too reliable since it represents the lower energy limit at which nuclear emulsion data can be used. Both spectra are represented quite well by the function $e^{-E}\sinh(2E)^{\frac{1}{2}}$. However, there is a tendency for the two sets of data to show somewhat more high energy neutrons than the above empirical relation. It is difficult to know if this is a real effect on account of the small number of proton recoil tracks measured in the high energy region.

The author wishes to thank Julia Carlson and Shirley Suttman for their services in reading the nuclear plates used in this experiment.

PHYSICAL REVIEW VOLUME 88, NUMBER 4 NOVEMBER 15, 1952

Nuclear Photoprocesses at High Energy*

J. G. BRENNAN AND R. G. SACHS
Department of Physics, University of Wisconsin, Madison, Wisconsin
(Received July 28, 1952)

In view of recent interest in the influence of pion production on the high energy photonucleon cross sections, an attempt is made to give a systematic discussion of the background effects, i.e., those high energy effects which are not directly concerned with either the production of pions or relativistic corrections to the nucleon motion. An appropriate definition of each high energy (irreducible) multipole moment is given. It is shown that the Siegert theorem does not apply, so even the electric dipole transition may be affected by (adiabatic) exchange currents. These and other high energy effects are found to contribute about 5 percent to the electric dipole photodisintegration cross section of the deuteron. Larger corrections are anticipated for heavier nuclei. It is shown that the corrections are calculated most readily by using the usual form of the multipole moment operators, rather than the formally correct irreducible operators.

1. INTRODUCTION

It is generally recognized that the possibility of pion photoproduction should have a marked influence on nuclear photodisintegration cross sections at photon energies of the order of 140 Mev or larger.[1] The separation of this influence from the "ordinary" process of photodisintegration can be accomplished only if a reliable theoretical value of the cross section for the ordinary process is available. The natural procedure, and the one that has recently been followed,[2] is to use the electric dipole cross section for this purpose. The dipole moment operator is usually[3] taken to be the static moment operator $D=\sum_{\alpha}e_{\alpha}(\mathbf{u}\cdot\mathbf{r}_{\alpha})$, where $\mathbf{u}$ is the direction of polarization of the photon. Justification of this procedure has been based on the Siegert theorem, which asserts that that form of the electric moment may be used as long as the dynamics of the nuclear system can be described in terms of nuclear variables alone. Thus any observed deviation from the calculated curve is interpreted as an indication of the effects that depend explicitly on the pion variables, in other words, as the influence of the "pion polarizability" of the system. In particular, this interpretation has recently been given[2] to the deviation of the observed photodisintegration cross section of the deuteron from the calculated curves of Schiff[4] and of Marshall and Guth,[5] curves which are based on the deuteron electric moment, $\frac{1}{2}e(\mathbf{u}\cdot\mathbf{r})$.

Our purpose is to point out that the Siegert theorem is *not* valid at high energy; in fact, for the deuteron it breaks down at energies in the neighborhood of 50 Mev. Therefore, the above interpretation of the data could in principle be erroneous, but we shall see below that the errors are quite small.

2. REDUCIBLE MULTIPOLE MOMENTS

To understand the failure of the Siegert theorem, it is necessary to reconsider the problem of defining the multipole moments of a system. The most elementary definition involves an expansion of its interaction with the electromagnetic field in powers of kr, where k is the propagation vector of the radiation and r is a distance of the order of the linear dimensions of the radiating nucleus. At low energies, this expansion converges rapidly, so only the lowest of the terms which contribute to a given transition need be considered. That term is fixed by the specification of the angular momentum and parity change associated with the transition. Thus the lowest term contributing to a transition $\Delta j=l$ with parity change equal to $(-1)^l$ is defined as the electric 2^l pole moment, and that contributing with the opposite change of parity is the magnetic 2^l pole moment. These are the definitions for which the proof of the Siegert theorem has been given.[6]

* This work was supported in part by the AEC and in part by the Wisconsin Alumni Research Foundation.

[1] S. Kikuchi, Phys. Rev. **85**, 1062 (1952).

[2] T. S. Benedict and W. M. Woodward, Phys. Rev. **85**, 924 (1952); R. R. Wilson, Phys. Rev. **86**, 125 (1952); R. Littauer and J. Keck, Phys. Rev. **86**, 1051 (1952); B. Bruno and S. Depkin, Phys. Rev. **86**, 1054 (1952).

[3] J. Levinger, Phys. Rev. **84**, 43 (1951)

[4] L. Schiff, Phys. Rev. **78**, 733 (1950).

