Stripping and the Nuclear Shell Model

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It has previously been pointed out[1] that the angular distributions from certain (d, p) and (d, n) nuclear reactions should give a sensitive measure of the accuracy of the nuclear shell model in ascribing definite orbital angular momentum states to nucleons in a nucleus. The possibility of this is due to the fact that these reactions proceed via the stripping process,[2] and that their angular distributions are characterized by the orbital angular momenta l with which the captured particle can be accepted into the final state. Each allowed value of l produces a peak in the angular distribution, the peaks resulting from different l values being quite separate. In cases when the selection rules allow more than one value of l, therefore, the purity of the final nuclear states involved can be investigated by a measurement of the relative heights of the peaks resulting from the different l values. For this it is possible to choose examples[1] which are very sensitive to deviations from pure independent particle nuclear states.

Some of the experiments suggested in reference 1 have now been performed,[3,4] and in order to make quite clear their interpretation, we will discuss in this note precisely what nuclear property is being measured in these experiments, i.e., what nuclear property determines the relative heights of the maxima resulting from different l values.

The most convenient starting point for this is to write the stripping differential cross section (for a (d, p) reaction, for example), as obtained by the method of reference 2, in the form

$$\sigma(\theta_p)=\frac{\pi^2C^2m_p{}^*m_d{}^*k_p(2J+1)}{qK_dm_n{}^{*2}(2j_0+1)}\times\sum_{Mm_0\mu_n}\left|\sum_{l_nm_n}a_0(Jj_0l_n,\,Mm_0m_n\mu_n)F_{l_n}(\theta_p)\right|^2, \quad (1)$$

where

$$F_{l_n}(\theta_p)=\left[\frac{1}{K^2+a^2}-\frac{1}{K^2+(a+b)^2}\right]\left\{\sum_{r=0}^{l_n}\frac{(l_n+r)!}{r!(l_n-r)!(2\kappa r_0)^r}\times\left[\left(\kappa+\frac{r}{r_0}\right)+\frac{\partial}{\partial r_0}\right]\left(\frac{r_0}{Z}\right)^{\frac{1}{2}}J_{l_n+\frac{1}{2}}(Zr_0)\right.$$

Here $m_d{}^*$, $m_p{}^*$, and $m_n{}^*$ are the appropriate reduced masses of the deuteron, proton, and captured neutron, respectively, q is a number close to unity,[5] and all other factors are as defined in reference 2 except that the spin of the initial nucleus has been written j_0 (with orientation m_0).

The factor in (1) depending on the wave functions of the initial and final nuclei is the $a_0(Jj_0l_n, Mm_0m_n\mu_n)$, which is defined as follows: Let the wave function $v(J, M; \xi, \mathbf{r}_n)$ of the final nucleus be expanded in terms of states t of the initial nucleus, i.e.,

$$v(J,M;\xi,\mathbf{r}_n)=\sum_{tm_t\mu_nl_nm_n}u_t(j_t,m_t;\xi)\psi_n(\mu_n)Y_{l_nm_n}(\theta_n\varphi_n)\times\frac{f_t(Jj_tl_n,\,Mm_tm_n\mu_n;\,r_n)}{r_n}. \quad (2)$$

Here the $u_t(j_t, m_t; \xi)$ are the wave functions for the states t of the initial nucleus (with spins j_t and orientations m_t) and $\psi_n(\mu_n)$ is the neutron spin function. Then $a_0(Jj_0l_n, Mm_0m_n\mu_n)$ is directly proportional to the wave function $f_0(Jj_0l_n, Mm_0m_n\mu_n; r_n)$ taken at the nuclear surface; we have in fact

$$a_0(Jj_0l_n,\,Mm_0m_n\mu_n)=f_0(Jj_0l_n,\,Mm_0m_n\mu_n;\,r_0)/g_{l_n}(r_0),$$

where

$$g_{l_n}(r_0)=\left\{\sum_{r=0}^{l_n}\frac{(l_n+r)!}{r!(l_n-r)!(2\kappa r_0)^r}\right\}.$$

It may be mentioned that the above simple form for the nuclear dependence is obtained from Eq. (29) of reference 2 by expressing the factor db/dk_p appearing there in terms of the factor $a(k_p)$, using the method outlined in Sec. 2 of reference 2.

By making use of the symmetry properties of the wave functions $f_0(Jj_0l_n, Mm_0m_n\mu_n;r_n)$, the summation over spin orientations in (1) can now readily be performed, and the cross section becomes

$$\sigma(\theta_p)=\frac{\pi^2C^2m_p{}^*m_d{}^*k_p(2J+1)}{qK_dm_n{}^{*2}(2j_0+1)}\sum_{l_n}\frac{|f_0(Jj_{0l_n};\,r_0)|^2}{g_{l_n}{}^2(r_0)}|F_{l_n}(\theta_p)|^2. \quad (3)$$

Here $|f_0(Jj_0l_n; r_0)|^2$ is the probability density for finding a neutron with orbital angular momentum l_n at the surface of the final nucleus, when the nucleus it leaves behind (core) is in the ground state.

