(a) $n=1$: We multiply the numerator and the denominator of the integrand by $(\omega_1-\omega_2)$:

$$J_1(r)=\frac{2}{\pi r^2}\int_0^\infty\int_0^\infty\left(\frac{1}{\omega_2}-\frac{1}{\omega_1}\right)\frac{\sin k_1 r\,\sin k_2 r k_1 k_2}{k_1^2-k_2^2}dk_1dk_2. \tag{7a}$$

By exchanging the integrations over k_1 and k_2 in the second part of the right-hand side, we obtain

$$J_1(r)=\frac{4}{\pi r^2}\int_0^\infty\frac{\sin k_2 r}{\omega_2}F_1(k_2,r)k_2dk_2, \tag{8a}$$

where the function F_1 is defined by

$$F_1(k_2,r)=\int_0^\infty\frac{\sin k_1 r k_1}{k_1^2-k_2^2}dk_1=\frac{\pi}{2}\cos k_2 r \text{ for } r>0. \tag{9a}$$

It follows, therefore, that

$$J_1(r)=\frac{1}{r^2}\int_0^\infty\frac{\sin 2k_2 r}{\omega_2}k_2dk_2=\frac{\mu}{r^2}K_1(2\mu r), \tag{10a}$$

where use has been made of the definition of $I_0(r)$ in the previous paragraph.

(b) $n=2$: We differentiate $J_2(\mu r)$ with respect to μ:

$$\frac{\partial}{\partial\mu}J_2(\mu r)=-\frac{\mu}{(2\pi)^3}\int\frac{e^{-i(\mathbf{k}_1+\mathbf{k}_2)\mathbf{r}}}{\omega_1^3\omega_2^3}d\mathbf{k}_1d\mathbf{k}_2$$

$$=-\mu[I_1(r)]^2=-\frac{2\mu}{\pi}[K_0(\mu r)]^2. \tag{11a}$$

Since $J_2(\mu r)\rightarrow 0$ for $\mu\rightarrow\infty$, it follows therefore that

$$J_2(\mu r)=\frac{2}{\pi r^2}\int_{\mu r}^\infty[K_0(x)]^2xdx. \tag{12a}$$

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$C^{12}(p,pn)C^{11}$ Cross Section from Threshold to 340 Mev*

R. Lee Aamodt,† Vincent Peterson,‡ and Robert Phillips
Radiation Laboratory, Department of Physics, University of California, Berkeley, California

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The excitation function for the reaction $C^{12}(p,pn)C^{11}$ has been measured from threshold to 340 Mev using the Berkeley 40-ft linear accelerator and 184-in. cyclotron. Absolute cross-section measurements were made at various energies, using a Faraday cup and calibrated beta-counter. The threshold occurs at 18.5±0.3 Mev. The cross section has a broad maximum of 100 millibarns near 45 Mev and decreases to 43 millibarns at 340 Mev.

I. INTRODUCTION

THE formation of radioactive C^{11} from C^{12} by high energy particles (protons, neutrons, deuterons, and alpha-particles) has been widely used at this laboratory as a monitor and detector.[1-6] These reactions have thresholds near 20 Mev and therefore discriminate against low energy background. The positron activity of C^{11}(0.97 Mev, 20.5 min)[7] is convenient for short activation and counting periods. Carbon targets are readily available in the form of graphite or polystyrene.

Knowledge of the variation of cross section with energy and the absolute value of the cross section is important to the extensive use of such reactions. The $C^{12}(p,pn)C^{11}$ reaction is of particular interest because of the number of existing proton accelerators. A considerable amount of work, both experimental and theoretical, has already been done at the Radiation Laboratory on this reaction. Before the 184-in. cyclotron was converted from deuteron to proton acceleration, Chupp and McMillan[8] measured the relative excitation curve up to 140 Mev using protons "stripped" from 190-Mev deuterons inside the cyclotron vacuum tank. By using this proton source, McMillan and Miller[9] determined the absolute cross section at 62 Mev. More recently Panofsky and Phillips,[10] working with the Berkeley 32-Mev proton linear accelerator, established the excitation curve up to 27 Mev. In particular they studied the region just above the threshold in

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* This work was sponsored by the AEC.

† Now at Los Alamos Scientific Laboratory, Los Alamos, New Mexico.

‡ Now at California Institute of Technology, Pasadena, California.

[1] Helmholz, McMillan, and Sewell, Phys. Rev. **72**, 1003 (1947).