[5] J. F. Marshall and E. Guth, Phys. Rev. **78**, 738 (1950).

[6] R. G. Sachs and N. Austern, Phys. Rev. **81**, 705 (1951).

Clearly the above definition fails for radiation of such high energy that kr becomes comparable to unity. Then not only is there a contribution from the aforementioned lowest term, but an infinite series of higher terms must be included for given Δj and parity change. Furthermore, the clear separation of electric and magnetic effects no longer follows from the above rule; for example, the above-defined magnetic quadrupole moment contributes to the electric dipole transition $\Delta j=1$, yes. Since the Siegert theorem does not apply to magnetic transitions, it cannot be applicable here.

3. REDUCED MULTIPOLE MOMENTS

The multipole moments at high energy are best defined on the basis of the angular momentum and parity of the radiation field produced in the transition. This has the advantage that only a finite number of moments can contribute to a transition of a given Δj and parity change, instead of the previously mentioned infinite series. In particular, the transition $j=0$ to $j=l$ can only radiate angular momentum l, so only the 2^l pole moment plays a role therein.

The assignment of a given angular momentum to the photon requires an expansion of the radiation field in terms of spherical waves.[7] The expansion is usually expressed directly in terms of the solutions $\phi_l^m(k\mathbf{r})$ of the scalar wave equation

$$\phi_l^m=f_l(kr)Y_l^m,$$

where Y_l^m is the spherical harmonic and f_l is the spherical Bessel function.[8] The set of vector potentials which provide the electric multipole fields then have the form[9]

$$\mathbf{A}_l^m(\text{elect})=\text{grad}\chi_l^m+k\mathbf{r}\phi_l^m, \tag{1}$$

with

$$\chi_l^m=k^{-1}[\phi_l^m+(\mathbf{r}\cdot\text{grad}\phi_l^m)]. \tag{2}$$

The magnetic multipole contribution arises from the set

$$\mathbf{A}_l^m(\text{mag})=\text{curl}\mathbf{r}\phi_l^m. \tag{3}$$

If the Hamiltonian of the nuclear system in interaction with the radiation is expanded in powers of the vector potential $\mathbf{A}$, only the linear term $H_1\{\mathbf{A}\}$ is important for the emission and absorption processes with which we are concerned. The transition probability for an electric 2^l pole process is proportional to

$$|\langle i|H_1\{\mathbf{A}_l^m(\text{elect})\}|f\rangle|^2,$$

where the matrix element is taken between the initial (i) state and the final (f) state of the radiating system. The electric moment is usually (i.e., at low energies) defined in such a way that its time derivative appears explicitly in the interaction. Therefore, it is convenient to introduce an operator $\dot{E}_l^m$ such that E_l^m plays the role of the electric 2^l pole moment. Then $\dot{E}_l^m$ is proportional to $H_1\{\mathbf{A}_l^m(\text{elect})\}$, and the constant of proportionality can be determined by the condition that the result agree in the low energy limit with the definition given on the basis of an expansion in powers of kr:[10]

$$\dot{E}_l^m=-ck^{1-l}[2\pi(2l+1)/l(l+1)]^{\frac{1}{2}}H_1\{\mathbf{A}_l^m(\text{elect})\}. \tag{4}$$

Similarly, it is possible to formally relate the 2^l pole magnetic moment M_l^m to H_1 as

$$M_l^m=k^{-l}[2\pi(2l+1)/l(l+1)]^{\frac{1}{2}}H_1\{\mathbf{A}_l^m(\text{mag})\}. \tag{5}$$

Questions concerning the Siegert theorem are most easily discussed in the formalism of reference 6. There it is shown that gauge invariance of the Hamiltonian has the consequence

$$H_1\{\mathbf{F}+\text{grad}G\}=-c^{-1}\sum_\alpha e_\alpha\dot{G}(\mathbf{r}_\alpha)+H_1\{\mathbf{F}\},$$

where $\mathbf{F}(r)$ and $G(r)$ are arbitrary functions and $\mathbf{r}_\alpha$ is the coordinate of a particle (nucleon) of charge e_α. Thus introduction of Eq. (1) into Eq. (4) leads to a natural division of the electric moment into the sum

$$E_l^m=P_l^m+S_l^m,$$

where the primary term is

$$P_l^m=k^{-l}[2\pi(2l+1)/l(l+1)]^{\frac{1}{2}}\times[\sum_\alpha e_\alpha Y_l^m\{f_l+(\mathbf{r}\cdot\text{grad}f_l)]_{\mathbf{r}=\mathbf{r}_\alpha}, \tag{6}$$

and the time derivative of the secondary term is[11]

$$\dot{S}_l^m=-ick^{2-l}[2\pi(2l+1)/l(l+1)]^{\frac{1}{2}}H_1\{\mathbf{r}f_l(kr)Y_l^m\}. \tag{7}$$

