Proton Triton Interaction*,†

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A phase shift analysis of the Los Alamos data on p-T scattering for incident protons of 0.708 to 2.548 Mev is carried out using singlet and triplet s and p phase shifts. The large p wave phase shifts obtained are shown to be consistent with the suggested ^1P resonant level of the α-particle. The effect of absorption associated with the T(p,n)He^3 reaction is taken into account by means of a schematic resonance model. A satisfactory fit is obtained with attractive singlet and triplet potentials of depths ~46 Mev and ~11 Mev, respectively. The singlet depth is chosen to give the observed bound state of the α-particle.

INTRODUCTION

DURING the last few years extensive experimental investigations of the interactions of protons and tritons have been carried out at Los Alamos and the University of Minnesota. Differential cross sections for elastic scattering, T(p,p)T, have been reported by Hemmendinger *et al.*[1,2] for proton energies in the range 0.708 to 2.548 Mev. The energy range has been extended to 3.5 Mev by Claasen *et al.*[3] The angular distribution of neutrons from the reaction T(p,n)He^3, which becomes energetically possible for proton energies above 1.019 Mev, has been reported by Jarvis *et al.*[4] for proton energies up to 2.487 Mev. Argo *et al.*[5] have obtained the angular distribution and energy dependence for the reaction T(p,γ)He^4.

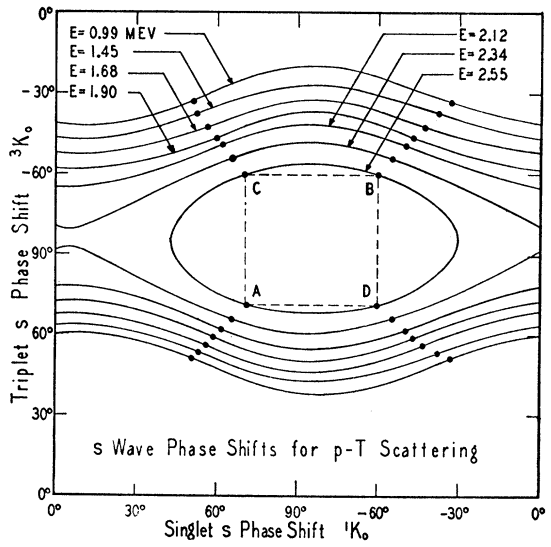

FIG. 1. Triplet s phase shift, $^3K_0$, *vs* singlet s phase shift, $^1K_0$.

In Sec. I a phase shift analysis of the scattering data previously reported[6] is carried out. The differential cross section is fitted with singlet and triplet s and p wave phase shifts with tensor and spin-orbit forces neglected. The absorption of protons from the incident beam, connected with the reaction $T(p,n)$He^3, is ignored in the present treatment, but the effect of absorption on the s wave scattering is considered in Sec. III using the schematic resonance treatment of Breit.[6]

In Sec. II the large ^1P phase shifts resulting from the phase shift analysis are related to the resonant state of He^4 suggested by Jarvis *et al.*[4] from the T(p,n)He^3 data and by Argo *et al.*[5] from the T(p,γ)He^4 data.

The contents of this paper have been presented in part at the 1952 Washington meeting of the American Physical Society.[7]

I. PHASE SHIFT ANALYSIS

As a first attempt to fit the p-T scattering data, it is assumed that there is a spin dependent central force between the proton and triton which allows for a different interaction in singlet and triplet spin states. A serious defect in this model is that no account is taken of the reaction T(p,n)He^3 which competes with elastic scattering for incident proton energies above 1.019 Mev. A schematic resonance treatment to take account of the absorption due to the reaction is discussed in Sec. III.

On the assumption of only spin dependent central forces the cross section for p-T scattering may be written as

$$\sigma(\theta)=\tfrac{1}{4}\cdot{}^1\sigma(\theta)+\tfrac{3}{4}\cdot{}^3\sigma(\theta), \quad (1)$$

where

$$\sigma^j(\theta)=\left|-(\eta/2ks^2)+k^{-1}\sum_{L=0}^{\infty}(2L+1)P_L(\cos\theta)\sin{}^jK_L \times\exp i[{}^jK_L+2(\sigma_L-\sigma_0)+\eta_l\ln s^2]\right|^2, \quad j=1,3 \quad (2)$$

and θ=scattering angle in the center-of-mass system, v=velocity of incident proton, μ=reduced mass of

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[1] Taschek, Jarvis, Hemmendinger, Everhart, and Gittings, Phys. Rev. 75, 1361 (1949).

[2] Hemmendinger, Jarvis, and Taschek, Phys. Rev. 76, 1137 (1949).

[3] Claasen, Brown, Freier, and Stratton, Phys. Rev. 82, 589 (1951).

[4] Jarvis, Hemmendinger, Argo, and Taschek, Phys. Rev. 79, 929 (1950).

[5] Argo, Gittings, Hemmendinger, Jarvis, and Taschek, Phys. Rev. 78, 691 (1950).

[6] G. Breit, Phys. Rev. 69, 472 (1946).

[7] McIntosh, Gluckstern, and Sack, and Freeman, Phys. Rev. 87, 237 (1952).

TABLE I. p-T scattering phase shift fits for possibilities A, B, C, D. All angles are expressed in degrees.