[2] Chupp, Gardner, and Taylor, Phys. Rev. **73**, 742 (1948).

[3] Cook, McMillan, Peterson, and Sewell, Phys. Rev. **75**, 7 (1949).

[4] Bratenahl, Fernbach, Hildebrand, Leith, and Moyer, Phys. Rev. **77**, 597 (1950).

[5] S. B. Jones and R. S. White, Phys. Rev. **78**, 12 (1950).

[6] W. J. Knox, Phys. Rev. **81**, 687 (1951).

[7] E. Siegbahn and E. Born, Arkiv Mat. Astron. Fysik **30B**, No. 3 (1944).

[8] W. W. Chupp and E. M. McMillan, Phys. Rev. **72**, 873 (1947).

[9] E. M. McMillan and R. D. Miller, Phys. Rev. **73**, 80 (1948).

[10] W. K. H. Panofsky and R. Phillips, Phys. Rev. **74**, 1732 (1948).

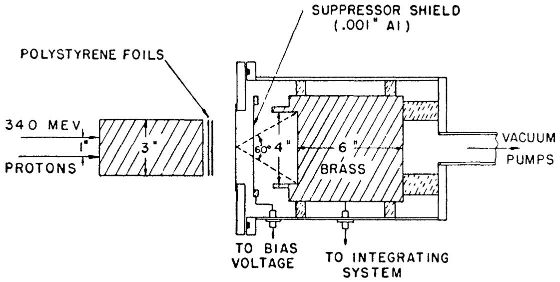

FIG. 1. Deflected beam geometry.

detail where the reaction was shown to be $C^{12}(p,d)C^{11}$. Heckrotte and Wolff[11] have calculated the excitation curve to be expected for both the (p,pn) and $(n,2n)$ reactions up to 100 Mev using a model of the nucleus proposed by Serber.

The new experimental work described here was performed mainly after an external deflected beam of 340-Mev protons became available from the 184-in. cyclotron. Absolute measurements of the (p,pn) cross section at proton energies of 340 Mev and below were then possible with a well-collimated beam. Additional work with the linear accelerator has extended the previous excitation curve up to 32 Mev, and an absolute determination of the cross section at 32 Mev has been made with improved accuracy. Considerable effort has been spent in the preparation and calibration of a beta-ray standard in order to establish the absolute values of the cross section over the entire energy range. Preliminary results of this work have been published.[12]

II. EXPERIMENTS USING 340-MEV PROTONS

A. Description of Apparatus

The experimental arrangement used in the high energy absolute cross-section determinations is shown in Fig. 1. A well-collimated beam of 340-Mev protons from the Berkeley 184-in. cyclotron was used to activate thin foils of polystyrene. The proton current incident on the foils was then collected by a Faraday cup and integrated. The C^{11} activity produced in the foils was counted with an end-window Geiger-Mueller counter and compared to the activity of various calibrated beta-ray sources (see Sec. IV).

If the incident proton flux were to remain constant while traversing its range in an absorber, the relative activities of foils interspersed in the absorber could be used to construct an excitation function. However, the range of 340-Mev protons is such ($\sim$93 g/cm^2 in copper) that nuclear processes significantly attenuate the flux. Supplementary measurements indicate that the flux is reduced to about one-half in 90 g/cm^2 of copper. Therefore it was necessary to determine the activation flux for each point at high energies.

The energy of the proton beam was reduced by interposing copper absorbers before the target foils. The initial angular spread (about 0.004 radian) and diameter (1 in.) of the beam is thereby increased as the energy is reduced, due to multiple Coulomb scattering. Calculations of the root-mean-square plane projected scattering angle and lateral displacement show that at a residual mean energy of 100 Mev, the values in copper are 5.4° and 0.35 cm, respectively. The exposure pattern observed on x-ray films sandwiched in the absorber stack confirmed that the spreading out of the beam is not yet serious at this energy. The angular spread, however, increases rapidly with decreasing residual range and it becomes difficult to collect all the current. Furthermore, the statistical straggling in range of protons initially nearly monochromatic produces an energy spread which is about 5 percent (rms) at 100 Mev and rises very steeply at lower residual energies. Therefore absolute measurements made with the high energy proton beam were not extended below a minimum energy of 93 Mev.