Thus the experiments suggested in reference 1 measure deviations in the shell model, expressed in terms of the relative sizes of the various $|f_0(Jj_0l_n; r_0)|^2$ factors for allowed values of l_n. This gives directly, therefore, an idea of the relative admixtures of orbital angular momentum states for the captured neutron when the core is in the ground state. It should be noted though that these experiments in no way measure admixtures of states in the final nucleus which are due to excitation of the core.

I am indebted to Dr. N. Austern for discussions in connection with this work.

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[5] The number q is given, to a very good approximation, by $\int|v(J,\,M;\,\xi,\,\mathbf{r}_n)|^2d\xi d\mathbf{r}_n$.

Energy Levels Associated with the Radioactive Decay of Gd^{153} and Tb^{161}†

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The irradiation of gadolinium with slow neutrons in the pile yields two long-lived radioactive emitters. One activity, first observed[1] by Fajans and Voigt, has a half-life reported as from 150 to 236 days and is believed[2] to be due to Gd^{153}, being formed by neutron capture from Gd^{152}, whose normal abundance is only 0.2 percent. Another activity of about 7-day half-life is really in Tb^{161}, being formed[3] by beta-gamma decay from a short-lived Gd^{161}. From the conversion electrons observed in high resolution magnetic spectrometers, a single gamma-ray appears to be associated with each activity.

The 103.7-kev gamma-ray in the Eu^{153} nucleus, following K capture in Gd^{153}, is found to have a K/L ratio of 5.2 ± 1. From the empirical relations[4] of Goldhaber and Sunyar this indicates either an $M2$ or a mixture of $E2$ and $M1$ transitions. However, from the long radiation lifetime (0.001 sec) required for the $M2$ radiation, which has not been observed, it appears preferable to assume the existence of the mixture. This same gamma-ray together with

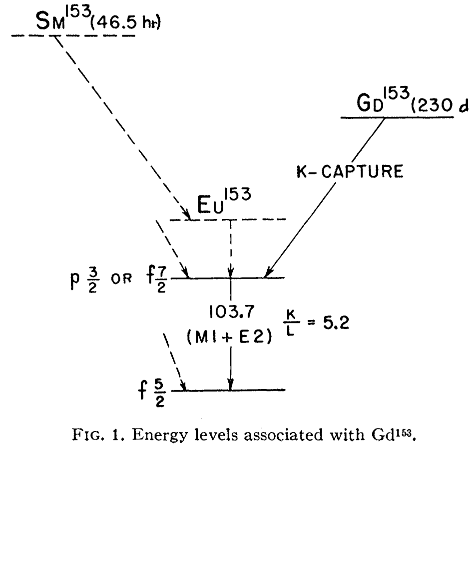

FIG. 1. Energy levels associated with Gd^{153}.

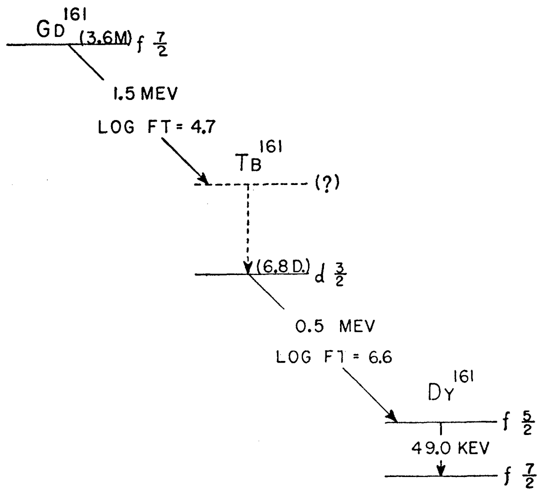

FIG. 2. Energy levels associated with Gd^{161} (from shell theory).

others has been observed in the beta-decay of Sm^{153}. It must follow in the ordering of the levels of Eu^{153} that the 103.7-kev transition leads directly to the ground state of the nucleus. It has been noted[5] that the measured parity of this ground state ($f_{5/2}$) is not in agreement with the value expected from shell theory ($d_{5/2}$).

In the Dy^{161} nucleus a single gamma-ray of energy 49.0 kev is found. At the low energy no K conversion electron line could occur. The L/M conversion ratio is found to be 3.7 ± 0.8. Since conversion occurred mainly in the L_1 level, the radiation[6] is quite probably $M1$. The half-life of the Tb^{161} activity was followed through six octaves and was found to be 6.8 ± 0.1 days. The beta-energies and the gamma-energy in Gd^{161} have not been re-evaluated in this investigation.

From shell theory for odd-even nuclei it is possible to assign spin and parity values to the observed levels of the isobar 161, with one exception. The $\log ft$ value for the beta-decay in Gd^{161} is 4.8. This indicates an allowed transition with no parity change. Shell theory indicates the ground state of Gd^{161} to be $f_{7/2}$ and the only low-lying excited state of Tb^{161} of the same parity is an $h_{11/2}$ state, which would require a spin change of two. It may be that the data leading to the ft value are inaccurate. Otherwise, this particular transition cannot be regarded as in satisfactory agreement with the extreme single particle theory. The complete data are shown graphically in Fig. 1 for Gd^{153} and in Fig. 2 for Gd^{161}.