E_{Mev}	$^3K_0$	$^1K_0$	$^3K_1$	$^1K_1$	$^3K_1$	$^1K_1$[a]
			Possibility A			
0.990	51	51	$-15\frac{1}{2}$	25	2	$-38\frac{1}{2}$
1.108	$49\frac{1}{2}$	$49\frac{1}{2}$	-8	19	$4\frac{1}{2}$	-23
1.236	49	49	$-10\frac{1}{2}$	$21\frac{1}{2}$	4	-28
1.450	$52\frac{1}{2}$	$52\frac{1}{2}$	$-11\frac{1}{2}$	24	4	-31
1.678	$56\frac{1}{2}$	$56\frac{1}{2}$	$-12\frac{1}{2}$	23	$3\frac{1}{2}$	-32
1.900	60	60	-14	25	3	-36
2.117	62	62	-16	33	4	-46
2.335	66	66	-17	32	2	-47
2.548	71	71	-18	34	1	-52
			Possibility B			
E_{Mev}	$^3K_0$	$^1K_0$	$^3K_1$	$^1K_1$	$^3K_1$	$^1K_1$[a]
0.990	-33	-33	19	-10	$5\frac{1}{2}$	$34\frac{1}{2}$
1.108	-32	-32	11	-3	4	$18\frac{1}{2}$
1.236	-33	-33	13	-6	4	23
1.450	$-37\frac{1}{2}$	$-37\frac{1}{2}$	$14\frac{1}{2}$	-10	3	$27\frac{1}{2}$
1.678	$-42\frac{1}{2}$	$-42\frac{1}{2}$	$15\frac{1}{2}$	-12	3	30
1.900	$-46\frac{1}{2}$	$-46\frac{1}{2}$	$17\frac{1}{2}$	-15	3	35
2.117	$-49\frac{1}{2}$	$-49\frac{1}{2}$	20	-23	2	45
2.335	$-54\frac{1}{2}$	$-54\frac{1}{2}$	21	-21	$3\frac{1}{2}$	47
2.548	-60	-60	23	-23	4	51
			Possibility C			
E_{Mev}	$^3K_0$	$^1K_0$	$^3K_1$	$^1K_1$[a]	$^3K_1$	$^1K_1$
0.990	-33	51	$2\frac{1}{2}$	$-28\frac{1}{2}$	22	16
1.108	-32	$49\frac{1}{2}$	$2\frac{1}{2}$	-13	13	13
1.236	-33	49	3	-19	15	15
1.450	$-37\frac{1}{2}$	$52\frac{1}{2}$	$1\frac{1}{2}$	-24	16	18
1.678	$-42\frac{1}{2}$	$56\frac{1}{2}$	1	-28	$16\frac{1}{2}$	18
1.900	$-46\frac{1}{2}$	60	$1\frac{1}{2}$	-32	18	20
2.117	$-49\frac{1}{2}$	62	0	-42	20	29
2.335	$-54\frac{1}{2}$	66	$1\frac{1}{2}$	$-43\frac{1}{2}$	22	24
2.548	-60	71	2	-49	24	26
			Possibility D			
E_{Mev}	$^3K_0$	$^1K_0$	$^3K_1$	$^1K_1$[a]	$^3K_1$	$^1K_1$
0.990	51	-33	$-1\frac{1}{2}$	44	$-12\frac{1}{2}$	-18
1.108	$49\frac{1}{2}$	-32	3	28	-7	-11
1.236	49	-33	3	32	-9	-13
1.450	$52\frac{1}{2}$	$-37\frac{1}{2}$	3	35	$-10\frac{1}{2}$	-16
1.678	$56\frac{1}{2}$	$-42\frac{1}{2}$	2	35	-12	-18
1.900	60	$-46\frac{1}{2}$	1	38	$-13\frac{1}{2}$	-19
2.117	62	$-49\frac{1}{2}$	2	49	-15	-29
2.335	66	$-54\frac{1}{2}$	0	49	-17	-27
2.548	70	-60	-2	52	-18	-31

[a] "Resonance" fits (see Sec. II).

system $=M_PM_T/(M_P+M_T)\cong\frac{3}{4}M$, $k=\mu v/\hbar$, $\eta=e^2/\hbar v$, $s=\sin(\theta/2)$, $L=$ orbital angular momentum, $^jK_L=$ phase shift for L and $j=1, 3$, $\sigma_L=$ Coulomb phase shift.

It is not possible to fit the data with s wave phase shifts alone because of the large observed cross section for backward scattering. For this reason p wave phase shifts ($^1K_1$, $^3K_1$) have to be included in the analysis in addition to the two s wave phase shifts ($^1K_0$, $^3K_0$), and the total of four adjustable parameters is available to fit the data.

The scheme for determining the best phase shifts at each energy is as follows:

(a) The cross section at $\theta=90°$, where the p wave contributions vanish, determines a relation between the two s wave phase shifts at each energy. Curves of $^1K_0$ *vs* $^3K_0$, with minor adjustments explained in (c), are plotted in Fig. 1 for the experimental proton energies.

(b) Calculations for the best p wave phase shifts are then carried out by trial and error for various points on each $^1K_0$ *vs* $^3K_0$ curve of Fig. 1. Four arbitrary pairs of $^1K_0$, $^3K_0$ chosen for convenience are the two pairs for $^1K_0=^3K_0$ (points denoted by A and B) and the two corresponding pairs for $^1K_0$ and $^3K_0$ opposite in sign (points denoted by C and D, with $^1K_0(C)=^1K_0(B)$, $^3K_0(C)=^3K_0(A)$ and $^1K_0(D)=^1K_0(A)$, $^3K_0(D)=^3K_0(B)$. For each pair ($^1K_0$, $^3K_0$), two distinct pairs ($^1K_1$, $^3K_1$) are found which fit the data within the quoted experimental error.

(c) Slight readjustments are made in $^1K_0$, $^3K_0$, $^1K_1$ and $^3K_1$ (as mentioned in (a)) to minimize the mean square deviation of the calculated fits from the experimental cross sections.

The adjusted values of $^1K_0$, $^3K_0$ are the ones plotted in Fig. 1 and the values of ($^1K_0$, $^3K_0$), ($^1K_1$, $^3K_1$) for possibilities A, B, C, D are given in Table I. Additional calculations have been carried out at $E=0.990$, 1.450 and 2.548 Mev to determine the values of ($^1K_1$, $^3K_1$) for values of ($^1K_0$, $^3K_0$) other than those corresponding to

TABLE II. p-T scattering phase shift fits for $E=0.990$, 1.450, 2.548 Mev. All angles are expressed in degrees.