The Faraday cup[13] used to collect the proton current was a 6-in. long brass cylinder 6 inches in diameter mounted on insulators inside a vacuum chamber (see Fig. 1). The proton beam enters through a 2-mil Be-Cu window. The collected current was integrated using calibrated condensers and a null-method electrometer-tube voltmeter. Loss of protons from the cup due to nuclear scattering was found to be negligible by placing nuclear emulsions around the outside surface. The pressure inside the cup-chamber was kept below 0.1 micron, a factor of 100 below the pressure at which ionization effects were observed to become important. Numerous experiments were made to measure the emission of secondary electrons from the internal surfaces of the cup both by the use of a suppressor foil to which bias voltages were applied and by means of magnetic fields. It was concluded that the correction to the current measured without electrical bias or magnetic field was less than 1 percent for full energy protons.

Considering all sources of error in making an absolute measurement of charge collected by the Faraday cup, it is felt that the results are good to ±2 percent. In relation to the difficulties in determining the absolute number of beta-particles emitted by the radioactive foils (discussed in Sec. IV), the charge measurement is a minor source of error in determination of the absolute cross section.

B. Experimental Results—340 Mev to 93 Mev

Five-mil polystyrene $(C_8H_8)_n$ foils, placed behind copper absorbers as close as possible to the Faraday

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[11] W. Heckrotte and P. Wolff, Phys. Rev. **73**, 264, 265 (1948).

[12] Aamodt, Peterson, and Phillips, Phys. Rev. **78**, 87 (1950); and Aamodt, Peterson, and Phillips, University of California Radiation Laboratory Report 526 (1949) and University of California Radiation Laboratory Report 1400 (1951) (unpublished).

[13] Details concerning the Faraday cup and its use in measuring currents of high energy charged particles will be included in an article submitted to the *Review of Scientific Instruments*.

cup window, were bombarded for periods of 10 to 20 minutes with proton currents of 10^{-10} to 10^{-12} ampere. The resulting C^{11} activity was counted for several half-lives, and extrapolated back to the end of bombardment in the usual manner. In later runs, the calculation of the cross section was simplified by adjusting the RC of the integrating circuit to equal the mean life of the C^{11} activity. In this case fluctuations in the proton current during bombardment do not enter, and

$$\sigma(mb)=0.00612R(t)/Q(t)w,$$

where w is the foil thickness in mg/cm^2, $R(t)$ the number of C^{11} disintegrations per second determined at the same time as the charge $Q(t)$ (in microcoulombs).

The decay curves showed no evidence for contaminating radioactivity, agreeing well with the published half-life.[7] In particular, foils placed adjacent to the copper absorber showed the same activity as foils protected by an additional 5 mils of polystyrene.

Neutrons are certainly produced in the 48-in. brass collimator and in the copper absorbers by the incident protons. Since the $C^{12}(n,2n)C^{11}$ cross section[14] is comparable to the (p,pn) cross section, one might expect an appreciable background activity. However the high threshold energy discriminates against neutrons below 20 Mev, and the angular distribution of neutrons produced by high energy protons is quite broad.[15] Experimentally the C^{11} activity produced by neutrons is about 2 percent of the initial activity, as measured by foils placed beyond the end of the proton range.

Absolute measurements of the $C^{12}(p,pn)C^{11}$ cross section were made at 8 different proton energies at or below 340 Mev. The values are given in Table I, and are plotted in Fig. 6. The probable errors of the mean value given are due mainly to counting statistics and do not include the absolute error involved in beta-source calibration.

C. Intermediate Energy Region—32 Mev to 100 Mev

The energy spread of the proton beam from the Berkeley cyclotron rapidly increases at degraded energies so that only the general outlines of the excitation curve can be observed. Fortunately, N. M. Hintz has investigated the $C^{12}(p,pn)C^{11}$ reaction using magnetically focused 110-Mev protons in the Harvard cyclotron. This proton energy is ideally suited to provide data in the region 32 to 100 Mev, and Hintz has kindly consented to allow us to include his results. The relative excitation curve was corrected for nuclear absorption in brass absorbers taken to be one-half the total cross section for 70-Mev neutrons in copper (1.1 barns).[16] It was then fitted to the Berkeley data. For the fit, both curves were plotted using a logarithmic scale for the cross section and a linear scale for the energy. Adjustment was made along both cross section and energy scales to achieve the best match of the shapes in the steeply rising region from 20 to 30 Mev. When this is done, the curve also agrees within the probable error with the Berkeley cyclotron data at 93 Mev. The composite curve is shown in Fig. 6.

