Letters to the Editor

PUBLICATION of brief reports of important discoveries in physics may be secured by addressing them to this department. The closing date for this department is five weeks prior to the date of issue. No proof will be sent to the authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents. Communications should not exceed 600 words in length and should be submitted in duplicate.

The Current Density in Quantum Electrodynamics

H. C. CORBEN*
Institute of Physics, University of Genoa, Genoa, Italy
(Received September 5, 1952)

IF e_0, m_0 denote the charge and rest mass of the positron, the equation for a particle of chage e,

$$\{\gamma_i D_i + i(e/e_0)\kappa\gamma_5\}\psi = 0, \quad (1)$$

where $D_i = \partial_i - (ie/\hbar c)A_i$ and $\kappa = m_0c/\hbar$, yields the Dirac Hamiltonians

$$H = -e_0\varphi - c\boldsymbol{\alpha}\cdot(\mathbf{p}+e_0\mathbf{A}/c) - \rho_3 m_0c^2,$$
$$H = e_0\varphi - c\boldsymbol{\alpha}\cdot(\mathbf{p}-e_0\mathbf{A}/c) + \rho_3 m_0c^2,$$
$$H = -c\boldsymbol{\alpha}\cdot\mathbf{p}, \quad (\boldsymbol{\alpha} = \rho_1\boldsymbol{\sigma}),$$

for electrons, positrons, and neutrinos, respectively, provided that we take

$$\gamma_k = \rho_3\sigma_k(k=1,2,3), \quad \gamma_4 = \rho_2, \quad \gamma_5 = \rho_1, \quad (2)$$

rather than the usual expressions of the γ_μ in terms of the Dirac operators. The latter are in fact obtained from (2) under the transformation $\rho_1 \to \rho_1$, $\rho_2 \to \rho_3$, $\rho_3 \to -\rho_2$. Equation (1) requires that the charge e be a pseudoscalar and that the potentials A_i form a pseudovector.

In addition to the usual current-density pseudovector,

$$j_i = ec\psi^\dagger\gamma_i\psi, \quad (\psi^\dagger = i\psi^*\rho_2),$$

there exists the pseudovector density,

$$j_i^* = -\tfrac{1}{2}i(e_0\hbar/m_0)(\psi^\dagger\gamma_5 D_i\psi - D_i^*\psi^\dagger\gamma_5\psi),$$

which is also conserved in virtue of Eq. (1). Since j_i^* is also gauge-invariant, one could use it instead of j_i to specify the current density.

In the absence of an electromagnetic field, j_i^* reduces to j_i; and in general, it follows from Eq. (1) that j_i^* may be alternatively written as follows:

$$j_i^* = j_i + (c/4\pi)\partial_j h_{ji}, \quad (3)$$

where

$$h_{ij} = -h_{ji} = -(ie_0h/m_0c)\psi^\dagger\gamma_5(\gamma_i\gamma_j - \delta_{ij})\psi. \quad (4)$$

If j_k generates the field f_{ik},

$$\partial_i f_{ik} = -(4\pi/c)j_k, \quad (5)$$

the new current density j_k^* generates a new field f_{ik}^*:

$$\partial_i f_{ik}^* = -(4\pi/c)j_k^*, \quad (6)$$

where

$$f_{ik}^* = f_{ik} - h_{ik}. \quad (7)$$

In the same way that the new current density reduces to the old in the absence of an electromagnetic field, the new electromagnetic field reduces to the old in the absence of a particle field.