The quantity P_l^m has the properties associated with the Siegert theorem; its form is independent of any characteristic of the nucleon other than its charge and position. The secondary term S_l^m depends explicitly on the form of the Hamiltonian. For small kr it is one order smaller than P_l^m; hence it is usually neglected. This is the condition under which the Siegert theorem applies. But for $kr\approx 1$, it cannot be neglected; so, then, the form of the electric moment depends explicitly on the form of the Hamiltonian. Under all conditions the magnetic moments depend explicitly on the form of the Hamiltonian.

---

7 W. Heitler, Proc. Cambridge Phil. Soc. **32**, 112 (1936).

8 Normalization of f_l is such that asymptotically as $kr\to\infty$,

$$f_l(kr)\to(kr)^{-1}\sin(kr-\tfrac{1}{2}l\pi).$$

9 J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill Book Company, Inc., New York, 1941), Chap. VII.

10 The E_l^m are defined in such a way as to be proportional to Y_l^m. Then, for example, the components of the electric dipole moment vector $\mathbf{E}$, are related to the E_1^m by relationships of the form $E_1^{\pm1}=2^{-\frac{1}{2}}(E_x\pm iE_y)$, etc. The coefficients are most easily obtained by use of the expansion of a plane transverse wave in spherical waves. The required expansion is given in reference 9.

11 Any given matrix element of S_l^m is to be obtained from the matrix element of $\dot{S}_l^m$ by the Heisenberg relation

$$\langle 1|\dot{S}_l^m|2\rangle=i\omega_{12}\langle 1|S_l^m|2\rangle;$$

hence the form of the operator S_l^m can only be obtained from a complete knowledge of the dynamics of the system. An essential feature of the Siegert theorem is that such knowledge is not required to obtain P_l^m. It should be emphasized that $\dot{S}_l^m$, not S_l^m, is required for the calculation of a transition probability.

4. CORRECTIONS TO ELECTRIC MOMENTS AT HIGH ENERGY

Two quite different features of the Hamiltonian lead to contributions to S_l^m which may be important at high energy. One is due to the influence of mesons on the coupling of the nucleons with the electromagnetic field; it is intimately related to the interaction effects on magnetic moments, those effects that lead to nonadditive contributions to nuclear moments.[12] The other feature is present whether or not there are meson currents; it is concerned only with the distribution of (additive) electric currents between orbital and spin motion of the nucleon. This is determined by the ordinary part of H_1,

$$H_1(\text{ord}) = -(2Mc)^{-1}\sum_\alpha[e_\alpha(\mathbf{p}_\alpha\cdot\mathbf{A}_\alpha + \mathbf{A}_\alpha\cdot\mathbf{p}_\alpha) + e\hbar\mu_\alpha(\boldsymbol{\sigma}_\alpha\cdot\text{curl}\mathbf{A}_\alpha)], \quad (8)$$

where $\mathbf{p}_\alpha$ is the momentum of the α^{th} nucleon, μ_α is its magnetic moment, and $\mathbf{A}_\alpha = \mathbf{A}(\mathbf{r}_\alpha)$. When Eq. (7) is applied to H_1(ord), the contribution to the secondary electric moment is found to be

$$\dot{S}_l^m(\text{ord}) = -ck^{2-l}[2\pi(2l+1)/l(l+1)]^{\frac{1}{2}} \times\sum_\alpha(e_\alpha\hbar/2Mc)\left[\mu_\alpha\boldsymbol{\sigma}_\alpha\cdot(\mathbf{L}Y_l^m)f_l + Y_l^m\left\{3f_l + r\frac{\partial f_l}{\partial r} + 2f_l r\frac{\partial}{\partial r}\right\}\right]_{\mathbf{r}=\mathbf{r}_\alpha}. \quad (9)$$

Such terms are included in the expressions for the electric moments given by Weisskopf,[13] but in a quite different form.