TABLE I. Absolute cross-section measurements (Berkeley cyclotron).

Proton energy (Mev)	Number of runs	Cross-section mean value (millibarns)	P.E. of mean (millibarns)
340	12	41.2	0.6
313	1	47.6	2.1
293	5	47.7	1.0
263	1	50.5	2.6
245	4	49.8	1.2
194	3	52.0	1.5
144	3	56.5	1.5
93	1	70.5	3.6

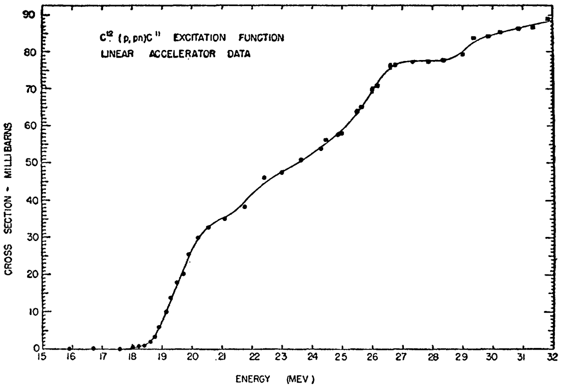

FIG. 2. $C^{12}(p,pn)C^{11}$ excitation function linear accelerator data.

III. EXPERIMENTS WITH 32-MEV PROTONS

A. Relative Excitation Curve

Previously reported measurements[10] include data on the relative excitation curve from threshold (18.5±0.3 Mev) to 27 Mev using the Berkeley 32-Mev proton linear accelerator. Using the same stacked foil technique the excitation curve has now been extended up to full beam energy.

Stacks of 0.010-in. polystyrene foils were bombarded by the full energy beam, and the foils were then counted with an end-window Geiger counter. In order to obtain good counting statistics, only enough foils were counted to obtain a sufficient overlap with the previously established curve. The curve of the run which yielded the highest activity, fitted to the data from threshold to 27 Mev is shown in Fig. 2. The probable errors resulting from counting statistics are too small to be shown on the scale used for Fig. 2. The two curves overlap for about $2\frac{1}{2}$ Mev in a region where a bend permits accurate activity and energy normalization.

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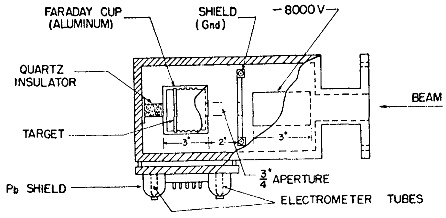
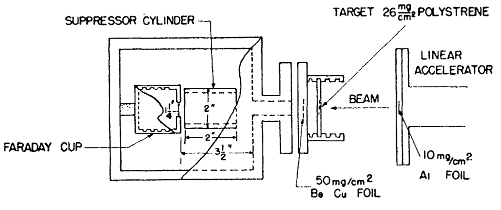
[14] E. M. McMillan and H. F. York, Phys. Rev. **73**, 262 (1948); R. L. Mather and H. F. York (private communication). The $(n,2n)$ cross section is 20±4 millibarns for neutrons of mean energy of 90 Mev.

[15] Miller, Sewell, and Wright, Phys. Rev. **81**, 374 (1951).

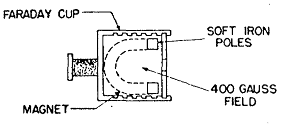
[16] J. DeJuren, Phys. Rev. **81**, 919 (1951).



(A) EARLY RUNS



(B) INTERMEDIATE RUNS



(C) FINAL RUNS

FIG. 3. Faraday cup geometry for measurements at 32 Mev.

The fitting of the two curves does not permit a shift of more than ±0.15 Mev, which introduces an uncertainty of only 2 millibarns at the full energy. Although the incident energy of the beam may vary downward as much as 1 Mev, the best fit results from assuming the incident energy to be the same as for the lower part of the curve (32.0±0.1 Mev).

A foil placed beyond the proton range as a neutron monitor showed that error from this source could be considered negligible.

Because of the comparatively small thicknesses of absorbers used to slow down 32-Mev protons, no nuclear absorption correction has been applied to the activity curves. Furthermore due to the small beam diameter (¼ in.) relative to foil diameter (1 in.), no scattering loss is expected.