With this field f_{ik}^* may be associated an energy density,

$$u^* = (E^{*2}+H^{*2})/8\pi = (E^2+H^2)/8\pi + Y + Z,$$

where

$$Y = -(e_0\hbar/2m_0c)[\psi^*\rho_3(\boldsymbol{\sigma}\cdot\mathbf{H} - i\boldsymbol{\alpha}\cdot\mathbf{E})\psi],$$
$$Z = \tfrac{1}{2}\pi(e_0\hbar/m_0c)^2[(\psi^*\rho_3\boldsymbol{\sigma}\psi)^2 + (\psi^*\rho_2\boldsymbol{\sigma}\psi)^2].$$

Free-particle solutions of Eq. (1) may be characterized by the sign of the charge,

$$e = \pm(e_0/m_0c^2)(w^2 - p^2c^2)^{\frac{1}{2}},$$

and for the positive energy solutions corresponding to zero momentum,

$$\rho_3\psi = (e/e_0)\psi.$$

The term Y therefore contains the interaction energy of the field f_{ij} with the magnetic and electric moments of the electron-positron field. For atomic electrons the term Z yields on integration an energy of fine structure order, although if an electron were to be confined to distances of the order of nuclear dimensions, integration of Z would yield an energy of several hundred Mev.

If we use f_{ik}^* and j_i^* instead of f_{ik} and j_i in the Lagrangian that leads to (1) and (5), i.e.,

$$L = \tfrac{1}{2}i\hbar c(\psi^\dagger\gamma_i\partial_i\psi - \partial_i\psi^\dagger\gamma_i\psi) - (e/e_0)m_0c^2\psi^\dagger\gamma_5\psi + j_i^*A_i/c - f_{ij}^*f_{ij}^*/16\pi,$$

where f_{ij}^*, j_i^* defined by (3), (4), and (7), the Lagrangian equations which result with A_i, $\psi^\dagger$, ψ as independent variables are Eq. (6) and the nonlinear equation,

$$\gamma_i D_i\psi + (ie/e_0)\kappa\gamma_5\psi = -(e_0/4m_0c^2)h_{ij}\gamma_5\gamma_i\gamma_j\psi.$$

This modification does not destroy the conservation property of j_i, j_i^*. Some applications of this equation will be published in *Il Nuovo Cimento*.

* On leave of absence from Carnegie Institute of Technology, Pittsburgh, Pennsylvania.

Atomic Oxygen *g*-Factors*

E. B. RAWSON† AND R. BERINGER
Sloane Physics Laboratory, Yale University, New Haven, Connecticut
(Received September 15, 1952)

WE have recently studied the Zeeman splittings of the ground $^3P_2$ and metastable $^3P_1$ states of atomic oxygen in an attempt to determine the effects of the electron spin-moment anomaly in a complex atomic state. The microwave resonance apparatus is similar to that used for hydrogen[1] and paramagnetic gases.[2] Transitions between adjacent M levels of a given atomic term are observed in a stream of vapor which originates in a discharge tube and passes through a microwave cavity. The cavity is in a proton-controlled magnetic field, and a second proton-resonance coil, whose mineral-oil sample volume simulates the atomic vapor stream, is later fitted into the cavity and its proton-resonance frequency measured. The six microwave transitions in oxygen are labeled $a(M=1\to0$ in $^3P_1)$, $b(M=0\to-1$ in $^3P_1)$, $c(M=2\to1$ in $^3P_2)$, $d(M=1\to0$ in $^3P_2)$, $e(M=0\to-1$ in $^3P_2)$, and $f(M=-1\to-2$ in $^3P_2)$. The six transition energies are slightly different due to Paschen-Back terms. On LS coupling they are symmetrical about $g_J\mu_0H$ in pairs (a,b), (c,f), (d,e). Provided only that this symmetry exists, g_J can be calculated from frequencies alone. For the pair (a,b),

$$g_J = g_p\nu(1+f_a^2/f_b^2)/[f_a(1+f_a/f_b)],$$

with[3] $g_p = g_s/(g_s/g_p) = 2(1.0011454)/658.2271$ for our cylindrical sample of mineral oil. The f's are simulator proton-resonance frequencies, and ν is the fixed microwave frequency.

If the coupling in the ^3P term is pure LS, one expects a simple contribution[4] to g_s from the electron-spin anomaly with $g_J=3/2+0.0011454$. Unfortunately, the ground $(2p)^4$ configuration in oxygen departs considerably from LS coupling, and the electrostatic and LS splittings cannot be fitted without configuration mixing. We have not calculated configuration-interaction contributions to the Paschen-Back effect.

