electron energy; and k is a constant of the order of unity.

With this approximation the formula for the extrapolated thickness t_0 [see Eq. (7)] becomes

$$|t_0|=\frac{2S}{D}=\frac{2\int_n^1 N(\gamma)\frac{d\gamma}{\gamma}\phi\left(\gamma,\frac{n}{\gamma}\right)}{\int_n^1 \Psi(E_s,\gamma)\frac{d\gamma}{\gamma}\phi\left(\gamma,\frac{n}{\gamma}\right)}, \quad (9)$$

where n is the ratio of the positron energy to the incident electron energy, E_s; Ψ and ϕ are the Bethe-Heitler formulas[3,4] for bremsstrahlung and pair production, respectively; and N and γ are the same as in Eq. (8). Substituting the result of this experiment for t_0 the arbitrary constant k in the formula for the virtual quanta spectrum was calculated:

$$k=1.6\pm0.2. \quad (10)$$

This is in agreement with the Weizsaker-Williams approximation.

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Symmetrical Pseudoscalar Meson Theory of Nuclear Forces*

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Nuclear forces yielded by the symmetrical pseudoscalar theory are discussed in terms of a perturbation expansion. It is shown that, up to terms in the square of the coupling constant, the pseudoscalar coupling is equivalent to a scalar pair coupling of the pseudoscalar field plus a pseudovector coupling of that field. The dominant contribution to the fourth-order nucleon potential is then obtained in a simple way by using this result.

INTRODUCTION

THE nuclear force given by the symmetrical pseudoscalar meson theory with pseudoscalar coupling has been re-examined in recent years by a number of authors[1] treating the meson-nucleon interaction as weak. They have shown that the contribution to these forces due to processes involving transport of momentum between nucleons by a pair of mesons are larger than those due to a single meson. The pseudoscalar character of the meson implies that simultaneous emission or absorption of S-state meson pairs by a single nucleon cannot involve nucleon spin change. Similarly the symmetrical theory implies that there can be no isotopic spin change for these processes. As a consequence the forces due to them are spin and charge independent.

The importance of these effects suggests that one should be able to exhibit them explicitly in the meson-nucleon interaction Hamiltonian. In what follows the nuclear forces are obtained in a simple manner by taking advantage of this possibility. Besides having the aforementioned properties it is shown that these forces are highly singular and have a range of half the meson Compton wavelength.

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[1] K. M. Watson and J. V. Lepore, Phys. Rev. **76**, 193 (1949) and Phys. Rev. **76**, 1157 (1949); H. A. Bethe, Phys. Rev. **76**, 191 (1949); Y. Nambu, Prog. Theoret. Phys. **5**, 4, 614 (1950); R. P. Feynman, California Institute of Technology lecture notes (unpublished).

I. TRANSFORMATION OF THE HAMILTONIAN

The equation of motion, in the interaction representation,[2] of the state vector, $\Psi[\sigma]$, of the coupled meson and nucleon fields is determined by the coupling Hamiltonian

$$H(x)=if\bar{\psi}(x)\gamma_5\tau_\alpha\psi(x)\phi_\alpha(x). \quad (1)$$

Here and in the following ψ, $\bar{\psi}$, ϕ represent the nucleon and meson field variables, τ_α is the nucleon isotopic spin and $\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4$ is the Dirac pseudoscalar. Units are chosen so $\hbar=c=1$ and the meson and nucleon masses are μ and K_0, respectively.

If one now applies the transformation $e^{iS[\sigma]}$ used by Dyson,[3]

$$S[\sigma]=-\frac{if}{2K_0}\int d\sigma_\mu\bar{\psi}(x)\gamma_\mu\gamma_5\tau_\alpha\phi_\alpha(x)\psi(x), \quad (2)$$

to the state vector $\Psi[\sigma]$,

$$\Psi[\sigma]=e^{iS[\sigma]}\Psi'[\sigma], \quad (3)$$

one finds that the new Hamiltonian is, up to terms in f^3,

$$H'(x)=H(x)-i[S[\sigma],H(x)]+\frac{\delta S[\sigma]}{\delta\sigma(x)}-\frac{i}{2}\left[S[\sigma],\frac{\delta S[\sigma]}{\delta\sigma(x)}\right]. \quad (4)$$

[2] J. S. Schwinger, Phys. Rev. **74**, 1439 (1948).
[3] F. J. Dyson, Phys. Rev. **73**, 929 (1948).