B. Absolute Cross Section at 32 Mev

The absolute cross section measurements at 32 Mev previously reported[10] have been extended and improved, primarily as regards beta-standardization (Sec. IV) and determination of the effects of secondary electrons. Thin (0.010-in.) polystyrene foils were bombarded at full beam energy, and the current was collected in a Faraday cup. Due to high average beam intensity ($\sim 10^{-8}$ amp) it is possible to use short bombardments (6 to 120 sec) and therefore render negligible any error due to inconstancy of the beam current. The electrometer tubes and integration condensers are contained within the cup vacuum chamber.

A total of 34 separate runs were made on the absolute cross section at full beam energy. Three different target Faraday cup arrangements were used as shown in Fig. 3 in order to assess and eliminate the error due to secondary electron emission. In the early runs as illustrated in Fig. 3(a), a cylinder at high (8000 v) negative potential removed most low energy charged particles from the beam before it entered the Faraday cup. The cup aperture was only ¾-in. diameter, with the target foil inside. Since the solid angle of this aperture to particles emitted from the target area is only 1½ percent (of 4π), this geometry might be expected to trap nearly all of the secondary electrons. However, it does not readily provide a means of measuring the magnitude of the effect.

Intermediate runs as shown in Fig. 3(b) were made with the target foil outside the cup chamber and a secondary-suppressing cylinder before the cup. The cylinder bias was kept at 200 to 400 volts negative. A series of foils was bombarded with different voltages on the cylinder. The results indicate that secondary electron emission from the Faraday cup decreases by approximately 5 percent as the bias is increased from 0 to 400 volts. This figure agrees with the secondary emission check made by Cork[17] for 32-Mev protons.

The final runs as shown in Fig. 3(c) were made with a permanent magnet which produced a field of 400 gauss inside the Faraday cup. A variation of suppressor voltage from 0 to 400 volts produced no observable change in the cross section.

The average values of the measured cross section using the three methods of integration are shown in Table II. The agreement between methods, combined with the voltage effects noted above, indicates that most of the secondary electrons are of low energy and that the suppression measures employed were adequate. The estimated systematic error in cross section due to secondary electron effects is 3 percent.

The finally adopted value at 32 Mev is

$$\sigma_{32\ \mathrm{Mev}} = 89 \pm 4 \text{ millibarns.}$$

The uncertainty includes errors due to integration and counting statistics but does *not* include the probable error in beta-standard calibration.

IV. ABSOLUTE BETA-ACTIVITY DETERMINATION

The absolute values of the cross section are dependent on a knowledge of the number of C^{11} nuclei produced

TABLE II. Average values of the measured cross section at 32 Mev.

Method	Number of runs	Mean value (millibarns)	P.E.[a] (millibarns)
A	23	88.1	1.5
B	4	84.0	4.6
C	7	92.6	1.6
Combined		88.5	1.0

[a] Probable error of mean from spread in measured values.

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[17] B. Cork, Phys. Rev. **80**, 321 (1950).

in the target foil, and therefore, the disintegration rate at a later time. This rate is found by comparing the counting rate of the target foils with the counting rate of various β-emitters of known disintegration rate under identical geometrical conditions.

Ideally, foils used for absolute beta-activity determination should be very thin to minimize self-absorption and scattering in the source. The necessity for sufficient activity in short bombardments sets a lower limit on foil thickness, however, and 0.005-in. ($\sim$13 mg/cm^2) and 0.010-in. polystyrene foils were used in the cyclotron and linear accelerator runs. The foils were counted on the lower shelves of an end-window Geiger-Mueller counter of 2 mg/cm^2 window thickness. The geometry is shown in Fig. 4.

Significant corrections for absorption and scattering of low energy beta-particles in the source, air, and counter window must be made if the geometry or beta-spectrum of the target foil and standard are different. These corrections are difficult to assess and involve dubious extrapolations to zero thickness. Here such corrections have been avoided by choosing beta-standards of nearly the same energy as C^{11} and counting them in geometries as nearly like that of the C^{11} foil as possible. Half-thicknesses of polystyrene foils were placed above and below the active deposit in counting standards, and the same foil holders were used. Since the standards were effective point sources, a small geometrical correction for the finite extent of the active region of the cyclotron-bombarded foils had to be made. This correction was based on a series of measurements, as shown in Fig. 4, of counting rate as a function of lateral position off the counter axis, made by Mrs. Beverly Lee in this Laboratory. The counting rate of a uniform distribution of activity over a one-in. diameter centered circle, as compared to a point source on the axis is 6 percent low on the third shelf, and 3 percent low on the fourth shelf.