$^3K_0$	$^1K_0$	$^3K_1$	$^1K_1$	$^3K_0$	$^1K_0$	$^3K_1$	$^1K_1$
			$E=0.990$ Mev				
-42	9	10	-26	-42	9	9	35
-33	51	$2\frac{1}{2}$	$-28\frac{1}{2}$	-33	51	22	16
-22	-81	$16\frac{1}{2}$	$-19\frac{1}{2}$	-22	-81	19	$22\frac{1}{2}$
-33	-33	19	-10	-33	-33	$5\frac{1}{2}$	$34\frac{1}{2}$
60	9	$-1\frac{1}{2}$	-35	60	9	$-1\frac{1}{2}$	45
51	51	2	$-38\frac{1}{2}$	51	51	$-15\frac{1}{2}$	25
40	-81	$-10\frac{1}{2}$	-24	40	-81	$-13\frac{1}{2}$	$27\frac{1}{2}$
51	-33	$-12\frac{1}{2}$	-18	51	-33	$-1\frac{1}{2}$	44
			$E=1.450$ Mev				
$^3K_0$	$^1K_0$	$^3K_1$	$^1K_1$	$^3K_0$	$^1K_0$	$^3K_1$	$^1K_1$
-47	$7\frac{1}{2}$	8	-22	-47	$7\frac{1}{2}$	6	29
-45	$27\frac{1}{2}$	2	$-23\frac{1}{2}$	-45	$27\frac{1}{2}$	$13\frac{1}{2}$	22
-37	52	$1\frac{1}{2}$	-24	-37	52	16	18
-30	$77\frac{1}{2}$	7	-23	-30	$77\frac{1}{2}$	15	19
-28	$-82\frac{1}{2}$	$11\frac{1}{2}$	$-17\frac{1}{2}$	-28	$-82\frac{1}{2}$	$12\frac{1}{2}$	22
-30	$-62\frac{1}{2}$	14	-13	-30	$-62\frac{1}{2}$	$8\frac{1}{2}$	$26\frac{1}{2}$
$-37\frac{1}{2}$	$-37\frac{1}{2}$	$14\frac{1}{2}$	-10	$-37\frac{1}{2}$	$-37\frac{1}{2}$	3	$27\frac{1}{2}$
-40	-30	$14\frac{1}{2}$	-10	-40	-30	2	$27\frac{1}{2}$
-45	$-12\frac{1}{2}$	14	$-12\frac{1}{2}$	-45	$-12\frac{1}{2}$	2	28
43	$-82\frac{1}{2}$	$-7\frac{1}{2}$	$-22\frac{1}{2}$	43	$-82\frac{1}{2}$	-9	$26\frac{1}{2}$
45	$-62\frac{1}{2}$	$-10\frac{1}{2}$	$-17\frac{1}{2}$	45	$-62\frac{1}{2}$	$-4\frac{1}{2}$	$32\frac{1}{2}$
$52\frac{1}{2}$	$-37\frac{1}{2}$	$-10\frac{1}{2}$	-16	$52\frac{1}{2}$	$-37\frac{1}{2}$	3	35
60	$-12\frac{1}{2}$	-8	$-21\frac{1}{2}$	60	$-12\frac{1}{2}$	$4\frac{1}{2}$	35
62	$7\frac{1}{2}$	$-\frac{1}{2}$	$-29\frac{1}{2}$	62	$7\frac{1}{2}$	1	36
60	$27\frac{1}{2}$	$4\frac{1}{2}$	$-30\frac{1}{2}$	60	$27\frac{1}{2}$	-7	$30\frac{1}{2}$
$52\frac{1}{2}$	$52\frac{1}{2}$	4	-31	$52\frac{1}{2}$	$52\frac{1}{2}$	$-11\frac{1}{2}$	24
45	$77\frac{1}{2}$	$-2\frac{1}{2}$	-29	45	$77\frac{1}{2}$	$-11\frac{1}{2}$	$23\frac{1}{2}$
			$E=2.548$ Mev				
$^3K_0$	$^1K_0$	$^3K_1$	$^1K_1$	$^3K_0$	$^1K_0$	$^3K_1$	$^1K_1$
71	71	1	-52	71	71	-18	35
71	-60	-18	-30	71	-60	-2	52
-85	-30	-21	24	-85	-32	-22	30
-60	-60	4	51	-60	-60	23	-23
-60	71	24	26	-60	71	2	-49
-85	42	22	28	-85	42	-21	-21
71	71	1	-52	71	71	-18	35

TABLE III. Rough calculation of phase shifts from Minnesota data. All angles are expressed in degrees.

E_{Mev}	$^3K_0$	$^1K_0$	$^3K_1$	$^1K_1$	$^3K_0$	$^1K_0$	$^3K_0$	$^1K_1$
2.55	64	64	−16	46	64	64	3	−60
2.74	66	66	−17	49	66	66	1	−65
3.03	66	66	−8	68	66	66	4	−73
3.25	67	67	−2	87	67	67	−2	−92
E_{Mev}	$^3K_0$	$^1K_0$	$^3K_1$	$^1K_1$	$^3K_0$	$^1K_0$	$^3K_1$	$^1K_1$
2.55	−53	64	−4	−53	−53	64	19	41
2.74	−55	66	−4	−58	−55	66	18	45
3.03	−55	66	−9	−60	−55	66	9	62
3.25	−57	67	−8	−69	−57	67	6	73
E_{Mev}	$^3K_0$	$^1K_0$	$^3K_1$	$^1K_1$	$^3K_0$	$^1K_0$	$^3K_1$	$^1K_1$
2.55	−53	−53	18	−39	−53	−53	−2	55
2.74	−55	−55	18	−42	−55	−55	−1	59
3.03	−55	−55	11	−58	−55	−55	−6	63
3.25	−57	−57	9	−67	−57	−57	−4	71
E_{Mev}	$^3K_0$	$^1K_0$	$^3K_1$	$^1K_1$	$^3K_0$	$^1K_0$	$^3K_1$	$^1K_1$
2.55	64	−53	0	60	64	−53	−16	43
2.74	66	−55	−1	64	66	−55	−17	46
3.03	66	−55	2	72	66	−55	−11	62
3.25	67	−57	−1	81	67	−57	−8	73

points A, B, C, D. The results are given in Table II.

In the case of scattering without a Coulomb field there is an indeterminacy in the sign of the calculated phase shifts. Specifically, if the data are fitted by

$$(^1K_0, ^3K_0), (^1K_1, ^3K_1),$$

they will be equally well fitted by

$$(^1K_0, -^3K_0), (^1K_1, -^3K_1);$$
$$(-^1K_0, -^3K_0), (-^1K_1, -^3K_1);$$
$$(-^1K_0, ^3K_0), (-^1K_1, ^3K_1).$$

An analogous situation exists when a Coulomb field is present. Because of the interference between the nuclear and Coulomb scattered waves, the phase shifts of one class are not so simply related to those of another class and corresponding fits in different classes are not equally good. Thus the presence of the Coulomb field

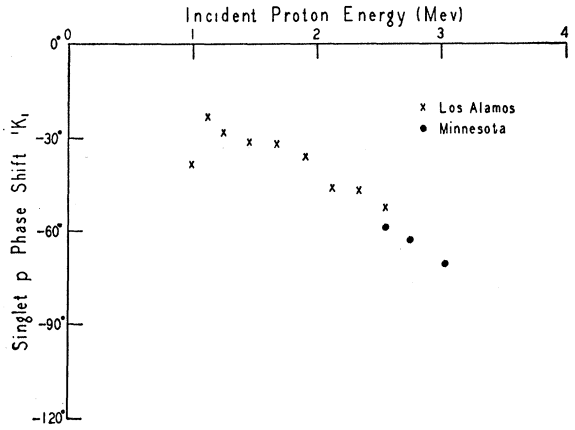

FIG. 2. Singlet p "resonant" phase shift *vs* energy for $^3K_0=^1K_0>0$ (possibility A).

appears to offer the possibility of choosing between the many possible values of the phase shifts.

