

## The Electronic Component at Low Altitudes Produced by $\pi$ -Meson Decay\*†

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The intensity of the electronic component at low altitudes produced by  $\pi$ -meson decay (designated as the  $E_\pi$  component) has been obtained as a function of zenith angle and altitude by subtracting from the experimentally observed soft component those electrons arising from the collision and decay processes of  $\mu$ -mesons (designated as  $E_\mu$  electrons). Extensive auxiliary studies of the corrections required in such soft component studies are described, and experimental requirements are stated for a precise telescopic study of the soft component. In particular, it is shown that the usual method of correcting a Geiger counter telescope for side showers is incorrect, and also that the effect of wall-generated secondaries should be considered. It is shown that the  $E_\pi$  component of energy greater than 30 Mev can be represented as a function of zenith angle  $\theta$  and atmospheric depth  $h$  by the expression

$$I(E_\pi, h, \theta) = 0.70 \exp(-h/160 \cos \theta) \text{ cm}^{-2} \text{ sec}^{-1} \text{ sterad}^{-1},$$

for  $\theta \leq 60^\circ$  and  $h \geq 700 \text{ g cm}^{-2}$ .

### I. INTRODUCTION

ALTHOUGH the bulk of the electronic component at low altitudes arises as a secondary radiation from  $\mu$ -mesons by collision and decay processes,<sup>1-5</sup> the strong dependence of the soft component intensity on zenith angle and altitude<sup>6-8</sup> and the small disagreement between observed and predicted electron rates on the basis of two or three particle  $\mu$ -meson decay schemes all indicate the presence of another electronic component<sup>9, 10</sup> which is now understandable in terms of the decay properties of the  $\pi$ -meson. In order to separate these components, the electron intensity at low altitudes was determined by Geiger counter telescope as a function of zenith angle, altitude, and energy. This was done for the entire electronic portion of the soft component by first measuring the total intensity as a function of zenith angle and altitude and then subtracting meson and proton intensities as observed in independent measurements. The dependence upon these factors for the electronic components lying in various energy intervals was obtained by the use of carbon absorbers. A separation of the electronic component into two subcomponents was then effected by the use of a computed  $E_\mu$  spectrum, the latter being

based on the known properties and spectrum of the  $\mu$ -meson.

### II. EXPERIMENTAL ARRANGEMENT AND PROCEDURE

The counter telescopes and absorbers were arranged as shown in Fig. 1. In order to reduce scattering and absorption of the soft electrons, aluminum-walled Geiger counters of 0.016-inch wall thickness and one-inch outer diameter were used throughout. Counters *a* to *g* (small counters) were of ten inch over-all length with an effective length of  $7.75 \pm 0.05$  inches, while the *h* counters (large counters) were eighteen inches long with an effective length of sixteen inches. This type of counter has been described elsewhere.<sup>11</sup> The solid angle of each telescope was defined by counter *c* and those counters above *c*, while underneath counter *c* was placed a curved carbon absorber. Absorber thicknesses of 2.62, 5.62, and  $8.18 \text{ g cm}^{-2}$  were used. It was also possible to insert  $4.28 \text{ g cm}^{-2}$  of carbon absorber above counter *c*; this, however, led to an increase in the effective width of counter *c*, which will be considered later (Sec. III-L). Tray *h*, located beneath the carbon absorber, was arranged so that a particle traversing the absorber

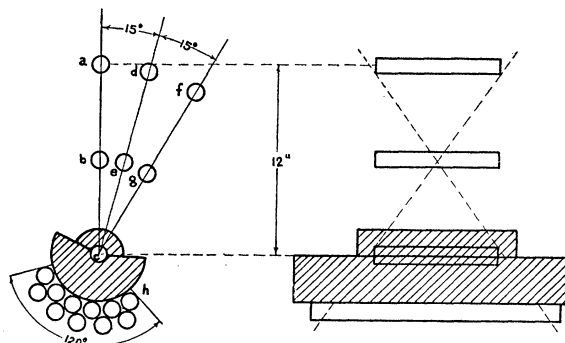


FIG. 1. Axial and side views of arrangement of Geiger counter telescopes and carbon absorbers.

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<sup>1</sup> W. Heitler, Proc. Roy. Soc. (London) **161**, 261 (1937).

<sup>2</sup> H. Euler and W. Heisenberg, Ergeb. exakt. Naturw. **17**, 1 (1938).

<sup>3</sup> B. Ferretti, Ricerca sci. **7-8**, 736 (1939).

<sup>4</sup> H. J. Bhabha, Proc. Roy. Soc. (London) **164**, 256 (1938).

<sup>5</sup> D. Lyons, Physik. Z. **42**, 166 (1941).

<sup>6</sup> K. Greisen, Phys. Rev. **61**, 212 (1942).

<sup>7</sup> (a) Bernardini, Cacciapuoti, Feretti, Piccioni, and Wick, Phys. Rev. **58**, 1017 (1940); (b) Bernardini, Cacciapuoti, and Querzoli, Phys. Rev. **73**, 328 and 335 (1948).

<sup>8</sup> Azimov, Veksler, Dobrotin, Zhdanov, and Lubimov, J. Phys. (U.S.S.R.) **10**, 507 (1946).

<sup>9</sup> K. Greisen, Phys. Rev. **63**, 323 (1943).

<sup>10</sup> B. Rossi and K. Greisen, Phys. Rev. **61**, 121 (1942).

<sup>11</sup> D. R. Corson and R. R. Wilson, Rev. Sci. Instr. **19**, 207 (1948).

could undergo a deflection of at least  $45^\circ$  from its original path and still be detected by the tray.

The material in the vicinity of the telescope was minimized as follows. The framework of the telescope was constructed of pressed wood ( $0.4 \text{ g cm}^{-2}$ ) located at the extreme ends of the counters. The whole apparatus was housed at Ithaca (275 meters) in a small wooden hut with walls and roof of pressed wood ( $0.4 \text{ g cm}^{-2}$ ). The hut projected from a top window of the laboratory and the telescope was located four feet from the walls of the building; the top counters being on the same level as the flat roof of the building. At Echo Lake (3240 meters) the wooden hut was replaced by a canvas tent  $0.03 \text{ g cm}^{-2}$  thick.

The following coincidence rates were observed: threefold, *abc*, *dec*, and *fgc*; fourfold, *abch*, *dech*, and *fgch*; displaced center counter rates, *aec*, *dbc*, *dgc*, and *fec*. These latter rates were used to obtain a correction for the effects of extensive air showers. The electronic circuits were of standard form, the resolving time of each channel being four microseconds, while the dead time of the mechanical recorders was 0.07 second.

For each of the four absorber thicknesses, measurements were made at two separate telescope positions, an upper position where the zenith angles for the *abc* and *fgc* telescopes were  $0^\circ$  and  $30^\circ$ , respectively, and a lower position, where these angles were  $30^\circ$  and  $60^\circ$ , respectively.

Measurements in the upper telescope position were carried out for approximately one-hundred hours with each absorber (yielding a standard statistical error of somewhat less than one percent in the total intensity measurements at zero zenith angle) and for approximately seventy hours with each absorber in the lower position of the telescope. The total data for each particular arrangement of the telescope and absorber were obtained in several different runs during the course of the measurement rather than in one continuous run. This was done in the expectation that the fluctuations arising from meteorological causes would sometimes compensate one another. Furthermore, the similarity of the data obtained in several different setting of a given set of experimental conditions served as a check on the over-all operation.

At Ithaca the counter axes were in the north-south direction and the principal telescope axis was tilted towards the East. At Echo Lake the measurements were all made with the counter axes in the north-south direction, but were equally divided between east and

TABLE I. Average rates ( $n$ ) and resultant dead time inefficiencies for Geiger counters, resulting from counter dead time of 300 microseconds.

	Ithaca		Echo Lake	
	$n(\text{sec}^{-1})$	Dead time inefficiency	$n(\text{sec}^{-1})$	Dead time inefficiency
Small counters	2.5	<0.10%	4.5	0.15%
Large counters	5.0	0.15%	9.0	0.30%

west inclinations of the principal telescope axis. As no statistically significant differences were observed, the readings for these two settings at Echo Lake were combined.

### III. TREATMENT AND CORRECTION OF THE OBSERVED DATA

The following factors were considered in the correction of the observed data.