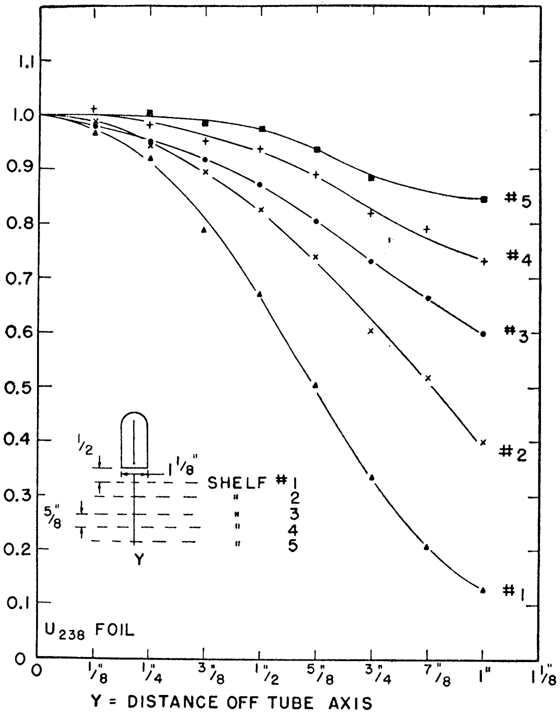

FIG. 4. Relative variation of counting efficiency with lateral displacement of a point source at various distances from a counter window.

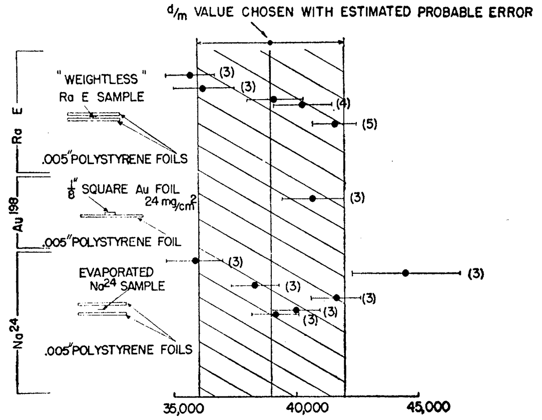

FIG. 5. Equiv. d/m of UO_2 Secondary β-Std for primary standards shown. Numbers in parenthesis indicate shelf on which counting was done.

Several beta-standards of energies bracketing that of C^{11}(0.97 Mev) were used: Au^{198}(0.92 Mev.), RaE(1.17 Mev), and Na^{24}(1.4 Mev).[18] Both Au^{198} and Na^{24} have well-established simple decay schemes[19] which make them suitable for calibration by the beta-gamma coincidence method.[20] The RaE was calibrated by counting the alphas form the daughter RaF in a standard geometry alpha-counter. The alpha-count was made several times over a period of four half-lives to insure that the RaF was in equilibrium with the RaE. Each of these standards was then used to calibrate the beta-counter for C^{11} betas under specific conditions of geometry and foil thickness.

If the standard is calibrated at N_s disintegrations per minute, and counts at the rate of N_1c/m, as compared with N_2c/m for the C^{11} foil, then we say $(N_2/N_1)N_s=N_c$, the disintegration rate of the C^{11} foil. It is convenient to interpose a long-lived secondary standard assigning it a number of "equivalent" d/m. If it counts at a rate N_3 c/m we say this number $N_{\mathrm{eq.}}=(N_3/N_1)N_s$. Then, if at a later time the C^{11} is counted in the same geometry, it is satisfactory to say $N_c=(N_2/N_3)N_{\mathrm{eq.}}$ since this equals (N_2/N_3) $(N_3/N_1)N_s$ $=(N_2/N_1)N_s$ as before.

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[18] G. T. Seaborg and I. Perlman, Revs. Modern Phys. **20**, 585 (1948).

[19] C. L. Peacock and R. G. Wilkinson, Phys. Rev. **74**, 297 (1948); Cook, Jurney, and Langer, Phys. Rev. **70**, 985 (1946).

[20] W. Siri, *Isotopic Tracers and Nuclear Radiations* (McGraw-Hill Book Company, Inc., New York, 1949), Chap. 13. The Na^{24} sample was kindly made available by Dr. C. A. Tobias, as was the coincidence-counting equipment.