Unfortunately it does not prove possible to choose a "best" set of phase shifts on the basis of the "best" fit to the experimental data. As was previously mentioned, all sets of phase shifts given in Tables I and II fit the data within the experimental error. However, the various fits do differ from one another significantly at low angles. (At $E=1.450$ Mev and $\theta=15°$ the possible phase shift fits lead to cross sections which differ by as much as a factor of two.) It therefore seems likely that data below $\theta=59°$ will eliminate many of the fits in Tables I and II. In addition it is expected that small d waves (up to 5°) will be necessary in order to obtain satisfactory fits to the low angle data.

The presence of spin-orbit and tensor interactions has been neglected. The introduction of a tensor force, for example, would split the triplet p phase shift into three, giving two additional fitting parameters. The

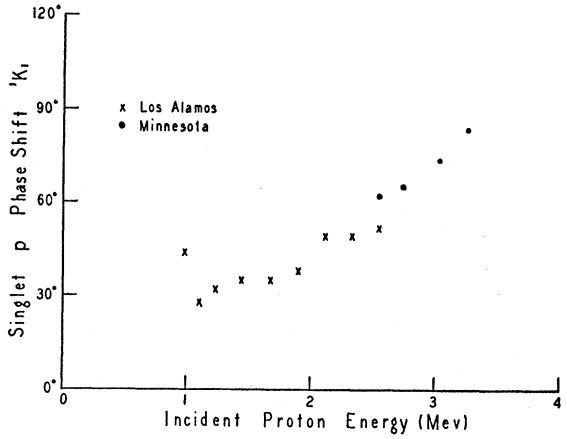

FIG. 3. Singlet p "resonant" phase shift *vs* energy for $^3K_0=^1K_0<0$ (possibility B).

accuracy of the experimental data does not seem to warrant an elaborate investigation employing p wave splitting at the present time.

II. POSSIBLE ^1P RESONANT LEVEL OF He^4

A feature of the results in Tables I and II which is surprising is that the magnitude of the p wave phase shifts required for the best fits is considerably larger than would be expected for scattering from a simple potential well. However, these p phase shifts may not be unreasonable if there exists a ^1P resonant state of He^4 as suggested by Argo *et al.*[5] from considerations of the $T(p,\gamma)He^4$ reaction data. Subsequent measurements of Falk and Phillips[8] are not in agreement with the results of Argo *et al.* and seem to indicate that if a resonant state exists, it occurs at a proton energy above 3.5 Mev. The disagreement may be due to a difference in targets and to the choice of the type of curve

[8] C. E. Falk and G. C. Phillips, Phys. Rev. **83**, 468 (1951).

drawn through the experimental points. In addition Flowers and Mandl[9] have shown that it may be possible to explain the large slope of the yield *vs* energy curve without assuming a resonance. Thus although the existence of the resonant state is far from certain, it may prove helpful to determine whether the large p waves are consistent with the resonance. Such a ^1P resonant state would permit large values of $^1K_1$ but would require small $^3K_1$. Half of the fits in Table I (denoted by *) are of this character and are referred to as "resonance" fits, while the others are called "non-resonance" fits.

In order to investigate the variation of $^1K_1$ with energy for the "resonance" fits, rough calculations have been performed with the data of Claasen *et al.*[3] at proton energies from 2.12 to 3.5 Mev. For simplicity calculations were performed only for regions A, B, C, D although it is quite unlikely that the true variation of $^1K_0$ and $^3K_0$ with energy will follow any of these possibilities exactly. The results are given in Table III and they join reasonably well with the results obtained from the Los Alamos data.[2]

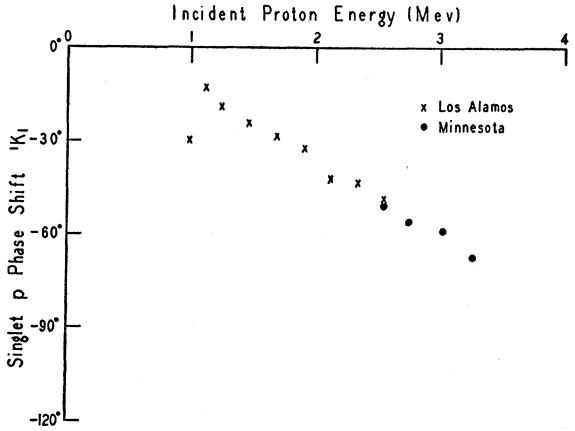

FIG. 4. Singlet p "resonant" phase shift *vs* energy for $^3K_0<0$, $^1K_0>0$ (possibility C).

The "resonant" values of $^1K_1$ are plotted against energy in Figs. 2–5 for possibilities A, B, C, D. Although wide variation of $^1K_1$ is evident, the absence of points above 3.5 Mev does not permit any definite conclusions about the resonant state. If such a state does occur the variation of $^1K_1$ with energy suggests that it occurs at about 3–4 Mev with a half-width of about 1 Mev. However only possibilities B and D seem consistent with the usual expectation of a phase shift which increases as the energy increases through the resonance.

From Figs. 3 and 5 it may be seen that possibilities B and D imply that the singlet s phase shift $^1K_0$ should be negative (equivalent to values between 90° and 180°), but do not imply a restriction on $^3K_0$. It can be shown that the singlet potential well necessary to permit the known ground state of He^4 leads to negative values of $^1K_0$ in the energy range 0–5 Mev, so that on the basis of the assumed model, a ^1P excited state of He^4 is consistent with the information used thus far.

