frac{4}{225}\left(\frac{f}{\mu}\right)^4 \frac{k^4}{\nu^2|1-R^2|^2}[(10+5\cos^2\theta) + |R|^2(4+3\cos^2\theta)], \tag{5}$$

with

$$R = (\nu^2 - a\mu^3)/\nu\nu_0 + iak^3/\nu\nu_0. \tag{6}$$

Here, k, ν, and μ are the momentum, energy, and mass of the meson, f is the coupling parameter, and θ the angle of scattering. The quantity a is a measure of the nucleon source size, and $\nu_0 = 3a\mu^2/2f^2$ is a measure of

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1 Y. Fujimoto and H. Miyazawa, Prog. Theoret. Phys. **5**, 1052 (1950).

2 K. A. Brueckner and K. M. Case, Phys. Rev. **83**, 1141 (1951).

3 R. E. Marshak, University of Rochester High Energy Physics Program, Final Report, July 1, 1949–August 31, 1950 (unpublished).

4 The term strong-coupling theory is used throughout this note in its generic sense. We are actually concerned with moderately strong or intermediate coupling.

5 Anderson, Fermi, Long, and Nagle, Phys. Rev. **85**, 936 (1952).

6 K. A. Brueckner, Phys. Rev. **86**, 106 (1952).

7 A phase shift analysis of the angular distributions of the scattering of 135-Mev mesons in hydrogen has been carried out by Anderson, Fermi, Nagle, and Yodh [Phys. Rev. **86**, 793 (1952)] under the assumption of charge independence. This has shown the $P_{\frac{1}{2}}$, $I=\frac{1}{2}$ interaction to be comparable to that in the $P_{\frac{3}{2}}$, $I=\frac{3}{2}$ state. More recently, R. E. Marshak (unpublished) and C. N. Yang [Report of the International Physics Conference, Copenhagen, June 3–17, 1952 (unpublished)] have pointed out that the analysis of the preceding authors is not at all unique and that one can equally well have the predominant interaction in the $P_{\frac{1}{2}}$, $I=\frac{3}{2}$ state. In all of these analyses, there is also a large interaction in the $S_{\frac{1}{2}}$, $I=\frac{3}{2}$ state.

8 The quantum-mechanical problem is discussed by G. Wentzel, Phys. Rev. **86**, 437 (1952).

9 J. R. Oppenheimer and J. Schwinger, Phys. Rev. **60**, 150 (1941).

10 W. Pauli and S. M. Dancoff, Phys. Rev. **62**, 85 (1942).

11 For details of the method of calculation, see W. Pauli, *Meson Theory of Nuclear Forces* (Interscience Publishers, Inc., New York, 1946).

the spacing between isobaric states. These formulas involve but two independent parameters which we can take to be ν_0 and a.

If $a\mu \ll 1$ and $a\nu \ll 1$, one can write

$$|1-R^2|^2 \cong (1-(\nu/\nu_0)^2)^2 + 4(ak^3/\nu\nu_0)^2, \qquad (7)$$

whereupon the scattering cross sections will have a resonance at $\nu \sim \nu_0$ with a half-width $\Gamma \sim ak_0^3/\nu_0$. This is precisely the width found by Marshak[3] for the corresponding metastable nucleon state with precession frequency ν_0 in the classical treatment of the neutral pseudoscalar field.

It is interesting to compare the predictions of the classical theory (near the resonance) with those of the quantum-mechanical strong- and weak-coupling approximations (with neglect of nucleon recoil). For the ratio of the integrated cross sections $\sigma_{\text{ord}}/\sigma_{\text{exch}}$, one finds 3, 2.5, and 1.5 for the three theories, respectively; for the angular distributions for ordinary scattering, one has $(1+1.2\cos^2\theta)$, $(1+3\cos^2\theta)$, constant; and, finally, for the angular distribution for exchange scattering, one has $(1+0.57\cos^2\theta)$, $(1+3\cos^2\theta)$, $\cos^2\theta$. We note the qualitative correspondence between the classical and quantum-mechanical strong-coupling theories.

Now, we recall that, in applying the classical strong-coupling theory to the neutral photomeson production process, Brueckner and Case[2] have shown that, for a resonance frequency of 250 Mev and a source radius of the order of the nucleon Compton wavelength, the excitation function is roughly in agreement with experiment. The angular distribution is $\sim(2+1.5\sin^2\theta)$ which is to be compared with the $(2+3\sin^2\theta)$ distribution observed by Silverman and Cocconi[12] for 290-Mev photons. In view of the crudity of the classical model, the general agreement is good.

In Fig. 1 we have plotted the integrated meson-nucleon scattering cross sections as predicted by the classical strong-coupling theory for $\nu_0 = 1.77\mu$ and $a = 1/4\mu$ (the corresponding resonance energy is 250 Mev with 63-Mev half-width); the choice of ν_0 and a corresponds closely to the parameters of Brueckner and Case.[13] If we assume that the charge-exchange scattering cross section of π^- mesons in hydrogen is about twice that of the ordinary scattering of π^- mesons,[14] we can make a rough comparison of σ_{ord} and σ_{exch} with the experimental data. While the energy variation of σ_{ord} and σ_{exch} is reasonably well given by the theory (except in the low energy region where the S-wave scattering is predominant), the absolute magnitudes of the cross sections are uniformly too small by a factor of 2. Although a detailed comparison of the theoretical angular distributions with experiment is difficult to make since our crude model includes no S-wave interaction, the general features of the predictions of the strong-coupling theory are far more encouraging than those of the weak-coupling approximation.

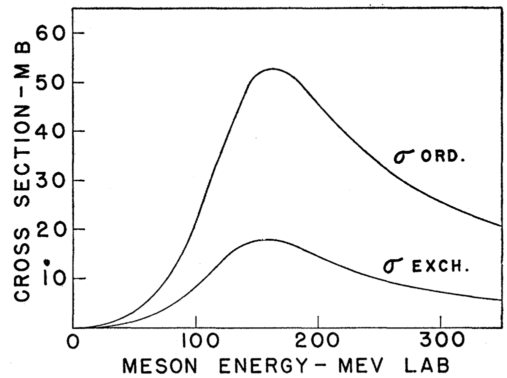

FIG. 1. The variation of the ordinary and exchange scattering cross sections with incident meson energy in the laboratory system. The resonance energy is 250 Mev and the nucleon source radius is $1/4\mu$.

In conclusion, we should like to re-emphasize that, while we have obtained some qualitative agreement with experiment in terms of the classical strong-coupling picture, the evidence for the existence of isobars is hardly definite at the present time, and it may ultimately turn out that the essential connection with reality of the model which we have been using is that it leads to a strong interaction in the $J=\frac{3}{2}$, $I=\frac{3}{2}$ meson-nucleon state.

I should like to thank Professor R. E. Marshak, who stimulated this investigation, for useful discussions.

[12] A. Silverman and G. Cocconi, Report of the International Physics Conference, Copenhagen, June 3–17, 1952 (unpublished).

[13] The strong-coupling condition can be expressed in the form $\nu_0 \ll \mu$; it is therefore clear that we are in fact dealing with an intermediate-coupling theory, as is also the case in the treatment of the photomeson process (see reference 2).

[14] Fermi, Anderson, Lundby, Nagle, and Yodh, Phys. Rev. **85**, 935 (1952); see also the more recent measurements at 135 Mev of Anderson, Fermi, Nagle, and Yodh, Phys. Rev. **86**, 793 (1952).