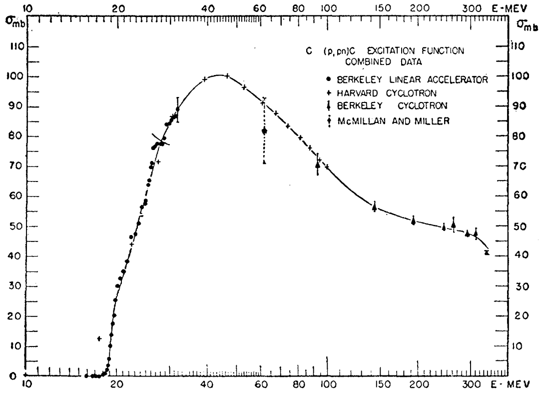

FIG. 6. $C^{12}(p,pn)C^{11}$ excitation function combined data.

Obviously this number $N_{eq.}$ must be redetermined when the target thickness or the geometry is changed.

Values of $N_{eq.}$ for the various standards with geometry shown appear in Fig. 5, where the probable errors, including the standardization statistical probable error, are shown. It is assumed that the proper value of $N_{eq.}$ for a known C^{11} activity would lie somewhere in the shaded area (±7.5 percent probable error).

V. DISCUSSION OF RESULTS

The combined data from the Berkeley linear accelerator, Havard cyclotron, and Berkeley cyclotron is plotted in the final curve of Fig. 6. The probable errors shown are those of absolute measurements but only relative to one another, i.e., they do not include the probable error of $7\frac{1}{2}$ percent in beta-standard calibration.

An independent absolute measurement at 62 Mev by McMillan and Miller,[9] in which the present uranium "intermediate" beta-standard was used as the primary standard, may be corrected in view of a more accurate calibration of the equivalent C^{11} beta-activity of the uranium. The corrected value is 82 ± 11 millibarns and is plotted for comparison in Fig. 6.

The earlier experiments of Chupp and McMillan,[8] using stacks of graphite plates bombarded internally in the cyclotron with protons stripped from deuterons, indicated that the excitation curve was flat from 140 to 60 Mev. No correction for nuclear absorption was made in these earlier experiments. However, a correction based on a nuclear absorption cross section as large as 0.5 barn (approximately the average total cross section for neutrons in this energy region)[12,16] in carbon apparently is not sufficient to account for the discrepancy. The remaining discrepancy results from some other source of error, probably a contamination of the proton beam inside the cyclotron tank from lower energy particles.

The shape of the excitation curve has several features of interest. As shown earlier by Panofsky and Phillips,[10] the low threshold energy provides definite evidence for the existence of a (p,d) reaction in this region. Evidence for the persistence of the (p,d) reaction with good probability for protons of 32 Mev occurs in the independent experiments of Levinthal *et al.*[21] In this energy range the (p,pn) and (p,d) reactions represent inelastic processes best described by the theory of the compound nucleus. The low energy excitation curve, Fig. 2, exhibits irregularities which are experimentally significant but difficult to analyze in terms of the capture process. The peak of the curve occurs at 45 Mev, in agreement with the calculations of Heckrctte and Wolff,[11] but the width of the peak is considerably greater than their evaporation theory estimates.

Furthermore, at higher energies where noncapture processes may be expected to become more important, the experimental curve does not show a plateau but decreases with energy approximately as $E^{-\frac{1}{2}}$. This energy dependence is very similar to that of the neutron total cross section in carbon.[16]

The dip in the $C^{12}(p,pn)C^{11}$ cross section at 340 Mev is apparently real. This was checked by bombarding foils placed on both sides of the first absorber. The results, corrected for nuclear absorption, verify the dip. A calculation based on measurements of the neutron production cross section[6] and angular distribution[22] and checked by Serber's theory[23] indicates that neutrons formed in the absorber can at most account for a 2 percent change. No evidence of copper recoil activity was found in the beta-counting. This deviation from a gradual decrease in cross section with energy makes it unwise to extrapolate the excitation curve to higher energies.

We wish to thank Professor W. K. H. Panofsky for his constant advice and encouragement throughout the history of this experiment. The generosity of Dr. N. M. Hintz in allowing us to incorporate his data is greatly appreciated.

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[21] Levinthal, Martinelli, and Silverman, Phys. Rev. **78**, 199 (1950).

[22] J. DeJuren, Phys. Rev. **81**, 458 (1951).

[23] R. Serber, Phys. Rev. **72**, 1008 (1947).