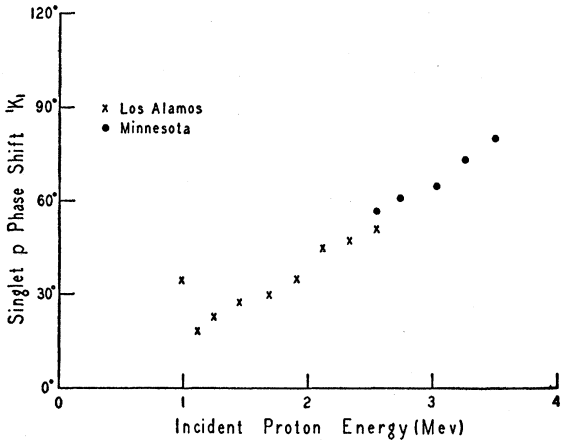

FIG. 5. Singlet p "resonant" phase shift *vs* energy for $^3K_0>0$, $^1K_0<0$ (possibility D).

Another point consistent with the "resonance" fits comes from consideration of the strong cusps in the $\sigma(\theta)$ *vs* E curves in Fig. 4 of Hemmendinger *et al.*[2] for low angle scattering. It has been pointed out by Wigner[10] that at the threshold of a reaction, the energy dependence of the elastic scattering cross section may show a cusp. In the present case the cusp is approximately located at 1 Mev, the threshold of the T(p,n)He^3 reaction. One expects that near threshold, the neutrons, which are predominantly s wave, come directly from the s wave protons, and therefore the cross section for scattering should break sharply downward. This is not the case for T(p,n)He^3, where the scattering cross section seems to increase sharply. The increase can be accounted for by the removal of a large destructive s–p interference term which arises from large p waves. This is indeed the case for angles less than 90° for the "resonance" fits. These fits correspond to values of $^1K_0$ and $^1K_1$ which are always opposite in sign, while $^3K_1$ is sufficiently small so that the triplet s–p interference may be neglected for the immediate purpose.

Consideration of the angular distribution of neutrons from the reaction T(p,n)He^3 furnishes further evidence favoring the "resonance" fits. Jarvis *et al.*[4] in their paper reporting the experimental results, analyzed the angular distribution in a power series in $\cos\theta$ and found that the coefficient of $\cos^2\theta$ rises rapidly with energy, suggesting a p wave resonance. The question may be examined more quantitatively with the use of a schematic resonance treatment. It is assumed for the energies here considered (<1.35 Mev above threshold) that only s and p wave protons contribute to the reaction, although as pointed out in Sec. III, there is probably some d wave contribution for the higher energies.

[9] B. H. Flowers and F. Mandl, Proc. Roy. Soc. (London) **A206**, 131 (1951).

[10] E. Wigner, Phys. Rev. **73**, 1002 (1948).

TABLE IV. Resonance widths for assumed ^1P resonance at $E^R=2.0$ Mev.

E(Mev)	$(E^R-E')/\Gamma_p$	Γ_p(Mev)
1.62	1.10	0.73
1.83	0.81	0.80
2.05	0.58	0.83
2.20	0.45	0.82
2.36	0.31	0.81

The further assumption is made that all the p wave neutrons are produced by a single ^1P resonant level of the He^4 system. The reaction cross section may then be written

$$^1\sigma_{pn}(\theta)=(1/k^2)\,|\,a_0+3\cos\theta a_1\,|^2, \tag{3}$$

where a_0 is the s wave contribution and a_1 is given by

$$a_1=(\Gamma_p\Gamma_n)^{\frac{1}{2}}\exp i(\delta_p+\delta_n)/(E^R-E'-i(\Gamma_p+\Gamma_n)), \tag{4}$$

where Γ_p and Γ_n are the level widths for protons and neutrons, respectively, δ_p and δ_n are protons and neutron phase shifts, E^R is the resonant energy associated with the compound level, and E' is the energy in the center-of-mass system. The effects of the s wave are eliminated by considering

$$^1\sigma_{pn}(45°)+{}^1\sigma_{pr}(135°)-2\,{}^1\sigma_{pn}(90°)=(9/k^2)\,|\,a_1\,|^2. \tag{5}$$

For the "resonance" fits, $^3K_1\approx 0$ and therefore

$$^3\sigma_{pn}(45°)+{}^3\sigma_{pn}(135°)-2\,{}^3\sigma_{pn}(90°)\cong 0. \tag{6}$$

Since $\sigma_{pn}=(3/4)\,{}^3\sigma_{pn}+(1/4)\,{}^1\sigma_{pn}$,

$$\sigma_{pn}(45°)+\sigma_{pn}(135°)-2\sigma_{pn}(90°)=(9/4k^2)\,|\,a_1\,|^2$$
$$=(9/4k^2)(\Gamma_n/\Gamma_p)(\Gamma^2{}_p/[(E^R-E')^2+\Gamma_p{}^2]), \tag{7}$$

where Γ_p is written in place of $\Gamma=\Gamma_p+\Gamma_n$ since $\Gamma_n\cong 0$ near threshold. One now writes

$$\Gamma_n/\Gamma_p\approx(v_n/v_p)^{2L+1}/[(1+\eta^2)2\pi\eta/(\exp 2\pi\eta-1)], \tag{8}$$

where the factor in η is related to the Coulomb barrier factor $C_1{}^2$ for p waves. In view of other uncertainties no attempt is being made to be precise about taking into account the correction factor for the Coulomb field. Substitution of (8) into (7) gives a roughly constant value of Γ_p if E^R is set equal to 2.0 Mev, corresponding to a laboratory energy of 2.7 Mev as shown in Table IV.

The angular distribution of the reaction neutrons thus suggests a broad ^1P resonance at a laboratory energy of $\sim$2.7 Mev with Γ_p in the laboratory system $\approx$1.1 Mev. These figures are consistent with the broad resonance, $\Gamma\approx$1 Mev indicated by the "resonance" fits of the p-T scattering data [Figs. 2–5]. The above calculation further agrees with the scattering analysis in placing the ^1P resonant level of He^4 somewhat higher than the 2.5 Mev suggested by Argo *et al.*[5] and Jarvis *et al.*[4] The possibility of d wave contributions to the neutron cross section, discussed in Sec. III, makes the numerical results somewhat uncertain.