#### A. Meteorological Fluctuations

Owing to large barometric fluctuations existing at sea level, the Ithaca data were not corrected by the use of the barometric coefficient, but by the use of data from a wide-angled monitor telescope which ran with a counting rate of one hundred times the greatest counting rate observed with the arrangement of Fig. 1. The reduction of the Ithaca data to a mean intensity was accomplished by multiplying the number of coincidences during a given run by the ratio of the average monitor rate during the whole measurement to the average monitor rate during the given run.

At Echo Lake the average barometric pressure during any given run never deviated from the long time average by more than 1.7 mm Hg (and in most cases the deviation was much less than this); hence, no meteorological corrections were applied.

#### B. Variation of Zenith Angle over the Telescope

The conversion of observed coincidence rates into absolute intensities is dependent not only upon the telescope geometry but also upon the zenith angle of the radiation. For telescopes of the dimensions used here, the observed coincidence rate  $N(\theta)\text{sec}^{-1}$  is related to the absolute intensity  $I(\theta)\text{cm}^{-2}\text{sterad}^{-1}\text{sec}^{-1}$  by the expression<sup>6</sup>

$$N(\theta) = kI(\theta) = kI(0)\cos^n\theta,$$

where  $k = 2.37; 2.28; 2.22; 2.17; \text{ and } 2.12$  for  $n = 0, 1, 2, 3, \text{ and } 4$ , respectively. For nonintegral values of  $n$ ,  $k$  is obtained by interpolation.

#### C. Failure of the Particle to Produce an Ion Pair in the Counter

For our counters, with an average primary specific ionization of thirty ion pairs  $\text{cm}^{-1}\text{atmosphere}^{-1}$  and a pressure of 0.13 atmosphere, a calculation similar to that performed by Montgomery,<sup>12</sup> but including a very small correction to allow for the inclination of the particle paths to the telescope axis, yields an inefficiency of 1.2 percent per counter. This type of inefficiency is normally offset by the effect of particles lying just outside the solid angle of the telescope creating secondaries in the counter walls.<sup>13</sup> Owing to the thinness of our walls, there was considerable uncertainty as to the

<sup>12</sup> R. A. Montgomery, Phys. Rev. **75**, 1407 (1949).

<sup>13</sup> K. Greisen and N. Nereson, Phys. Rev. **62**, 316 (1942).

magnitude of this compensation; hence an auxiliary study of the generation of secondaries in thin strips of material was made by means of cloud chamber and Geiger counter telescope.<sup>14</sup> From these studies the contribution of wall-generated secondaries to apparent increase in the Geiger counter efficiency can readily be shown to amount to only 0.1 percent in the case of our thin-walled aluminum counters; hence the final correction to be applied is +1.1 percent per counter. This correction is not applicable to the middle counters of our three counter telescopes, since relatively few particles lying in the telescope traverse the edges of these counters; nor does it apply to the counters of tray *h*, owing to overlapping of the edges of those counters.

#### D. Dead Time of Counters

The dead time of these counters was 300 microseconds; the inefficiencies introduced here are as shown in Table I. For the counters of tray *h*, where three-quarters of the particles traversed two counters, this inefficiency will be negligible.

#### E. Chance Coincidences

Chance coincidences were negligible, in the worst case amounting to no more than 0.1 percent of the total rate.

#### F. Circuit Inefficiencies

The finite resolving time of the mechanical recorders was approximately 0.07 sec in all cases. This never gives rise to a correction of more than 0.2 percent at sea level.

#### G. Special Correction for Tray *h*

Owing to the overlapping of the counters of the bottom tray in addition to their being electrically isolated from one another, an inefficiency in this tray of less than 0.1 percent is expected. It was observed, however, when all the absorber was removed from the telescope (both at Ithaca and Echo Lake) that the number of particles traversing the top telescope and not detected by tray *h* was greater than this and was practically independent of zenith angle. This portion amounted to about one percent of the vertical rate and is interpreted as an extremely soft portion of the soft component, which fails to penetrate the walls of tray *h* counters, and which is so strongly scattered that it no longer possesses an observable zenith angle dependence.

#### H. Extensive Shower Correction

In order to obtain the contribution from extensive showers in the conventional manner (by displacement of the center counter out of the solid angle of the telescope) the following coincidence rates were observed: *ace*, *dbc*, *dgc*, and *fec*. Each of these four arrangements

<sup>14</sup> Brown, McKay, and Palmatier, Phys. Rev. **76**, 506 (1949).

TABLE II. Displaced center counter rates for various telescope zenith angles as observed at Ithaca and Echo Lake. The second column under each location shows the value given by the analytical expression adopted for that location.

Zenith angle of telescope	Ithaca		Echo Lake	
	Observed rate (hr <sup>-1</sup> )	Analytical expression (0.45 hr <sup>-1</sup> + 2.4 cos <sup>2</sup> θ hr <sup>-1</sup> )	Observed rate (hr <sup>-1</sup> )	Analytical expression (4.5 hr <sup>-1</sup> + 6.2 cos <sup>2</sup> θ hr <sup>-1</sup> )
0°	2.83±0.1	2.85	10.4±0.3	10.7
15°	2.7 ±0.1	2.6	10.9±0.3	10.3
30°	2.5 ±0.1	2.2	8.9±0.2	9.2
45°	1.5 ±0.2	1.6	7.6±0.3	7.6
60°	1.3 ±0.1	1.0	5.2±0.25	6.0

can be considered as being derived from some simple three counter telescope by a slight displacement of the center counter from its normal position. We shall refer to a given arrangement as being inclined at a certain zenith angle, namely, that of the telescope from which it is derived. Thus the *dgc* arrangement with the lower telescope position is derived from the *dec* telescope in the lower position; hence the former will be said to possess a zenith angle of 45°

Now as will be seen from the data (Table II) the displaced center counter rate varies rather strongly with zenith angle, but only by a factor of 4 with altitude. Were this rate due to extensive showers, we would expect little if any zenith angle variation, but a rather strong altitude variation (factor of 10 between the two stations). The observed behavior is, however, quite reasonable if it is considered to arise primarily from secondaries generated in the roof, telescope, and counter walls by the total component. Since extensive showers increase by approximately tenfold between Ithaca and Echo Lake,<sup>15</sup> whereas the total component is observed to increase by a factor of 2.6 only, we may expect to fit the observed data by the expressions

$$c + d \cos^2\theta \text{ for Ithaca,}$$

and

$$10c + 2.6d \cos^2\theta \text{ for Echo Lake,}$$

where *c* refers to the contribution from extensive showers and *d* to the contribution from locally generated secondaries. Indeed, such expressions do fit the displaced counter data very well for the values *c*=0.45 hr<sup>-1</sup> and *d*=2.4 hr<sup>-1</sup> as is shown in Table II. It is quite apparent that any interpretation which assigns a large portion of the displaced counter rate at Ithaca to extensive showers must predict a displaced counter rate at Echo Lake which greatly exceeds that observed. We are forced to conclude that the standard method of displacing the center counter to estimate the shower rate leads to a considerable overcorrection.

The above conclusions are confirmed by the following independent considerations:

(a) Extensive shower measurements by Cocconi and Tongiorgi<sup>15</sup> show that the threefold coincidence rate for

<sup>15</sup> G. C. Cocconi and V. Cocconi Tongiorgi, Phys. Rev. **75**, 1058 (1949).

counters of 50 cm<sup>2</sup> area at sea level and separated by 4 m is 0.3 hr<sup>-1</sup>. This must certainly be a lower limit for our counters.

(b) The displaced center counter rate *ace*, when the zenith angle of our telescope is zero, is 2.8 hr<sup>-1</sup>, whereas the rate of *acf* is 0.5 hr<sup>-1</sup>. Were the *ace* rate due entirely to extensive showers, we would expect no appreciable difference in the two rates; hence we may take 0.5 hr<sup>-1</sup> as an upper limit to the extensive shower rate.

(c) Whenever a fourfold coincidence rate is observed in this experimental arrangement, such that no three counters are in line, we obtain a coincidence rate (which we may now more reasonably assume to be entirely due to extensive showers) of 0.25 hr<sup>-1</sup>. It has been observed experimentally<sup>16</sup> that for counters of 54 cm<sup>2</sup> area, the ratio of threefold to fourfold shower coincidence rate is 1.6; this yields a threefold shower coincidence rate of 0.4 hr<sup>-1</sup> for our counters, in good agreement with the two previous limits. Owing to the small solid angle of our telescope, only an extremely small portion of what we consider as the extensive shower correction will actually consist of shower particles with paths parallel to the axis of the telescope; thus overcorrection on this account need not be considered.