The formalism used here is capable of taking into account the meson effects for which the contribution H_1(exch) to H_1 depends only on nucleon variables (exchange effects). For example, such a term is known[14] to be associated with the space exchange potential, no matter what the source of the potential. Furthermore, there is very strong evidence[15] for the existence of a spin-exchange current which must be included in H_1(exch). The two effects suggest an H_1(exch) of the form[16]

$$H_1(\text{exch}) = (ie/\hbar c)\sum_{\pi,\nu}\left\{\left(\int_{r_\nu}^{r_\pi} A_s ds\right)J(r_{\pi\nu})P_{\pi\nu} + (\boldsymbol{\sigma}_\pi\cdot\text{curl}\mathbf{A}_\pi - \boldsymbol{\sigma}_\nu\cdot\text{curl}\mathbf{A}_\nu)\Phi(r_{\pi\nu})\right\}, \quad (10)$$

where $\mathbf{r}_\nu$ and $\mathbf{r}_\pi$ are neutron and proton coordinates, respectively, $J(r_{\pi\nu})P_{\pi\nu}$ is the space exchange potential, and $\Phi(r_{\pi\nu})$ is a short-range function of the neutron-proton distance which can be normalized in such a way as to give the correct magnetic moment for H^3 and He^3. The corresponding $\dot{S}_l^m$(exch) can be obtained directly by means of Eq. (7).

5. APPLICATION TO THE DEUTERON

Application of these considerations has been made to the electric dipole photodisintegration of the deuteron with a view to estimating corrections to the Schiff[4] and Marshall-Guth[5] results. In addition to the corrections associated with S_1^m(ord) and S_1^m(exch), it must be noted that P_1^m is not actually equal to $\frac{1}{2}e(\mathbf{u}\cdot\mathbf{r})$ because of the retardation effect. In fact,

$$P_1^m = (3\pi)^{\frac{1}{2}}ek^{-1}Y_1^m\left\{f_1(kr/2) + r\frac{\partial}{\partial r}f_1(kr/2)\right\},$$

and the Schiff and Marshall-Guth curves must be corrected for the difference $P_1^m - \frac{1}{2}eY_1^m$. Furthermore, contributions to the ${}^3S_1\rightarrow{}^3P_1$ transition may arise from the magnetic quadrupole moment, M_2^m, and contributions to ${}^3S_1\rightarrow{}^3P_2$ may arise from both M_2^m and the electric octopole term E_3^m.

A preliminary exploration of these corrections and the corrections due to S_1^m showed that an expansion of f_l in powers of kr led to a rapidly convergent series of matrix elements even for photon energies as high as 300 Mev. Convergence occurs because, at high energy, the short wavelength $\lambda\!\!\!^{-}$ of the outgoing deuteron wave limits the effective value of kr to $k\lambda\!\!\!^{-}\approx(E_\gamma/Mc^2)^{\frac{1}{2}}$. The only significant corrections[17] to the cross section turn out to be those of order k and k^2.

The validity of an expansion in powers of kr having been established, it is immediately evident that the detailed calculation of corrections is most easily made on the basis of the simple moment operators described in Sec. 2. The corrections then arise from the contributions to the dipole transition of the (reducible) magnetic quadrupole and electric octopole moments. The first of these yields a correction to the dipole matrix element of order k, the second of order k^2. Corrections of order k to the cross section are then introduced by interference of the magnetic quadrupole term with the usual electric dipole, and the k^2 corrections arise both from the magnetic quadrupole and from the interference of electric octopole with electric dipole terms. The great simplicity of this procedure is illustrated by the fact that all corrections associated with the space exchange

[12] N. Austern and R. G. Sachs, Phys. Rev. **81**, 710 (1951).

[13] V. Weisskopf, Phys. Rev. **83**, 1073 (1951).

[14] R. G. Sachs, Phys. Rev. **74**, 433 (1948).

[15] F. Villars, Helv. Phys. Acta **20**, 476 (1943); R. Avery and R. G. Sachs, Phys. Rev. **74**, 1320 (1948); R. G. Sachs and M. Ross, Phys. Rev. **84**, 379 (1951).

[16] The evidence for the spin dependent contribution arises entirely in connection with magnetic dipole moments; hence only the interaction with a uniform magnetic field has known properties. To fix the interaction with a nonuniform field, we have assumed that the spin-exchange moment results from a change in the intrinsic nucleon moments, i.e., that the corresponding current distribution is localized at the nucleon. The form used here then corresponds to the spin-antisymmetric moment of reference 12. See also R. K. Osborne and L. L. Foldy, Phys. Rev. **79**, 795 (1950); A. Bohr, Phys. Rev. **81**, 134 (1951); H. Miyazawa, Prog. Theoret. Phys. **6**, 263 (1951); A. DeShalit, Helv. Phys. Acta **24**, 296 (1951); F. Bloch, Phys. Rev. **83**, 839 (1951).