III. SCHEMATIC RESONANCE TREATMENT OF THE S WAVES

For incident protons of energy 1.019 Mev and higher, neutrons from the reaction $T(p,n)He^3$ are observed among the outgoing particles.[4] The reaction cross section rises sharply with energy, and for 2.5-Mev protons it is over 20 percent of the scattering cross section. Thus the effect of absorption from the proton beam is large, and the phase shift analysis of the preceding sections must be modified to take the reaction into account. It has been shown by Ostrofsky, Breit, and Johnson[11] and others[12] that the effect of absorption may be accounted for by the introduction of a complex potential energy, which leads to complex phase shifts. A somewhat more definite model is provided by the schematic resonance treatment of Breit.[6,13] From this point of view the total singlet or triplet cross section, in the case of no Coulomb field, for a particle incident in the pth mode and leaving in the qth mode is given by

$$\sigma_{p\to q}=(2L+1)(\Lambda_p{}^2/\pi\,|\,\delta_{pq}\exp(-i\delta_p)\sin\delta_p$$
$$+\textstyle\sum_j G_{qj}G_{pj}/(E_j{}^R-E'-i\sum_q\Gamma_{qj})\,|^2, \tag{9}$$

where the effects of relative spin orientations of the particles are neglected. Here the p and q modes may be taken to refer to s wave protons and neutrons, respectively, Λ_p is the wavelength of the incident protons, δ_p is the phase shift produced by the effective potential, E' is the energy in the center-of-mass system, $E_j{}^R$ is the real part of the energy corresponding to the jth compound level, Γ_{qj} is the partial level width of the qth mode and the jth level, and G_{qj} is related to the Γ_{qj} by the equation:

$$|\,G_{qj}\,|^2=\Gamma_{qj}\textstyle\sum_r|\,\xi_j{}^r\,|^2. \tag{10}$$

The sum of the squares of the transformation coefficients[6] $\sum_r|\,\xi_j{}^r\,|^2$ is never less than unity, reducing to unity in the one-level case. It is seen from Eq. (9) that the model gives a scattered wave consisting of two parts, one of which is such as would take place from a potential and another consisting of contributions associated with resonance levels. The manner in which the resonance

TABLE V. Values of ^1T, ^3T for $R=(^3T)^2/(^1T)^2=0.5$, 1.0, 2.0.[a]

Incident energy	$R=0.5$		$R=1.0$		$R=2.0$	
	^1T	^3T	^1T	^3T	^1T	^3T
0.990 Mev	0.309	0.223	0.244	0.244	0.185	0.262
1.108	0.328	0.236	0.259	0.259	0.196	0.277
1.236	0.346	0.249	0.273	0.273	0.206	0.293
1.450	0.381	0.275	0.301	0.301	0.228	0.322
1.678	0.416	0.300	0.329	0.329	0.249	0.352
1.900	0.450	0.325	0.356	0.356	0.269	0.381
2.117	0.483	0.348	0.381	0.381	0.288	0.409
2.335	0.517	0.374	0.409	0.409	0.310	0.439
2.548	0.551	0.398	0.435	0.435	0.329	0.467

[a] See Eqs. (17) and (18) for definitions of ^3T and ^1T.

[11] Ostrofsky, Breit, and Johnson, Phys. Rev. **49**, 22 (1936).
[12] H. A. Bethe, Phys. Rev. **57**, 1125 (1940).
[13] Breit, Condon, and Present, Phys. Rev. **50**, 825 (1936).

level parameters G, E^R, Γ are connected with the initial parameters of the model does not matter in the present application, the form of the answer being the main feature used below. The equations are applied here to s wave scattering, and it is assumed that s wave neutrons are produced by incident s wave protons. The subscripts p and n are henceforth used for the proton and neutron s wave modes, respectively. To obtain the s wave cross section the reaction data are taken at 90°, where the p wave contributions vanish. Introduction of Coulomb terms then leads to the differential cross section at 90°:

$$\sigma_{p\to q}(90°)=k^{-2}\{\tfrac{3}{4}|-\eta\exp[i(\eta\ln 2-2{}^3\delta_0)]+\exp(-i\,{}^3\delta_0)\sin({}^3\delta_0)+\textstyle\sum_j({}^3G_{pj})^2/({}^3E_j{}^R-E'-i\sum_q{}^3\Gamma_{qj})|^2+\tfrac{1}{4}|-\eta\exp[i(\eta\ln 2-2\,{}^1\delta_0)]+\exp(-i\,{}^1\delta_0)\sin({}^1\delta_0)+\textstyle\sum_k({}^1G_{pk})^2/({}^1E_k{}^R-E'-i\sum_q\Gamma_{qk})|^2\}. \quad (11)$$

$$\sigma_{p\to n}(90°)=k^{-2}\{\tfrac{3}{4}|\textstyle\sum_j({}^3G_{nj})({}^3G_{pj})/({}^3E_j{}^R-E'-i\sum_q{}^3\Gamma_{qj})|^2+\tfrac{1}{4}|\textstyle\sum_k({}^1G_{nk})({}^1G_{pk})/({}^1E_k{}^R-E'-i\sum_q{}^1\Gamma_{qk})|^2\}. \quad (12)$$

Since it would be hopeless to attempt to ascertain the parameters G, E^R, Γ for all terms in the expansion, the resonance terms are lumped into ${}^3T(E')\exp[i\,{}^3\tau(E')]$ and ${}^1T(E')\exp[i\,{}^1\tau(E')]$, and it is assumed that one may approximate

$$G_{nj}/G_{pj}=(v_n/v_p)^{\frac{1}{2}}C_0^{-1}, \quad (13)$$

where v_n and v_p are the neutron and proton velocities and

$$C_0^2=(2\pi\eta)/[\exp(2\pi\eta)-1]. \quad (14)$$

Equation (13) is reasonable since the G's vary approximately as $v^{\frac{1}{2}}$ [see Breit,[6] Eq. (5.4)] and C_0^2 is included to approximate the effects of the Coulomb barrier for protons. One then has

$$\sigma_{p\to p}(90°)=k^{-2}\{\tfrac{3}{4}|-\eta\exp[i(\eta\ln 2-2\,{}^3\delta_0)]+\exp(-i\,{}^3\delta_0)\sin({}^3\delta_0)+{}^3T\exp(i\,{}^3\tau)|^2+\tfrac{1}{4}|-\eta\exp[i(\eta\ln 2-2\,{}^1\delta_0)]+\exp(-i\,{}^1\delta_0)\sin({}^1\delta_0)+{}^1T\exp(i\,{}^1\tau)|^2\}, \quad (15)$$

$$\sigma_{n\to p}(90°)=k^{-2}(v_n/v_pC_0^2)\{\tfrac{3}{4}({}^3T)^2+\tfrac{1}{4}({}^1T)^2\}, \quad (16)$$

where

$${}^3T(E')\exp[i\,{}^3\tau(E')]=\textstyle\sum_j({}^3G_{pj})^2/({}^3E_j{}^R-E'-i\sum_q{}^3\Gamma_{qj}), \quad (17)$$

and

$${}^1T(E')\exp[i\,{}^1\tau(E')]=\textstyle\sum_k({}^1G_{pk})^2/({}^1E_k{}^R-E'-i\sum_q{}^1\Gamma_{qk}). \quad (18)$$