We conclude from all these considerations that for our telescopes the extensive shower rate at Ithaca is 0.4 hr<sup>-1</sup> for any orientation of the telescope, while for Echo Lake (where the extensive shower rate is ten times as great as at Ithaca) the rate is 4.0 hr<sup>-1</sup>.

### I. Losses Arising from the Simultaneous Traversal of the Telescope by Several Particles

The resolving time of the mechanical recorder was 0.07 second; hence two or more particles traversing the telescope within this time interval will be recorded as one particle only. For random events this effect is negligible (correction *F*). For genetically related particles also, the following considerations show that the effect is negligible.

The *fac* rate was observed to be  $0.53 \pm 0.05$  hr<sup>-1</sup>. Since the known contributions comprise the major portion of this rate (0.4 hr<sup>-1</sup> from extensive showers and 0.04 hr<sup>-1</sup> from counter wall- and roof-generated secondaries) we can say that at most a contribution of 0.1 hr<sup>-1</sup> arises from the simultaneous traversal of *ac* and *f* by two genetically related particles travelling in parallel paths. When we further restrict the paths of these particles to the solid angle of the telescope, it is apparent that the rate will be negligible. Montgomery<sup>12</sup> has reached the same conclusion by the use of two parallel telescopes.

### J. Contribution of Secondaries Generated in the Roof and Counter Walls

Using our uncorrected total intensity data, the known distribution of secondaries,<sup>14</sup> and the geometry of a

telescope located about four inches below the thin wooden roof, we have estimated the contribution from various roof and wall sources. Owing mainly to the scattering of the low energy secondaries in the counter walls, it is found that the only appreciable contribution arises when a cosmic-ray particle traverses the two upper counters (not the bottom one) and generates a secondary electron in the center counter wall, which then enters the bottom counter. This effect is found to give rise to a correction of  $-0.3 \cos^2\theta$  hr<sup>-1</sup> at Ithaca and  $-0.8 \cos^2\theta$  hr<sup>-1</sup> at Echo Lake for our type of counter walls and geometry. These corrections are to be applied to both the threefold and fourfold rates.

### K. Effect of Counter Walls upon Soft Component Intensity

In order to extrapolate to zero wall thickness, an auxiliary study was made of the counting rate as a function of aluminum wall thickness. It was found that an excellent fit of the data for the particular three counter telescope used in this auxiliary experiment was given by  $R(t) = (70 + 50e^{-0.18t})$  hr<sup>-1</sup> where  $R(t)$  is the soft component coincidence rate (in counts per hour) corrected for meteorological fluctuations only, and  $t$  is the wall thickness (for each wall) in mils of aluminum. This expression is valid for values of  $t$  from 16 to 140. Now on theoretical grounds the electron spectrum is not expected to diverge rapidly at low energies; hence over the small range of  $t$  the extrapolation to zero wall thickness does not seem unreasonable. We therefore consider that an approximate increase of ten percent in the soft component intensity (or three percent of the total intensity) results when all the counter wall thicknesses are reduced from 16 mils to 0.

### L. Errors Arising from the Use of Carbon Absorber above the *c* Counter

When carbon absorbers were located both above and below counter *c*, the absorption was considered as occurring in two stages. The true absorption occurring in the top section (4.28 g cm<sup>-2</sup> above *c*) was assumed to be equal to the absorption measured by placing an equal thickness of carbon below counter *c* with no absorber above *c*. This latter quantity is obtained directly from a linear interpolation between the rates obtained with the 2.62 g cm<sup>-2</sup> and the 5.62 g cm<sup>-2</sup> carbon absorbers in the bottom position. The absorption occurring in the bottom section (below *c*) was assumed to be equal to the difference of the *abc* and *abch* rates with both parts of the absorber in place. This seems quite a reasonable assumption, since any effects arising from the presence of the carbon absorber above *c* will occur almost identically in both telescopes.

That is, if  $x$  represents the amount of absorber above *c* and  $y$  the amount of absorber between *c* and *h*, then the total absorption occurring in a thickness  $x_1 + y_1$  is

<sup>16</sup> Cocconi, Loverdo, and Tongiorgi, Phys. Rev. **70**, 841 (1946).

equated to

$$\left. \begin{array}{l} abc-abch \\ y=x_1 \end{array} \right\} x=0 \quad + \quad \left. \begin{array}{l} abc-abch \\ y=y_1 \end{array} \right\} x=x_1$$

in which the first term refers to the absorption in  $x_1$  and the second term to the added absorption in  $y_1$ .

### M. Contribution of Soft Mesons to the Soft Component

The meson contribution to our telescope rates at Ithaca with  $12 \text{ g cm}^{-2}$  of carbon absorber has been determined from Kraushaar's measurements<sup>17-18</sup> as  $-0.4 \cos^3\theta \text{ hr}^{-1}$ . In view of the slight disagreements in the published measurements of the variation of soft meson intensity with altitude, we consider that the delayed coincidence method, which excludes the proton component, is the most reliable method for determining this factor. Using Kraushaar's value of 2, we obtain a meson correction to the soft component rate in our telescopes of  $-0.8 \cos^3\theta \text{ hr}^{-1}$  at Echo Lake. The corrections to be applied for the other absorber thicknesses will then be proportional to the absorber thickness.

The mesons contained in the entire soft component (all particles of range less than  $80 \text{ g cm}^{-2}$  air equivalent) were determined from Kraushaar's measurements. From the differential proton spectrum presented in the next section we estimate that his differential vertical intensity of mesons of range  $105 \text{ g cm}^{-2}$  air equivalent (which was obtained by an absorption method) contains a six percent contribution from protons. This correction, though quite insignificant for our purpose has been included in the final correction which is then  $-3.0 \cos^3\theta \text{ hr}^{-1}$  for our telescope rates at Ithaca.

The Echo Lake correction amounting to  $-6.0 \cos^3\theta \text{ hr}^{-1}$  is then obtained as before from the results of the delayed coincidence methods and should contain no appreciable contribution from protons.

### N. Contribution of Soft Protons to the Soft Component

The proton spectrum for Ithaca, which we have used to obtain the correction, is presented in Fig. 2. No great accuracy can be claimed for this spectrum owing to the uncertainties of interpretation of proton intensity measurements, especially in those instances where absorption and production of protons may occur in material above the apparatus. Owing to the small number of protons present at low altitudes, however, it seems reasonable that future revisions of this proton spectrum will not appreciably influence our final corrected intensities for the soft component. For momenta above  $1000 \text{ Mev}/c$  the observations of Mylroi and Wilson<sup>19</sup>

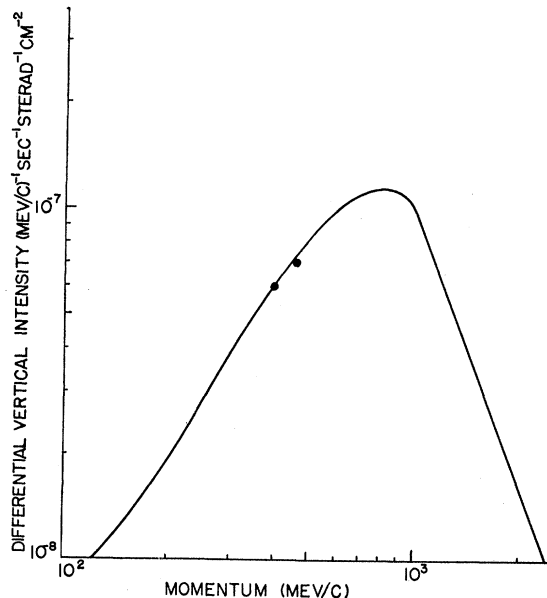


Fig. 2. Differential vertical intensity spectrum of protons at Ithaca as used in correction  $N$ .

have been used. The observations of Todd *et al.*<sup>20</sup> obtained at an altitude of 3400 meters, when converted to lower altitudes by the use of an absorption length of  $130 \text{ g cm}^{-2}$  yield a vertical differential intensity at Ithaca of  $6 \times 10^{-8} (\text{Mev}/c)^{-1} \text{ sec}^{-1} \text{ sterad}^{-1} \text{ cm}^{-2}$  for momenta of around  $400 \text{ Mev}/c$ . This latter value is in good agreement with the early measurement of Rochester and Bound,<sup>21</sup> whose observations yield a vertical differential intensity of  $5 \times 10^{-7} \text{ g}^{-1} \text{ sec}^{-1} \text{ sterad}^{-1}$  for protons of range  $20 \text{ g cm}^{-2}$  in Pb (or  $7 \times 10^{-8} (\text{Mev}/c)^{-1} \text{ cm}^{-2} \text{ sec}^{-1} \text{ sterad}^{-1}$  at momenta of around  $460 \text{ Mev}/c$ ). For the lower momenta values, the observations of Franzinetti<sup>22</sup> indicate that the number of protons is quite small as is expected on the basis of ionization loss considerations.