[17] Note that relativistic corrections to the nucleon motion may become significant at these energies, although they are not considered here.

term in Eq. (10) vanish for the deuteron since the exchange magnetic 2^l pole moment operators all vanish.[18] If the problem is formulated in terms of the reduced moments, the elimination of these terms occurs after a detailed calculation through a cancellation between $\dot{S}_l^m$(exch) and $\dot{P}_l^m$.

The magnetic quadrupole moment contains an orbital term, proportional to the orbital angular momentum operator $\mathbf{L}$, a term associated with the intrinsic spin moments, and a spin interaction term, corresponding to the spin-dependent contribution to Eq. (10). The last of these turns out to be negligible, so it will not be discussed further. Only the orbital part of the moment can lead to interference with the electric dipole term. Since the orbital magnetic quadrupole moment is[12]

$$\mathbf{M}_2(\mathrm{orb}) = (e/24Mck)[\mathbf{L}(\mathbf{k}\cdot\mathbf{r}) + (\mathbf{k}\cdot\mathbf{r})\mathbf{L}],$$

the ratio of its matrix element to the matrix element of the electric dipole moment is independent of any detailed features of the wave function. The relative correction to the dipole cross section due to interference between these terms is just $-E_\gamma/6Mc^2$ which is five percent at 300 Mev.

The spin term in M_2 is[19]

$$\mathbf{M}_2(\mathrm{spin}) = \tfrac{1}{2}(e\hbar/2Mck)(\mu_p\boldsymbol{\sigma}_p - \mu_n\boldsymbol{\sigma}_n)(\mathbf{k}\cdot\mathbf{r}),$$

so its matrix element is also proportional to that of the electric dipole moment. The interference term vanishes since $(\mathbf{k}\cdot\mathbf{r})$ yields a wave function orthogonal to that produced by $(\mathbf{u}\cdot\mathbf{r})$. The relative correction to the cross section for the $^3S\rightarrow{}^3P$ transition is $\frac{2}{3}([\mu_p-\mu_n]/4)^2(E_\gamma/Mc^2)^2$, which amounts to 10 percent at 300 Mev. Note that the angular distribution for this term is $\cos^2\theta$, rather than the usual $\sin^2\theta$ electric dipole distribution. The spin term also produces an electric dipole $^3S\rightarrow{}^1P$ transition, but the corresponding cross section is much smaller.

[18] The magnetic space exchange moments are shown in reference 12 to be proportional to $\mathbf{r}_\pi\times\mathbf{r}_\nu$, which vanishes for the deuteron since $\mathbf{r}_\pi = -\mathbf{r}_\nu$.

[19] This moment is obtained from the second term in Eq. (9) by the prescription outlined in reference 6. Its contribution to the transition has been calculated by Marshall and Guth, reference 5, but they failed to include the orbital term in the magnetic quadrupole matrix element.

The interference between the electric octopole and electric dipole terms cannot be obtained so readily, since the one involves the matrix element of r^3 while the other involves the matrix element of r. However, an estimate of the contribution indicates that it is about 0.1 percent at 300 Mev. Hence the only important correction to the dipole transition is that due to the magnetic quadrupole moment. Since the interference term is negative, the net correction at 300 Mev is only about 5 percent. This is of the same order as the contribution of the electric quadrupole transition.[4,5] Both are quite small compared to the observed deviations from the Schiff and Marshall-Guth curves.

6. CONCLUSIONS

At high energy there is some ambiguity as to the meaning of a given multipole moment. This ambiguity has been removed by introducing the "correct" (irreducible) moments of Sec. 3. Nevertheless, it is much more convenient to work with the usual (reducible) moments whenever an expansion in powers of kr can be justified. That is adequately illustrated by the simplicity of the above discussion of the deuteron photodisintegration.

The deuteron problem has been scrutinized for high energy corrections to the calculated photodisintegration cross section because there are large discrepancies between the observations and the calculated curves. However, all effects considered turn out to be small. Nevertheless, effects of the sort discussed here must be kept in mind because they may be appreciably larger for heavier nuclei.[20]

This discussion owes its existence in large part to conversations between the authors and Professor J. H. D. Jensen concerning the "best possible" definition of a multipole moment.

[20] This is suggested by the following facts: the nuclear radii may be larger than the effective radius of the deuteron; exchange contributions vanish just for the deuteron; and the partial cancellation which occurs between the interference term and the magnetic quadrupole cross section is probably a special property of the deuteron.