Six parameters, ${}^1\delta_0$, ${}^3\delta_0$, ${}^1T$, ${}^3T$, ${}^1\tau$, ${}^3\tau$ must now be determined from the experimental information available. It is assumed for the present that

$$R=({}^3T)^2/({}^1T)^2 \quad (19)$$

TABLE VI. Values of ${}^1\tau$, ${}^3\tau$ for $R=({}^3T)^2/({}^1T)^2=0.5$, 1.0, 2.0.[a]

Incident energy	$R=0.5$ ${}^1\tau$	$R=0.5$ ${}^3\tau$	$R=1.0$ ${}^1\tau$	$R=1.0$ ${}^3\tau$	$R=2.0$ ${}^1\tau$	$R=2.0$ ${}^3\tau$
0.990 Mev	18.0°	12.9°	14.1°	14.1°	10.7°	15.2°
1.108	19.1°	20.3°	22.4°	22.4°	16.7°	24.0°
1.236	35.5°	24.7°	27.3°	27.3°	20.2°	29.5°
1.450	44.9°	30.6°	33.9°	33.9°	25.0°	36.5°
1.678	54.0°	35.7°	39.8°	39.8°	28.9°	43.2°
1.900	64.0°	40.5°	45.3°	45.3°	32.5°	49.6°
2.117	79.1°	45.1°	50.8°	50.8°	35.9°	56.3°
2.335	···	50.3°	57.2°	57.2°	39.6°	64.4°
2.548	···	55.7°	64.4°	64.4°	43.1°	75.7°

[a] See Eqs. (17) and (18) for definitions of ${}^1T$, ${}^3T$, ${}^1\tau$, and ${}^3\tau$.

is a constant, that is, that the singlet and triplet resonance terms vary in the same way with energy. Actually this restriction is not a critical one and will be relaxed later. It is made temporarily in order not to deal with too many parameters at once. For any value of R, ${}^3T$ and ${}^1T$ can then be calculated and other quantities entering into the equation may be obtained from the known energies. The values of ${}^3\tau$ and ${}^1\tau$ are fixed for any given R by conservation of particles. Since it is assumed that there is no mixing between singlet and triplets, the wave function for protons with either spin state is given by

$$\sin(k_rr_p)+\exp(ik_pr_p)\{\exp(i\delta)\sin\delta+T\exp[i(2\delta+\tau)]\}=-(2i)^{-1}\exp(-ik_pr_p)+\exp(ik_pr_p)\times\{(2i)^{-1}\exp(2i\delta)+T\exp[i(2\delta+\tau)]\}. \quad (20)$$

For the practical case the Coulomb field is screened, so its contribution merely adds to δ, which drops out in the final result. Therefore one may here neglect the Coulomb term. Equating incoming and outgoing particle fluxes, one obtains

$$\sin\tau=T\{1+(v_n/v_pC_0^2)\} \quad (21)$$

separately for singlets and triplets. Since a choice of R fixes both ${}^1T$ and ${}^3T$, it also fixes both ${}^1\tau$ and ${}^3\tau$. Table V gives ${}^1T$ and ${}^3T$ for $R=\frac{1}{2}$, 1, 2, and Table VI gives the corresponding values of ${}^1\tau$ and ${}^3\tau$ for these choices of R.

The phase shifts ${}^1\delta_0$ and ${}^3\delta_0$ remain to be calculated. To find ${}^1\delta_0$, a square well potential model for the singlet s state of He^4 is employed. The range of force is taken to be

$$r_0{}^{(1)}=1.5\times10^{-13}\times4^{\frac{1}{3}}=0.238\times10^{-12}\text{ cm}.$$

The depth is chosen to give the bound ${}^1S_0$ state of He^4 at 20.5 Mev below the threshold of $T(p,n)He^3$ or 19.75 Mev below $p+T$, as determined by the mass differences. The well depth so determined is $V=-46$ Mev. Various triplet well depths with $r_0{}^{(3)}=r_0{}^{(1)}$ are assumed in order to calculate ${}^3\delta_0$. For all reasonable repulsive wells and for attractive wells, $V=0$ to ~-9 Mev, the phase shifts are too small to reproduce the experimental values of the scattering cross section at 90° for any value of R. If the attractive well were deeper than ~-14 Mev, there would exist a bound ${}^3S_0$ level of He^4, which is

TABLE VII. Values of triplet well depth necessary to fit the 90° scattering data.

Incident energy (Mev)	Depth of potential well (Mev)
0.990	−11
1.108	−11
1.236	−11
1.450	−12
1.678	−13

unlikely. This limits the triplet well depth to between −9 and −14 Mev. For the lower proton energies and for values of R ranging from $\frac{1}{2}$ to 2, the value of V which reproduces the 90° scattering cross section is $V\cong-11$ Mev. The variation of this triplet well depth with energy is given in Table VII.

In the present analysis the potential well is a substitute for the main effect of many body forces acting inside the nucleus. It takes account of general effects, varying smoothly with energy, while the resonance term is concerned with additional effects caused by the possibility of formation of a semistable "compound" state. It would not be reasonable to demand strict constancy of the depth and range parameters of the well since it is a substitute for many-body effects. One expects nevertheless a $d\mathfrak{F}/\mathfrak{F}dr$ monotonically decreasing with energy and an approximate fit to the known stable states of the four-particle system.

It may be noted from Table VII that for energies greater than 1.450 Mev the well appears to deepen. It is in fact not possible to obtain a reasonable fit for energies greater than 2 Mev. However, the calculations do not hold for these energies since the assumption that the 90° cross section is due to s waves alone may no longer be valid. Jarvis *et al.*[4] have shown that for the reaction T(p,n)He^3 the coefficient of $\cos^3\theta$ in the expansion of $\sigma(\theta)$ in powers of $\cos\theta$ may not be neglected above 1.45 Mev and becomes larger above 2 Mev. The indication of d waves in the reaction cross section at higher energies as well as the probable existence of d wave contributions to the scattering cross section above 2 Mev makes the simple calculation incorrect at these energies. For energies where the d waves are less important the well depth appears quite constant. For ${}^3V\cong-11$ Mev the potential phase shift, ${}^3\delta_0$ varies from +43° to +45° in the energy range 0.990 to 2.548 Mev.