In order to correct for protons at various zenith angles, the  $\cos^5\theta$  zenith angle dependence computed by Mylroi and Wilson<sup>19</sup> has been used. The soft component corrections to be applied to our observed rates are finally obtained as  $-0.4 \cos^5\theta \text{ hr}^{-1}$  at Ithaca and  $-4.0 \cos^5\theta \text{ hr}^{-1}$  at Echo Lake

### IV. HARD COMPONENT INTENSITY AT ITHACA

In order to ensure that the soft component could properly be obtained as the difference between the total intensities from our measurements and the hard component intensities from other measurements, a separate determination of the hard component vertical intensity was made at Ithaca. The arrangement of counters and lead was the same as shown in Fig. 1 with the exception

<sup>17</sup> W. Kraushaar, Phys. Rev. **76**, 1045 (1949).

<sup>18</sup> W. Kraushaar, thesis, Cornell (1948).

<sup>19</sup> M. G. Mylroi and J. G. Wilson, Proc. Phys. Soc. (London) **A64**, 404 (1951).

<sup>20</sup> Todd, Henderson, Miller, and Potter, Phys. Rev. **76**, 590 (1949).

<sup>21</sup> G. D. Rochester and M. Bound, Nature **146**, 745 (1940).

<sup>22</sup> C. Franzinetti, Phil. Mag. **41**, 86 (1950).

TABLE III. Classification of observed coincidences according to ranges of particles.

Type	Minimum range (gm cm <sup>-2</sup> )	Maximum range (g cm <sup>-2</sup> )	Type of correction applied
1	0.03 air	0.55 Al	Discussed in Sec. III K
2	0.55 Al	0.77 Al	Discussed in Sec. III G
3	0.55	0.55+2.62 C	A, B, C, M, N. Latter three corrections just compensate for one another
4	0.55	0.55+5.62 C	
5	0.55	0.55+8.18 C	
6	0.55	0.55+12.46 C	
7	0.55	80 air equivalent	
8	0.55	∞	Corrections for type 8 plus M and N A, B, C, D, F, H, J

that the vertical telescope only was used and 10 cm of lead was placed between counter *c* and tray *h*. In the following paragraph the observed data and corrections are given in some detail to illustrate the relative magnitudes of the various corrections used throughout.

During the period from October 30th to December 1st, 22,363 coincidences were observed during 317 hours of operation. This yields an observed rate of  $70.5 \pm 0.5$  hr<sup>-1</sup>, to which the following corrections (as discussed in the previous section) were applied:

	Correction
Meteorological	-0.5 percent
Failure to produce an ion pair	+2.2 percent
Dead time inefficiencies	+0.2 percent
Circuit inefficiencies	+0.15 percent
Extensive showers (owing to the large area of the bottom tray the shower correction here will be the same as with our previous arrangement)	-0.4 hr <sup>-1</sup>
Knock-on secondaries from roof and counter walls	-0.2 hr <sup>-1</sup>
Back-scattered secondaries from lead absorber (calculated from information of reference 13)	-0.3 hr <sup>-1</sup>
Total correction	+0.5 hr <sup>-1</sup>

Applying this correction and converting to a minimum range of 80 g cm<sup>-2</sup> of dense air<sup>14</sup> we obtain

$$I_v(80 \text{ g cm}^{-2} \text{ dense air}) = (0.886 \pm 0.006) \times 10^{-2} \text{ cm}^{-2} \text{ sterad}^{-1} \text{ sec}^{-1}.$$

TABLE IV. Summary of data (counts/hour) and corrections to data which were used in obtaining the type 7 component for Ithaca (electronic component of the soft spectrum), namely, those electrons capable of penetrating 0.55 g cm<sup>-2</sup> of Al but with ranges  $\leq 80$  g cm<sup>-2</sup> of air equivalent.

No. of item	Item	0°	15°	30°	45°	60°
1	Observed rate for type 8, corrected only for meteorological fluctuations	100.2±0.6	91.0±0.6	70.2±0.4	45.1±0.5	21.5±0.3
2	Correction C	2.2	2.0	1.6	1.0	0.5
3	Correction D	0.2	0.2	0.15	0.1	0.05
4	Correction F	0.2	0.15	0.1	0.04	—
5	Correction H	-0.4	-0.4	-0.4	-0.4	-0.4
6	Correction J	-0.3	-0.27	-0.20	-0.15	-0.07
7	Type 8 (corrected)	102.1±0.6	92.7±0.6	71.4±0.4	45.0±0.5	21.6±0.3
8	Hard component of range $\geq 80$ g cm <sup>-2</sup> of air	70.6±0.4	65.6±0.4	52.2±0.3	34.1±0.2	16.5±0.1
9	Experimentally defined soft component (item 7-item 8)	31.5±0.7	27.1±0.7	19.2±0.5	11.6±0.6	5.1±0.3
10	Mesonic and protonic component of soft spectrum correction M and N	3.4±0.3	3.0±0.3	2.1±0.2	1.2±0.1	0.4±0.05
11	Type 7 electronic component of soft spectrum item 9-item 10	28.1±0.8	24.1±0.8	17.1±0.6	10.4±0.6	4.7±0.3

To compare this properly with Kraushaar's published value, we must apply a slight correction for his counter inefficiencies after the manner of Sec. III C. This calculation yields a correction of +0.4 percent per counter or +0.8 percent for his telescope. Our final corrected value of his result is

$$I_v(80 \text{ g cm}^{-2} \text{ dense air}) = (0.894 \pm 0.005) \times 10^{-2} \text{ cm}^{-2} \text{ sterad}^{-1} \text{ sec}^{-1}.$$

The excellent agreement of these two intensity measurements lends support to the general validity of our scheme of correcting for showers and secondaries.

## V. TREATMENT OF DATA

In analyzing these data it was found convenient to divide the observed coincidences into classes according to the type of absorber used. In Table III these classes are defined and the types of corrections which have been applied to each class are indicated. Those types of corrections not included are considered to be negligible.

While we shall not present all the observations and corrections, but only the final intensities, it seems worth while to illustrate the procedure and the order of magnitude of these corrections for one component; hence in Table IV we have summarized the procedure followed in obtaining the type 7 component which consists essentially of all electrons with ranges between 0.55 g cm<sup>-2</sup> of Al and 80 g cm<sup>-2</sup> of air equivalent.

In Fig. 3 the corrected rates are shown for types 2 to 7 of the soft spectrum as obtained at Ithaca. The solid curves, which fit the data within the standard deviations, have the property that the differences of the ordinates of any two adjacent curves are expressible by some power of  $\cos\theta$ . This is convenient, for we may now readily express our corrected rates as intensities for various energy intervals by the method of Sec. III B. In Table V the results of this conversion are given.

For the Echo Lake observations the corrected rates

TABLE V. Summary of corrected rates and absolute intensities of various difference components at Ithaca.

Component	Rate (hr <sup>-1</sup> )	K	Absolute intensity (cm <sup>-2</sup> sec <sup>-1</sup> sterad <sup>-1</sup> )
7-6	13.0 cos <sup>3.8</sup> θ	2.12	0.170 × 10 <sup>-2</sup> cos <sup>3.8</sup> θ
6-5	3.8 cos <sup>2.7</sup> θ	2.18	0.048 × 10 <sup>-2</sup> cos <sup>2.7</sup> θ
5-4	2.2 cos <sup>2.5</sup> θ	2.19	0.028 × 10 <sup>-2</sup> cos <sup>2.5</sup> θ
4-3	4.3 cos <sup>2.4</sup> θ	2.20	0.054 × 10 <sup>-2</sup> cos <sup>2.4</sup> θ
3-2	3.2 cos <sup>1.9</sup> θ	2.22	0.040 × 10 <sup>-2</sup> cos <sup>1.9</sup> θ
2	1.0	2.37	0.010 × 10 <sup>-2</sup>
1	3.0	2.37	0.035 × 10 <sup>-2</sup>
8	{ 102.2 cos <sup>2.53</sup> θ 96.5 cos <sup>2.14</sup> θ	2.19 2.21	{ 1.29 × 10 <sup>-2</sup> cos <sup>2.53</sup> θ for 0 ≤ θ ≤ 30° 1.21 × 10 <sup>-2</sup> cos <sup>2.14</sup> θ for 30° ≤ θ ≤ 60°

are shown in Fig. 4, while the analytical expressions for the rates and absolute intensities are given in Table VI.