The choice of $S[\sigma]$ insures that the new Hamiltonian contains no term in $H(x)$, since

$$\frac{\delta S[\sigma]}{\delta\sigma(x)}=-H(x)-\frac{if}{2K_0}\bar{\psi}(x)\gamma_\mu\gamma_5\tau_\alpha\partial_\mu\phi_\alpha(x)\psi(x). \quad (5)$$

If this expression for $\delta S[\sigma]/\delta\sigma(x)$ is inserted in Eq. (4) one finds that the commutators can be readily evaluated to give[4]

$$\begin{aligned}H'(x)=&(f^2/2K_0)\bar{\psi}(x)\psi(x)\phi_\alpha{}^2(x)\\&-(if/2K_0)\bar{\psi}\gamma_\mu\gamma_5\tau_\alpha\partial_\mu\phi_\alpha(x)\psi(x)\\&+\tfrac{1}{2}(f/2K_0)^2[\eta_\mu\bar{\psi}(x)\gamma_\mu\gamma_5\psi(x)]^2\\&\quad-(f/2K_0)^2\bar{\psi}(x)\gamma_\mu\epsilon_{\alpha\beta\gamma}\tau_\alpha\phi_\beta(x)\partial_\mu\phi_\gamma(x)\psi(x).\end{aligned} \quad (6)$$

The main terms in $H'(x)$ correspond to a scalar pair coupling of a pseudoscalar field plus two terms corresponding to the usual pseudovector coupling. The important thing to notice is that only a single power of the nucleon rest mass is contained in the denominator of the pair coupling. A similar result has been obtained by Foldy[5] using a different canonical transformation. It can also be shown that the scalar pair coupling yields the dominant contribution to the nuclear forces in a strong coupling approximation.[6]

II. DETERMINATION OF THE NUCLEAR FORCES

The interaction Hamiltonian, $H'(x)$, may now be treated by standard Feynman-Dyson techniques to yield the nuclear forces. The second-order nuclear force due to the pseudovector coupling has been obtained by many authors[7] so only the force yielded by the pair coupling will be discussed here.

The term in the scattering matrix corresponding to the lowest order effect involving two nucleons of the scalar pair coupling is

$$S_4=\frac{-3f^4}{16K_0{}^2}\int dx_1\int dx_2\bar{\psi}(x_1)\psi(x_1)\bar{\psi}(x_2)\psi(x_2)\times\Delta_F{}^2(x_1-x_2). \quad (7)$$

$\Delta_F(x)$ is the Feynman Δ-function as defined by Dyson.[8]

The nuclear force may be inferred from this expression by replacing all quantities in Eq. (7) by their appropriate nonrelativistic approximation. One finds, in the center of momentum system, for initial and final nucleon momenta P_μ, Q_μ; P_μ', Q_μ':

$$S_4=-2\pi i\delta(P_0'+Q_0'-P_0-Q_0)(P'Q'|V_4(r)|PQ), \quad (8)$$

where

$$V_4(r)=-\frac{3f^4}{16K_0{}^2}\int_{-\infty}^{\infty}dt\Delta_F{}^2(t^2-r^2), \quad (9)$$

and $(r, t)=(x_1-x_2, t_1-t_2)$, the relative coordinates of the two nucleons.

The integral in Eq. (9) may be evaluated by transforming to momentum space or by using one of the integral representations[8] for $\Delta_F(x)$. One finds

$$V_4(r)=-\left(\frac{f^2}{4\pi}\right)^2\left(\frac{\mu}{2K_0}\right)^2\frac{3\mu}{\pi(\mu r)^2}\int_{\mu r}^{\infty}K_0(2\lambda)d\lambda+\text{contact term} \quad (10)$$

for the pair-exchange nuclear potential. There are, of course, other terms in the nuclear potential proportional to f^4; these are, however, much smaller and contribute less than ten percent to the nucleon-nucleon interaction.

At small distances $V_4(r)$ represents a highly singular potential:

$$V_4(r)\sim(\mu r)^{-1}\ln(\mu r), \quad (11)$$

whereas for large distances it has half the range of a Yukawa potential:

$$V_4(r)\sim(\mu r)^{-5/2}e^{-2\mu r}. \quad (12)$$

This potential has been studied by Lévy[9] in connection with a hard core model for the nuclear forces.

[4] J. V. Lepore, Phys. Rev. **87**, 209 (1952).
[5] L. L. Foldy, Phys. Rev. **84**, 168 (1951).
[6] J. V. Lepore, Phys. Rev. **82**, 310 (1951).
[7] W. Pauli, *Meson Theory of Nuclear Forces* (Interscience Publishers, Inc., New York, 1948), p. 7.
[8] F. J. Dyson, Phys. Rev. **75**, 486 (1949).
[9] M. Lévy, Phys. Rev. **86**, 806 (1952).