It is now necessary to examine Tables V and VI to see if a reasonable variation of T and τ with energy can be obtained. If R is chosen equal to unity for all energies, ${}^1\tau$ and ${}^3\tau$ are equal and increase from 14° at 0.990 Mev to 45° at 1.900 Mev. If a single resonant level is assumed, the variation of τ with energy indicates an s wave resonance level at ~3.5 Mev with a width of ~1 Mev. If the resonant terms are due to several levels, some of these levels must be located at low energies (less than 4 Mev) in order to account for the rapid variation of the τ's in the region from 1 to 2 Mev. The fact that these levels must be wide would probably make their detection by scattering methods impractical.

If the artificial restriction of identical singlet and triplet resonant terms ($R=1$) is removed, the qualitative features of the resonances still remain. For example if $R=2$, the one-level assumption leads to a singlet level of width ~2 Mev located at ~5 Mev and a triplet level of width ~1 Mev located at ~3 Mev. If the further restriction of constant R is removed either the singlet or triplet s wave level may be moved to higher energies by choosing entries in Table VI which give slow variations of ${}^1\tau$ or ${}^3\tau$ with energy. However the other s wave level will be moved to lower energies corresponding to more rapid variation of τ.

The presence of d waves should not alter the qualitative features of the above discussion although the resonant level energies will probably change somewhat.

It therefore seems that the schematic resonance model leads to one or more broad low energy s levels superposed on a slowly varying background in order to satisfy scattering and reaction data simultaneously.

IV. SUMMARY AND CONCLUSIONS

From the study of the angular distribution of the p-T scattering and the T(p,n)He^3 reaction cross sections the following conclusions have been reached:

(1) The elastic scattering data may be fitted within experimental error in the angular range available by singlet and triplet s and p phase shifts, ${}^1K_0$, ${}^3K_0$, ${}^1K_1$, ${}^3K_1$, for all incident proton energies up to 2.5 Mev. There are an infinite number of s wave phase shift pairs possible, for each of which there are two p wave phase shift pairs. The values of the phase shifts for four arbitrary s wave pairs are given in Table I for each energy. Additional fits at three of the energies are given in Table II.

(2) The p wave phase shifts are in all cases larger than expected from potential scattering but one of the fits is consistent with the ${}^1P$ resonant level of He^4 suggested from the T(p,γ)He^4 data. These fits, denoted by (*) in Table I, are called "resonance" fits and have large values of the singlet p phase shift, ${}^1K_1$, and reasonably small values of the triplet p phase shift, ${}^3K_1$.

(3) The results of a rough phase shift analysis of the Minnesota scattering data are given in Table III and the singlet p "resonance" phase shifts are shown to be consistent with the suggested singlet p resonant level at about 3 to 4 Mev, with a width of about 1 Mev.

(4) The asymmetry of the T(p,n)He^3 reaction data was taken into account using the one-level resonance formulation. The results of this calculation are also consistent with the suggested singlet p resonant level of He^4 corresponding to incident protons of about 3 Mev.

(5) The schematic resonance treatment of Breit[6] is used to combine the p-T scattering data and the T(p,n)He^3 reaction data. The model divides both the singlet and triplet scattering amplitudes into a contribution due to resonant levels $[T\exp(i\tau)]$ and a con-

tribution due to potential scattering, while the reaction amplitude is due only to the resonant term. The values of ${}^1T$ and ${}^3T$ for different assumptions of the relative contribution of the singlet and triplet levels are given in Table V and the corresponding values of ${}^1\tau$ and ${}^3\tau$ (related to ${}^1T$ and ${}^3T$ by conservation requirements) are given in Table VI. It is found that the large values of the reaction cross section lead to values of T and τ which require that there be one or more broad ($\sim$1 Mev) low energy ($<$4 Mev) resonant s levels.

(6) The singlet potential scattering is obtained from a square well chosen to give the known singlet bound state of He^4. The triplet phase shifts, obtained from the schematic resonance model, are shown to be consistent with an attractive triplet well about $\frac{1}{4}$ as deep as the singlet well. The variation of the calculated well depth with energy is shown in Table VII.

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Small Angle X-Ray Scattering from Compact Identical Particles*

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A general expression is deduced for the intensity distribution of x-rays scattered at small angles from a compact system of identical particles. The scattering element selected for this derivation includes several near neighbors with the origin particle, thus taking cognizance of short-range order. An inherently positive intensity distribution may then be deduced for any such system describable by an "average" radial electron density which may be approximated by linear segments. This approximation for a compact system of impenetrable spheres compares favorably with the meager data available.

SMALL angle scattering theory[1] indicates that the intensity of scattering from a dilute system of particles may be expressed as a simple sum of the intensities scattered by the individual particles. Theoretical and experimental considerations both reveal the error in extension of this simple summation to compact systems. Early attempts to evaluate the scattering of x-rays at small angles by a compact system have not proven wholly adequate. The first attempt[2] gave qualitative agreement with experiment, predicting the observed maximum of scattered intensity at a small angle. This theory, based on the earlier work of Debye,[3] shares the glaring fault of that work in predicting negative intensities for sufficient compactness. A second attempt,[4] although avoiding this negative intensity, seems to predict the intensity maximum at rather too small values of angle.[5]

We have undertaken to deduce an expression for the amplitude of the electric field scattered from any system for which a radial electron density ρ is conceivable. Such a theory is not confined to a two-phase system of fixed electron density, but may involve any number of fixed or varying electron densities. A system having a radial electron density which is significant for only a few particle radii is, as will be evident, a fair approximation to a typical system of particles. We can then conceive of a specimen composed of volume elements, each one containing a particle surrounded by its neighbors. If we integrate the scattering from the electrons within each volume element and square, we will then have an expression for the intensity scattered from each volume element, which, when added for M volume elements, gives us the intensity scattered by the specimen.

The scattered amplitude from one such volume element is given by

$$A=A_e\int_V \exp(-ikr\cos\alpha)dV, \tag{1}$$

where A_e=amplitude scattered per electron, $k=(4\pi/\lambda)\times\sin(\epsilon/2)$, λ=radiation wavelength, ϵ=angle between incident and scattered rays, r=magnitude of radius vector from the origin, and α=angle between r and the line bisecting the incident and the scattered rays.

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