## VI. VALIDITY OF RESULTS AND COMPARISON WITH OTHER MEASUREMENTS

### A. Estimated Validity of the Final Intensities

It is considered that the greatest source of error in the whole experiment is associated with the scattering of the soft component particles in the absorber. In this section, where we shall compare our results with those of other workers, this error does not enter, since the comparison will be made in terms of absorber thickness. It will be considered later, however, when our range spectrum is converted to an energy spectrum.

From a consideration of the relative magnitudes of the various remaining errors and the cross-checks used in the experiment, we have placed the following estimates on the validity of our results

#### Observed Rates

Types 1 and 2: The error here is probably no greater than 20 percent, although it must be recognized that the process of extrapolation to zero wall thickness may contain some unrecognized error.

Types 3, 4, 5, 6, and 8: The determinate errors are considered to be no greater than the standard deviations; thus we estimate the total errors to be no greater than 1.5 times the standard deviations.

Type 7: An error of from 5 to 10 percent (several times the standard deviation) may arise here, owing to the location of the carbon absorber above the *c* counter.

TABLE VI. Summary of corrected rates and absolute intensities of various difference components at Echo Lake.

Component	Rate (hr <sup>-1</sup> )	K	Absolute intensity (cm <sup>-2</sup> sec <sup>-1</sup> sterad <sup>-1</sup> )
7-6	61 cos <sup>4.7</sup> θ	2.095	0.81 × 10 <sup>-2</sup> cos <sup>4.7</sup> θ
6-5	20 cos <sup>3.6</sup> θ	2.14	0.26 × 10 <sup>-2</sup> cos <sup>3.6</sup> θ
5-4	12 cos <sup>3.3</sup> θ	2.16	0.15 × 10 <sup>-2</sup> cos <sup>3.3</sup> θ
4-3	21 cos <sup>2.1</sup> θ	2.21	0.26 × 10 <sup>-2</sup> cos <sup>2.1</sup> θ
3-2	14.5 cos <sup>2.1</sup> θ	2.21	0.18 × 10 <sup>-2</sup> cos <sup>2.1</sup> θ
2	2.5	2.37	0.029 × 10 <sup>-2</sup>
1	8.8	2.37	0.103 × 10 <sup>-2</sup>
8	{ 261 cos <sup>2.56</sup> θ 235 cos <sup>2.26</sup> θ	2.19 2.205	{ 3.31 × 10 <sup>-2</sup> cos <sup>2.56</sup> θ 0 ≤ θ ≤ 45° 2.96 × 10 <sup>-2</sup> cos <sup>2.26</sup> θ 45° ≤ θ ≤ 60°

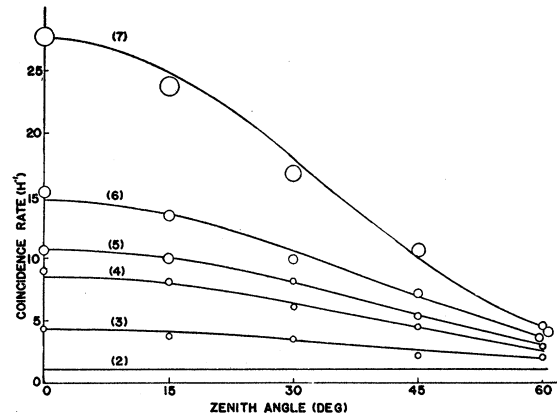


FIG. 3. Corrected rates for zenith angles out to 60° of types 2 to 7 of the electronic component at Ithaca.

### Analytical Expressions for Rates and Intensities

Owing to the manner in which data for many of the components were obtained (as the difference of two simultaneously observed rates) many of the errors are quite small. We consider that the average error in the expressions of the form  $R \cos^n \theta$  is no greater than 10 percent in the amplitude ( $R$ ) and no greater than 15 percent in the exponent ( $n$ ). For those components of large magnitude, the accuracy may be slightly better; for those of small magnitude it will possibly be worse.

### B. Comparison with other Measurements

In Table VII we have presented for comparison with our results, other observations which contain enough of the experimental details to permit correction in accordance with our scheme. It will be observed that good agreement is obtained among the first seven experiments, provided the correction for extensive showers is re-evaluated as explained above, instead of using directly the displaced center counter rate.

In the last four measurements of Table VII, a rather large error appears. This discrepancy, which is of the order of 20 percent of the soft component, is far greater

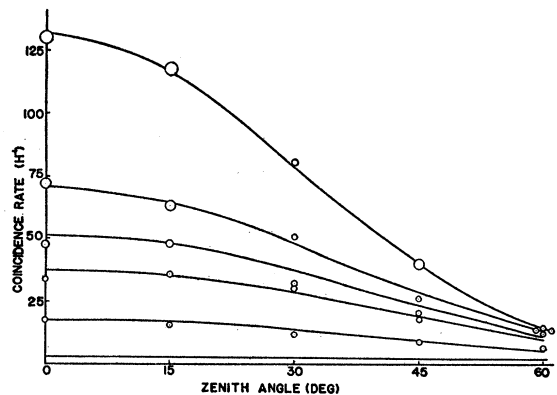


FIG. 4. Corrected rates for zenith angles out to 60° of types 2 to 7 of the electronic component at Echo Lake.

TABLE VII. Comparison of corrected results of various observers. *P* refers to results of this measurement.

Type of component	Altitude (meters)	Observer	Intensity		Comments on corrections
			$I$ in units of $\text{cm}^{-2} \text{sec}^{-1} \text{sterad}^{-1}$	$J_2$ in units of $\text{cm}^{-2} \text{sec}^{-1}$	
1. Hard-vertical	275	Kraushaar <sup>a</sup>	$I_v = (0.894 \pm 0.005) \times 10^{-2}$	1.	Correction <i>C</i> applied
2. Hard-vertical	275	<i>P</i>	$I_v = (0.886 \pm 0.006) \times 10^{-2}$		
3. Hard-integrated	275	Greisen <sup>b</sup>	$J_2 = (1.803 \pm 0.012) \times 10^{-2}$		4. Obtained from our observed vertical intensity and well established zenith angle dependence for mesons (17)
4. Hard-integrated	275	<i>P</i>	$J_2 = (1.811 \pm 0.12) \times 10^{-2}$	4.	
5. Total vertical	0	Montgomery <sup>c</sup>	$I_v = (1.215 \pm 0.01) \times 10^{-2}$	5.	A shower correction of $-1.0$ percent (rather than $-5.0$ percent) and a $-0.5$ percent correction for wall-generated secondaries have been applied
6. Total vertical	0	<i>P</i>	$I_v = (1.23 \pm 0.01) \times 10^{-2}$	6.	Corrected to sea level
7. Total vertical	0	Greisen <sup>d</sup>	$I_v = (1.22 \pm 0.01) \times 10^{-2}$	7.	This component has been corrected to sea level, to zero wall thickness and for an overcorrection made in the shower correction. Owing to the uncertainties of these corrections, the excellent agreement obtained is fortuitous
8. Integrated total of range 2.2 g $\text{cm}^{-2}$ brass	275	Greisen <sup>b</sup>	$J_2 = 2.51 \times 10^{-2}$		8(a). Recorrected by removal of shower correction
8(a). Integrated total of range 2.2 g $\text{cm}^{-2}$ brass	275	Greisen	$J_2 = 2.44 \times 10^{-2}$	8(a).	
9. Integrated total of range 2.2 g $\text{cm}^{-2}$ brass	275	<i>P</i>	$J_2 = 2.38 \times 10^{-2}$		
10. Soft-integrated	275	Greisen <sup>b</sup>	$J_2 = 0.71 \times 10^{-2}$	10.	Measurements 8-3
10(a). Soft-integrated	275	Greisen	$J_2 = 0.64 \times 10^{-2}$	10(a).	Measurements 8a-3
11. Soft-integrated	275	<i>P</i>	$J_2 = 0.57 \times 10^{-2}$	11.	Measurements 9-4

<sup>a</sup> See reference 17.  
<sup>b</sup> See reference 9.