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Total Cross Sections for 14-Mev Neutrons*

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TOTAL cross sections for 14-Mev, $T(d, n)He$ neutrons were measured for several elements. The deuterium ions were accelerated through a 130-kv potential drop against a tritiated zirconium target.

Detection of the neutrons was accomplished with an anthracene crystal and an RCA 5819 photomultiplier tube. The signal, after amplification, was biased so that recoil protons (in the anthracene crystal) of energy less than 12 Mev were not detected.

The usual corrections for background and room scattering were made. In-scattering corrections were made according to the method of McMillan and Sewell.[1] The results listed in Table I are, within statistical counting errors, in reasonable agreement with more accurate data by Coon, Graves, and Barschall.[2]

TABLE I. Total cross sections for 14-Mev neutrons (in units of 10^{-24} cm^2).

H	0.69 ± 0.06	Ti	2.2 ± 0.2	Zr	3.6 ± 0.2
D	0.82 ± 0.08	V	2.4 ± 0.1	Mo	3.6 ± 0.3
Be	$1.4\ \pm0.1$	Cr	2.3 ± 0.2	Sn	4.0 ± 0.4
C	1.20 ± 0.04	Fe	2.4 ± 0.2	Sb	4.6 ± 0.3
O	$1.5\ \pm0.1$	Ni	2.5 ± 0.1	W	4.8 ± 0.3
Mg	$1.8\ \pm0.2$	Cu	2.5 ± 0.1	Pb	5.1 ± 0.4
Al	$1.8\ \pm0.1$	Zn	3.0 ± 0.2	Bi	5.2 ± 0.5

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We would like to thank Dr. J. H. Coon for the privilege of seeing his report before it appeared in print.

* These data are part of a paper which was written to fulfill a partial requirement for a Ph.D. degree at the University of Chicago.

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Spin-Spin Relaxation in Ferromagnetic Resonance*

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WE demonstrate here the theoretical possibility of a nonvanishing line width in ferromagnetic resonance at 0°K arising from zero-point fluctuations of spin-spin magnetic dipole fields.

The Hamiltonian of the spin system is

$$\mathcal{H}=\mathcal{H}^0+\mathcal{H}^{++}+\mathcal{H}^{--}+\mathcal{H}^{+}+\mathcal{H}^{-}, \tag{1}$$

with

$$\mathcal{H}^0=-Hg\beta\Sigma_jS_j^z+\Sigma_{k>j}(A_{jk}\mathbf{S}_j\cdot\mathbf{S}_k+B_{jk}S_j^zS_k^z),$$
$$\mathcal{H}^{++}=\Sigma_{k>j}E_{jk}S_j^+S_k^+;\quad \mathcal{H}^{--}=\Sigma_{k>j}E_{jk}^*S_j^-S_k^-,$$
$$\mathcal{H}^{+}=\Sigma_{k\neq j}F_{jk}S_j^+S_k^z;\quad \mathcal{H}^{-}=\Sigma_{k\neq j}F_{jk}^*S_j^-S_k^z,$$

where

$$A_{jk}=-2J_{jk}+\tfrac{1}{2}D_{jk}(3\gamma_{jk}^2-1),$$
$$B_{jk}=-\tfrac{3}{2}D_{jk}(3\gamma_{jk}^2-1),$$
$$E_{jk}=\tfrac{3}{4}D_{jk}[\beta_{jk}^2-\alpha_{jk}^2+2i\alpha_{jk}\beta_{jk}],$$
$$F_{jk}=-\tfrac{3}{2}D_{jk}\gamma_{jk}(\alpha_{jk}-i\beta_{jk}).$$

In the above J_{jk} is the exchange integral between spins j and k; $D_{jk}=g^2\beta^2/r_{jk}^{-3}$ if purely magnetic dipole forces are operative; and α_{jk}, β_{jk}, γ_{jk} are direction cosines of r_{jk} taken with respect to the crystal axes. We shall assume for simplicity that the applied field H lies along a crystal axis.

The only part of $\mathcal{H}$ commuting with S^z is $\mathcal{H}^0$. The remainder of $\mathcal{H}$ breaks down the selection rules $\Delta M=\pm1$ in a resonance experiment, causing weak satellite lines to appear at $\Delta M=0, \pm2, \pm3$. In evaluating the second and fourth moments of the principal ($\Delta M=\pm1$) resonance line for a paramagnetic sample, therefore, Van Vleck[1] used the ingenious trick of limiting the Hamiltonian to $\mathcal{H}^0$.

If, however, $J>Hg\beta$, the energies of the many states of a given S^z are spread over a spin-wave spectrum larger than the Zeeman splitting of states of different S^z. States of different S^z are then nearly degenerate and are strongly mixed by the dipole terms. Thus M ceases to be even approximately a good quantum number, and the satellite picture breaks down. In paramagnetic resonance in a sample with large exchange forces, therefore, the moments should be evaluated by use of the entire Hamiltonian. This has the effect of multiplying Van Vleck's second moment by 10/3; and from general considerations it can be seen that the line width,