The total integrated cross sections are 2.8 Mev-barns for tantalum and 3.0 Mev-barns for bismuth, with the $(\gamma, 2n)$ contributing approximately 15 percent in each case. The sum rule prediction that the integrated cross section be given by

$$\int_0^\infty \sigma dE = 0.060 (NZ/A) (1+0.8x),$$

where x is the fraction of exchange force in the proton-neutron interaction, is in excellent accord with the results. The calculation of the fraction of exchange force, however, would require better techniques than the photon-difference method of obtaining cross sections from the yield data, especially as to any tail on the curves above 23 Mev, and also some determination of the importance of (γ, γ) processes. As can be seen, the shapes of the excitation functions of bismuth and tantalum are markedly different.

Work is continuing on other elements as well as on the detection of coincidences in the $(\gamma, 2n)$ process.

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Fourth-Order Vacuum Polarization

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W E have calculated the contribution from vacuum polarization to the fourth-order radiative correction to the motion of a slow electron in an external field.

Consider a single scattering of an electron in an electrodynamic potential varying in time and space as $\exp(iq_{\mu}x_{\mu})$; we write q^2 for $(q^2 - q_i^2)$, and *m* for the electron mass. If $q^2 \ll m^2$, the radiative corrections to scattering can be expanded in powers of (q^2/m^2) and of the fine structure constant α . The lowest term, which is due to vacuum polarization, in this expansion is the well-known Uehling¹ term whose ratio to the zero-order scattering is

$$(1/15)(\alpha/\pi)(q^2/m^2).$$
 (1)

The terms of order α^2 are the fourth-order radiative corrections. Various authors² have already discussed and evaluated all these terms, except for the contribution from vacuum polarization. The term in α^2 due to vacuum polarization comes from the three fourth-order diagrams shown in Fig. 1. To lowest order in q^2 these diagrams give a correction potential, whose ratio to the zero-order potential is of the form

$$K(\alpha/\pi)^2(q^2/m^2),$$
 (2)

where K is a dimensionless constant to be determined.



FIG. 1. The three Feynman diagrams contributing to the fourth-order vacuum polarization. Solid lines denote electrons, broken lines photons. Crosses denote the external potential, and the cross plus Δm denotes the mass renormalization operator.

The evaluation of the constant K was carried out along the lines of the Feynman formalism.²⁻⁵ To carry out the massrenormalization in an unambiguous way a regulation procedure^{3, 6} was used, involving "photons" of very large mass Λ . The calculations were arranged, making use of the principle of gauge invariance, so that charge renormalization and photon self-energy divergences were automatically excluded. The contributions to Kfrom the three diagrams in Fig. 1 were evaluated separately. The calculations were tedious but very much simpler than those of other fourth-order terms,² because in this case the final expressions to be integrated were always rational functions with a single simple denominator. The underlying reason for this simplification is not clear to us. No transcendentals appear in the final results, which are

$$K_a = (1231/8100) + (1/30) \log(\Lambda^2/m^2),$$
 (3a)

$$K_b = (23/450) - (2/15) \log(\Lambda^2/m^2),$$
 (3b)

$$K_c = -(1/20) - (1/10) \log(\Lambda^2/m^2), \qquad (3c)$$

$$K = K_a + K_b - K_c = 41/162.$$
⁽⁴⁾

The constant K is independent of the cutoff Λ .

The vacuum polarization terms have a nonzero expectation value for the 2S-state of hydrogenic atoms, zero expectation value for the 2P-state (to this order). They therefore contribute to the Lamb shift, the contribution of the fourth-order terms, Eq. (2), being -0.239 Mc/sec for hydrogen and deuterium, compared with -27.13 Mc/sec from the Uehling term, Eq. (1). All other fourth-order contributions to the Lamb shift have been calculated previously,² and also the corrections⁷ to the second-order terms produced by the failure of the approximation $q^2 \ll m^2$. Including these contributions, and corrections due to nuclear mass, radius, and structure,⁸ the theoretical values⁹ for the Lamb shift are (1057.19 ± 0.16) Mc/sec for hydrogen and (1058.49 ± 0.16) Mc/sec for deuterium. These values are about half a megacycle smaller than the corresponding experimental values.¹⁰

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Isomeric Levels in Pb²⁰¹ and Pb²⁰²

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SHORT period gamma-activities following proton bombard-ment of thallium targets been b ment of thallium targets have been studied using a sodium iodide scintillation spectrometer. Decay periods of 50 seconds and 5.6 seconds have been observed, and assignments have been made to isomeric levels of Pb^{201} and Pb^{202} , respectively. The γ -ray energies measured were 0.67, 0.42, and 0.25 Mev for the 50-second decay and 0.89 Mev for the 5.6-second decay. Channel analysis of the spectra was accomplished by brightening individual pulses with a circuit due to Watkins1 and displaying on a triggered oscilloscope. Photographing the screen on moving film yielded permanent records which were later projected and analyzed to give the pulse-height distributions. The 0.510-Mev γ -ray of Sr⁸⁵ was used as a convenient calibration line together with Co⁶⁰ lines.

The 5.6-second period was measured using a method due to Breckon and Martin.² For the 50-second activity it was possible to carry out chemical separation of the active lead from the thallium target. This showed conclusively that the activity was in