<sup>c</sup> See reference 12.  
<sup>d</sup> See reference 6.

than can be expected on the basis of statistical fluctuations of the observations. Since the hard component results of these measurements are in such good agreement (measurements 3 and 4 of Table VII) it appears that the trouble is in some way associated with the soft component. Considerations of the effects of extensive showers and secondaries enable us to indicate in a reasonable manner how these results may be reconciled; it has not been possible, however, to make a precise correction.

As shown in Sec. I of part III, Greisen's shower effect (two particles with parallel paths simultaneously traversing the telescope) has been found to be negligible by Montgomery<sup>12</sup> and ourselves. A computation of the contribution to be expected on the basis of extensive shower observations<sup>15</sup> confirms this conclusion. When Greisen's shower correction is made negligible his observed electronic component is decreased by 10 percent. This makes no difference to his hard component measurement, however, as the shower correction was negligible there.

The empirical method used by Greisen to determine his "collision" correction is based on the assumption that secondary electrons and showers originating in an absorber will not diverge to any great extent from their primary before emerging from the absorber; hence the arrangement of several counters used to determine this correction should form a highly efficient "collision" detector. We consider this assumption to hold quite well in the case of Pb and Fe; with carbon absorbers, however, it is probable that this assumption is not

strictly valid, and that the detector has a low efficiency. Such an effect will lead to an underestimation of the collision correction and an overestimation of the electronic component. It has not been possible to determine the value of the correction; we can only say that a further reduction of 10 percent in the soft component on this basis is not unreasonable.

#### VIII. CONVERSION OF RANGES TO ENERGY LIMITS

The number of electrons,  $N_0(t)$ , observed to traverse a telescope and absorber (of combined thickness  $t$ ) in unit time is given by

$$N_0(t) = - \int_0^\infty P(R', t) \frac{dN(R')}{dR'} dR',$$

where  $N(R)$  is the number of electrons of total range  $R$  or greater arriving in the solid angle of the telescope in unit time and  $P(R, t)$ , the penetration probability, is the probability that an electron capable of traveling a total path length  $R$  in the absorber, actually traverses a thickness  $t$  ( $t < R$ ) and is detected by the apparatus. In order to obtain an energy spectrum we must first convert  $N_0(t)$  to  $N(t)$  where the latter represents the number of electrons arriving in the solid angle of the telescope in unit time for which the total range is  $t$  g  $\text{cm}^{-2}$  (the actual thickness of absorber). The conversion to an energy spectrum is then made in a straightforward manner by means of a standard relationship between energy and total range. The initial step is



carried out by correcting  $N_0(t)$  for the number of absorbed electrons as follows:

$$N(t) = N_0(t) - \int_0^\infty \{1 - P(R', t)\} \frac{dN(R')}{dR'} dR'.$$

Now if the penetration probability is known,  $N(t)$  may be determined by the usual process of successive substitution of  $N_0(t)$  for  $N(t)$  in the integral, then the first-order corrected spectrum, and so on. It was found that the magnitude of the correction term to be applied to each observed rate depends primarily on the form of the penetration probability and that the differences between successive order correction terms varied in geometric progression. Thus the total correction term to be applied in each case could readily be ascertained.

The penetration probability was obtained as follows: For a thin beam of monoenergetic electrons of total range  $R$  in carbon, incident perpendicularly on a carbon plate, the radial distribution function  $F(t, r)$  for those particles lying at a depth  $t$  and possessing a perpendicular displacement  $r$  from the axis of the beam is found to be<sup>23,24</sup>

$$F(t, r) dr = \frac{1}{2A_2(R, t)} \exp\left(-\frac{r^2}{4A_2(R, t)}\right) r dr,$$

where

$$A_2(R, t) = \int_0^t \frac{(t-\eta)^2}{W^2(\eta)} d\eta \quad \text{and} \quad W(t) = \frac{2p\beta}{E_s}.$$

At each depth  $t$  there will exist an effective maximum displacement  $D(R, t)$  such that

$$P(R, t) = \int_0^{D(R, t)} F(t, r) dr.$$

While the correct value of  $D(t)$  is not known, it is certainly less than  $(R^2 - t^2)^{1/2}$ . We have taken  $D(t) = [R(R-t)]^{1/2}$ , which is the form obtained by Fowler *et al.*<sup>25</sup> We have used their method of correcting for the effect of radiation upon the penetration probability, i.e., by the use of a factor of the form  $\exp(-ht)$  where  $h = 0.018 \text{ g}^{-1} \text{ cm}^2$  for carbon. Thus our method is essentially the same as Fowler's with the added feature that the variation of energy loss has been handled by the method of Eyges.<sup>24</sup> The final form of the penetration probability is

$$P(R, t) = \exp(-ht) \{1 - \exp[-R(R-t)/4A_2]\}.$$

The actual computation was carried out for initial energies 5, 10, 15, 20, 26, 33, and 40 Mev. A total range  $R$  was obtained for these energies by means of Greisen's calculations<sup>26</sup> of the maximum ranges of electrons in

aluminum. These results were used as they agreed rather closely with experimental determinations of this quantity at lower energies.<sup>27</sup> The value of  $A_2(R, t)$  as a function of  $t$  was then obtained by numerical integration. The final curves for  $P(R, t)$  for various initial energies (or ranges) are shown in Fig. 5. The results for intermediate energies are readily obtained by interpolation. It must be noted that we have neglected the straggling of electrons about the maximum range  $R$  and have not considered the occasional increase in range occurring by production of photons which release new electrons at greater thicknesses.

The final results of our computations are given in Table VIII as energy limits to be associated with the absorber thicknesses and correction factors to be applied to the observed integrated intensities to obtain the integrated intensities for electrons of these lower energy limits. In Table IX are given the absolute intensities of the four most energetic groups of electrons at each altitude obtained by application of the previous results to the observed intensities of Tables V and VI.

#### IX. SEPARATION OF THE HIGH ENERGY ELECTRONIC COMPONENTS

As we have indicated initially, the  $E_\pi$  component can be obtained by removal of the computed  $E_\mu$  component from the total electronic component. This procedure was carried out as follows. Using the meson intensity measurements of Kraushaar<sup>17</sup> and the computations of Bernardini *et al.*,<sup>7</sup> corrected for a meson mass of 108 Mev/ $c^2$ , a half-life of  $2.2 \times 10^{-6}$  sec, and a three-particle  $\mu$ -meson decay scheme, the integrated vertical intensity of the  $E_\mu$  component of energy  $\geq 30$  Mev is readily obtained for both altitudes. Since our altitudes were identical with those of Kraushaar and quite close to those used by Bernardini and co-workers, a linear interpolation was used with the latter for the small altitude corrections required. These vertical  $E_\mu$  intensities were then subtracted from the total vertical electronic intensities obtained by taking the difference of the observed rates of types 6 and 7, correcting as

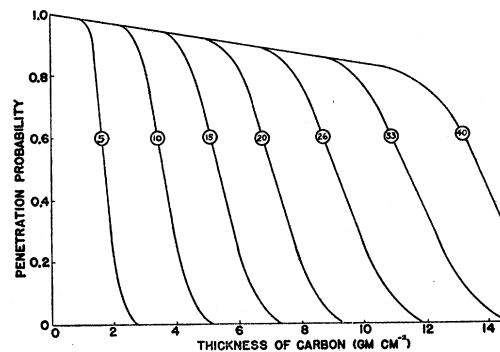


Fig. 5. Penetration probability to various depths in carbon absorber for electrons of initial energies of 5, 10, 15, 20, 26, 33, and 40 Mev.

<sup>23</sup> B. Rossi and K. Greisen, *Revs. Modern Phys.* **13**, 243 (1941).

<sup>24</sup> L. Eyges, *Phys. Rev.* **74**, 1534 (1948).

<sup>25</sup> Fowler, Lauritsen, and Lauritsen, *Revs. Modern Phys.* **20**, 265 (1948).

<sup>26</sup> K. I. Greisen, thesis, Cornell (1943).

<sup>27</sup> F. L. Hereford and C. P. Swann, *Phys. Rev.* **78**, 728 (1950).

TABLE VIII. Correction factors for conversion of integral range spectra to integral energy spectra.

Thickness of carbon absorber (g cm <sup>-2</sup> )	Lower energy limit Mev	Correction <sup>a</sup> factor
0.00	0.00	1.00
2.62	6.5	1.14
5.62	13.3	1.26
8.18	19.6	1.35
12.46	30.8	1.47

<sup>a</sup> This multiplying factor is to be applied to the integrated vertical electronic component for the given absorber, for both Ithaca and Echo Lake.

described previously and multiplying by the factor 1.47 to adjust the observed intensities to this energy limit (as discussed in the previous section). The procedure for the nonvertical directions was similar, the additional assumption being introduced that the  $E_\mu$  component varies as  $\cos^2\theta$ . This amounts essentially to neglecting the effect of multiple air scattering upon the zenith angle distributions of the high energy electronic components. We believe that this assumption does not lead to any considerable error for not only are most of the electrons of rather high energy, but as will be indicated later, the effect of scattering upon the zenith angle distribution is not very great at higher energies. In Fig. 6 the computed  $E_\mu$  component is shown by a solid line, while the  $E_\pi$  and total electronic components obtained by use of the data are indicated along with their standard deviations. The solid lines used to fit the data have been obtained in an identical manner from the smoothed out data of Figs. 3 and 4. The results at large angles of such a procedure are of uncertain value; owing to the large uncertainties in the small quantities involved, we obtain slightly negative values at 60°.

The high energy  $E_\pi$  component is seen to possess a very steep zenith angle distribution (steeper at Ithaca

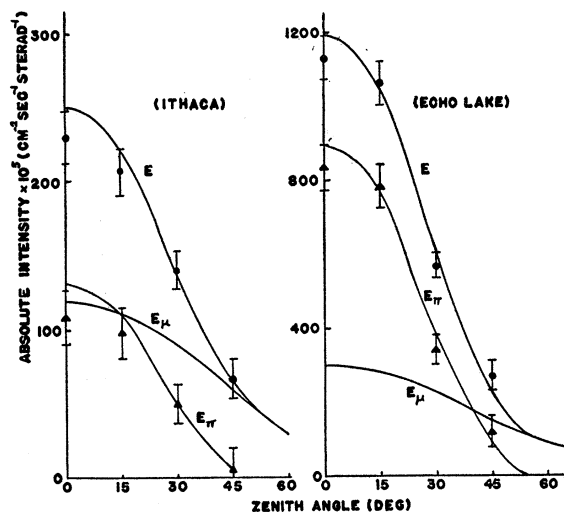


FIG. 6. Absolute intensities (multiplied by 10<sup>5</sup>) for zenith angles out to 45° of the  $E$ ,  $E_\pi$ , and  $E_\mu$  electronic components of energy greater than 30.8 Mev at Ithaca and Echo Lake.

than at Echo Lake). The results for the two altitudes may be related by plotting the logarithm of all the  $E_\pi$  intensities from 0° to 45° on one figure (Fig. 7) as a function of  $h/\cos\theta$ . The line joining the two vertical intensity points as obtained from the smoothed out data gives an excellent fit of the data, and we obtain for the absolute integrated intensity of the  $E_\pi$  component, at low altitudes, at atmospheric depth  $h$ , and zenith angle  $\theta$  for energies greater than 30 Mev, the following expression:

$$I_{E_\pi}(h, \theta) = 0.70 \exp(-h/160 \cos\theta) \text{ cm}^{-2} \text{ sec}^{-1} \text{ sterad}^{-1}.$$

#### X. SEPARATION OF THE LOW ENERGY ELECTRONIC COMPONENTS

The electronic components of the remaining bands, which are composed entirely of low energy electrons, have undergone rather severe atmospheric scattering, and cannot be separated without consideration of this phenomenon. The effect of multiple scattering upon the zenith angle distributions has been determined for two groups of 100-Mev electrons initially with  $\cos^2\theta$  and

TABLE IX. Absolute intensities of four most energetic bands of the electronic component at Ithaca and Echo Lake.

Energy limits of components (Mev)	Absolute intensities (cm <sup>-2</sup> sec <sup>-1</sup> sterad <sup>-1</sup> )	
	Ithaca	Echo Lake
6.5-13.3	(0.032 × 10 <sup>-2</sup> ) cos <sup>2.4</sup> θ	(0.15 × 10 <sup>-2</sup> ) cos <sup>2.1</sup> θ
13.3-19.6	(0.016 × 10 <sup>-2</sup> ) cos <sup>2.5</sup> θ	(0.10 × 10 <sup>-2</sup> ) cos <sup>3.2</sup> θ
19.6-30.8	(0.044 × 10 <sup>-2</sup> ) cos <sup>2.7</sup> θ	(0.25 × 10 <sup>-2</sup> ) cos <sup>3.0</sup> θ
30.8-	(0.25 × 10 <sup>-2</sup> ) cos <sup>3.8</sup> θ	(1.19 × 10 <sup>-2</sup> ) cos <sup>4.7</sup> θ

$\cos^6\theta$  dependences, representing, respectively, the  $E_\mu$  and  $E_\pi$  components of greater energies. The computation, which was performed numerically, consisted essentially in the repeated application of the standard scattering formulas<sup>23</sup> to obtain the zenith angle dependence at successively lower energies. In Table X are given the final results of the computations; the details will be given in a later publication dealing with the scattering of the extreme soft component. From Table X we may readily determine how the relative abundance of the  $E_\mu$  and  $E_\pi$  components, now known for energies greater than 30 Mev, changes as we proceed to the lower energy groups. This information, along with the computed zenith angle distribution for the  $E_\mu$  component, allows a separation to be effected. In Fig. 8 the total electronic component and the separated electron components for the three most energetic electron bands of the lower energy groups are shown for both altitudes. The resultant  $E_\pi$  curves compare favorably with the curves (shown by dotted lines) obtained by extending the computed  $E_\pi$  vertical intensity to large zenith angles in accordance with the results of our scattering calculations as summarized in Table X.

XI. CONCLUSIONS

Our conclusions can readily be arranged into two groups: those dealing with the experimental technique associated with a soft component study and those dealing with the properties of the various electronic components in the lower atmosphere. The first group are summarized in the following list of conditions which should be observed in any measurement of the soft component. This is an extension and in some places a modification of a list published by Azimov and his collaborators.<sup>8</sup>

(a) The multiple scattering of low energy electrons, both in the air and in the apparatus, must be considered throughout the analysis of the data. This effect leads to the most serious errors encountered in soft component measurements.

(b) Counter walls must be thin and of a low *Z* material, and all absorbers should be below the counters defining the solid angle of the telescope.

(c) A large solid angle of the telescope should be used; this will in general diminish the effects of side showers and scattering, although the angular resolution required will place an upper limit on the allowed solid angle.

TABLE X. Calculated zenith angle distributions of the electronic component slowed down to various energies from 100 Mev. The representations hold for  $\theta \leq 60^\circ$ .

Energy, Mev	Original form of $1.00 \cos^2\theta$	Original form of $\cos^2\theta$
100	$1.00 \cos^2\theta$	$1.00 \cos^2\theta$
68	$0.98 \cos^{1.99}\theta$	$0.965 \cos^{5.4}\theta$
40	$0.96 \cos^{1.96}\theta$	$0.908 \cos^{4.8}\theta$
30	$0.94 \cos^{1.94}\theta$	$0.865 \cos^{4.4}\theta$
20	$0.91 \cos^{1.85}\theta$	$0.793 \cos^{3.8}\theta$
15	$0.88 \cos^{1.68}\theta$	$0.733 \cos^{3.3}\theta$
12.25	$0.84 \cos^{1.53}\theta$	$0.688 \cos^{2.8}\theta$
10.35	$0.815 \cos^{1.46}\theta$	$0.647 \cos^{2.5}\theta$

(d) The hard and soft components must be corrected for side (extensive) showers. The best correction can be obtained directly from extensive shower measurements.<sup>15</sup> Determinations by simple "out of line" coincidence methods are seriously wrong.

(e) Corrections must be applied for locally generated showers and secondaries.<sup>14</sup>

(f) Meteorological corrections should not be made on the basis of barometric pressure changes only, unless the measurements have been taken continuously over a long period.

(g) Geiger counters of large cross section or high pressure with thin walls will reduce the corrections required for short path lengths in the gas.

With regard to the properties of the two electronic components, our assumption of two components and our method of separation are such that the discrepancies mentioned in the introduction now appear as the expected behavior of the soft component, at least for energies greater than 30 Mev. For energies lower than this value it is also necessary to consider the effect of multiple scattering in the air in order to account for the zenith angle distributions. That the scheme is reasonable, however, is indicated by the result that the absorption length ( $160 \text{ g cm}^{-2}$ ) obtained for the  $E_\pi$

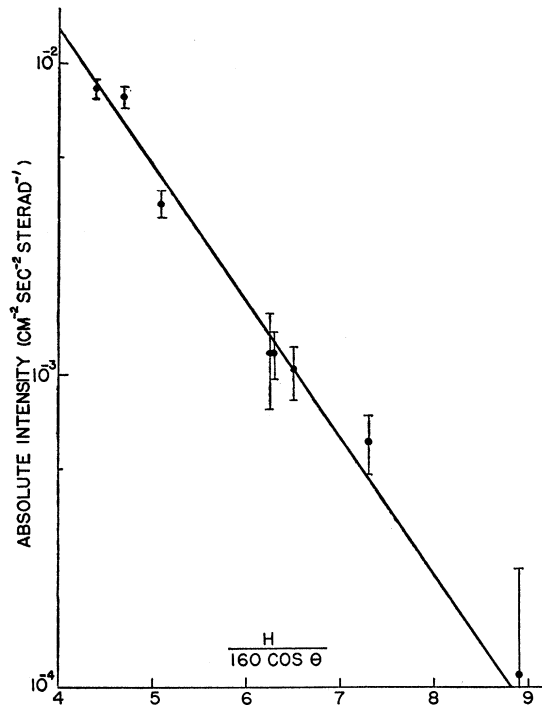


Fig. 7. Absolute intensity of  $E_\pi$  component of energy greater than 30.8 Mev at Ithaca and Echo Lake for zenith angles out to  $45^\circ$  plotted against  $h/160 \cos\theta$ .

component is certainly in accord with what is expected from our present knowledge of the altitude variation of

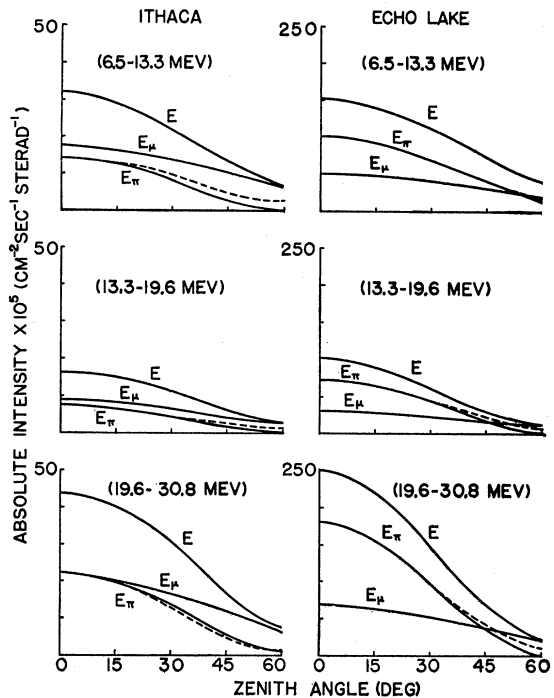


Fig. 8. Absolute intensities (multiplied by  $10^6$ ) for zenith angles out to  $45^\circ$  of the  $E$ ,  $E_\mu$ , and  $E_\pi$  electronic components of various energies less than 30.8 Mev at Ithaca and Echo Lake.

the  $N$  component and the properties of the neutral meson.

The author, in conclusion expresses his gratitude to Professor K. I. Greisen for his suggestion of this problem, and for his extensive help during the course of its

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## Six-Millimeter Spectra of OCS and $N_2O$ †

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The  $J=3\rightarrow 4$  pure rotational transition of OCS and the  $J=1\rightarrow 2$  transition of  $N_2O$  have been investigated. The molecules were in their normal isotopic abundances. The results are compared with those of previous investigators. Quadrupole hyperfine structure,  $l$ -type doubling, and Fermi resonance effects are analyzed.

### I. INTRODUCTION

**B**OTH the  $J=3\rightarrow 4$  pure rotational transition of the linear molecule OCS and the  $J=1\rightarrow 2$  rotational transition of the linear molecule  $N_2O$  occur in the wavelength region around 6 mm. These spectra have been measured using the apparatus previously described.<sup>1</sup>

### II. CARBON OXYSULFIDE

The OCS molecule has been extensively investigated at wavelengths longer than 6 mm, and some work has been done at shorter wavelengths. The OCS was prepared by the action of  $NH_4SCN$  on  $H_2SO_4$  in water.<sup>2</sup> All OCS isotopic species were observed in their normal

abundances. Ground-state lines were measured at  $T=195^\circ K$ , and excited vibrational state lines were measured at room temperature. The pressure was kept at about  $2\times 10^{-3}$  mm Hg. Line positions were measured to an accuracy of  $\pm 0.1$  Mc/sec.

Table I presents the experimental results obtained. No attempt was made to measure intensities, the relative intensities only being used as an aid in identifying the lines.

Only two quadrupole hyperfine components due to the  $O^{16}C^{12}S^{32}$  molecule were found. Using the value of the coupling constant ( $eqQ$ ) =  $-29.07\pm 0.01$  Mc/sec obtained by Eshbach, Hillger, and Strandberg,<sup>3</sup> and the center of gravity  $\nu_0=48,038.86$  Mc/sec, the calculated line positions agree almost exactly with those observed.

The calculated rotational constants and  $l$ -type doubling constant are shown in Table II. A value of  $1.28\pm 0.05$  kc/sec for  $D_0$ , the centrifugal distortion coefficient in the ground vibrational state of  $O^{16}C^{12}S^{32}$ , was used in the calculation. This value is obtained from the data of Johnson, Trambarulo, and Gordy.<sup>4</sup> Strandberg, Wentink, and Kyhl<sup>5</sup> obtained a value of  $D_0$  in  $O^{16}C^{12}S^{32}$  of  $1.60\pm 0.05$  kc/sec. The first value agrees very closely with the theoretically determined value.  $D_v$  in excited vibrational states and in all isotopes was taken equal to  $D_0$  in  $O^{16}C^{12}S^{32}$ . The values of  $\alpha_1$  and  $\alpha_2$  were obtained from the excited state data, corrections being made for Fermi resonance (see below). The value of  $\alpha_3$  was obtained from an analysis of the frequency shifts for isotopic changes of the masses of each of the three atoms of OCS. Townes, Holden, and Merritt obtained the relationship<sup>6</sup>

$$\alpha_1 + 2\alpha_2 + \alpha_3 = 0.00856B_0,$$

TABLE I.  $J=3\rightarrow 4$  rotational spectrum of OCS.

Molecule	Vibrational state and $F$ transition	Experimental frequency in Mc/sec	Previously determined frequency in Mc/sec	Calculated intensity <sup>b</sup> at $300^\circ K$ in $10^{-8}$ cm <sup>-1</sup>
$O^{16}C^{12}S^{32}$	0 0 0	48,651.40 $\pm$ 0.10	48,651.64 $\pm$ 0.05 <sup>a</sup>	436
	1 0 0	48,506.24		7.0
	0 1 0	48,710.80		34.9
	0 1 0	48,761.55		34.9
	0 2 0	48,801.08		2.7
$O^{16}C^{12}S^{34}$	0 2 0	48,819.92		5.2
	0 0 0	47,462.20	47,462.40 $\pm$ 0.05 <sup>a</sup>	17.8
$O^{16}C^{13}S^{32}$	0 0 0	48,494.76		4.7
$O^{16}C^{12}S^{33}$	0 0 0			
	3/2 $\rightarrow$ 5/2	48,038.19		1.0
	5/2 $\rightarrow$ 7/2			
	7/2 $\rightarrow$ 9/2	48,039.13		1.8
9/2 $\rightarrow$ 11/2				

<sup>a</sup> Strandberg, Wentink, and Kyhl, Phys. Rev. **75**, 270 (1948).

<sup>b</sup> Calculated using intensity relation of P. Kisliuk and C. H. Townes, J. Research Natl. Bur. Standards **44**, 611 (1950),  $\Delta\nu=6$  Mc/sec, and the dipole moments given in R. G. Shulman and C. H. Townes, Phys. Rev. **77**, 500 (1950). Those moments not measured were taken equal to 0.7085 debye.

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<sup>1</sup> S. J. Tetenbaum, Phys. Rev. **86**, 440 (1952).

<sup>2</sup> Stock, Siecke, and Pohland, Ber. deut. chem. Ges. **57**, 719 (1924).

<sup>3</sup> Eshbach, Hillger, and Strandberg, Phys. Rev. **85**, 532 (1952).

<sup>4</sup> Johnson, Trambarulo, and Gordy, Phys. Rev. **84**, 1178 (1951).

<sup>5</sup> Strandberg, Wentink, and Kyhl, Phys. Rev. **75**, 270 (1949).

<sup>6</sup> Townes, Holden, and Merritt, Phys. Rev. **74**, 1113 (1948).