Tentative results based on two runs agreeing to 4 parts in 10^6 are $g_J(a,b)=1.500971$, $g_J(c,f)=1.500905$, and $g_J(d,e)=1.500904$. The two values for the $^3P_2$ levels are sensibly identical, some 44 parts in 10^6 less than $g_J(a,b)$ and 161 parts in 10^6 less than the LS

coupling value. The Paschen-Back splittings of the four $^3P_2$ transitions are in good (± 0.06 percent) agreement with LS theory, but those for the $^3P_1$ transitions are not: 10.18 gauss between a and b as compared with 17.22 gauss for LS theory. The Paschen-Back difficulties may lie in an anomalous perturbation of the $(M=0, {}^3P_1)$ level by the $^3P_0$ term.

In all of the experiments another spectrum was found which consisted of three very narrow lines interleaved with the four $^3P_2$ transitions. The interval between these lines was the same as for the four $^3P_2$ lines, and their mean g value was lower than for the $^3P_2$ lines by 2 to 5 parts in 10^6. The origin of this spectrum is not established. It apparently cannot arise from any known oxygen term, but may be the result of argon contamination of the discharge tube. Further study of this spectrum is in progress.

* Assisted by the ONR.
† AEC Predoctoral Fellow. Part of a dissertation submitted to Yale University for the Ph.D. degree.

[1] R. Beringer and E. B. Rawson, Phys. Rev. **87**, 228 (1952).
[2] R. Beringer and J. G. Casle, Phys. Rev. **78**, 581 (1950); **81**, 82 (1951).
[3] Koenig, Prodell, and Kusch, Phys. Rev. **83**, 687 (1951).
[4] P. Kusch and H. M. Foley, Phys. Rev. **74**, 250 (1948).

X-Rays from the π-Mesic Atom*

A. M. L. MESSIAH AND R. E. MARSHAK
University of Rochester, Rochester, New York
(Received September 15, 1952)

It is well known that the nuclear absorption of slow μ^- mesons takes place primarily from the K shell of the appropriate μ-mesic atom and leads to the Z_{eff}^4 law[1] (Z_{eff} is the effective nuclear charge). The Z_{eff}^4 law is the result of a competition between the Z-independent μ^- meson decay and the Z_{eff}^4-dependent nuclear absorption for a bound $1s$ μ^- meson, the two becoming equally probable for $Z_{\text{eff}}\approx 10$. Since the probability for nuclear absorption is greatly reduced when the μ^- meson finds itself in a $2p$ state and since the $2p\rightarrow 1s$ optical transition probability is $1.3\times 10^{11}Z^4$ sec^{-1}, it follows that even for the heaviest elements all μ^- mesons reaching the $2p$ state (and this means the vast majority[2]) will emit K_α x-ray lines.

On the other hand, the strong interaction of π-mesons with nuclei alters the entire picture for π-mesons. The interaction is so strong that the majority of π-mesons will not reach the K shells for any but the two lightest elements, namely, hydrogen and helium. This can be seen as follows: For deuterium several authors[3] have shown that the probability for nuclear absorption from the $2p$ state, w_{abs}, is 20–30 times smaller than the $2p\rightarrow 1s$ optical transition probability, $w_{2p\rightarrow 1s}$. Now $w_{\text{abs}}\approx Z^6$ (restricting ourselves to light nuclei where $Z_{\text{eff}}\approx Z$) and $w_{2p\rightarrow 1s}\approx Z^4$, so that $w_{\text{abs}}\approx w_{2p\rightarrow 1s}$ when $Z^2\approx 20$–30 or $Z\approx 4$–5 provided that the nuclear absorption per deuteron stays the same in heavier nuclei. This, however, is not the case since the deuteron is such a loosely bound structure. An estimate of the effect of nuclear binding can be deduced from the relative cross sections for nuclear absorption at low positive kinetic energies of the π^- meson; this leads to the use of $5Z$ as the effective number of deuterons[4] and therefore to $5Z^2\approx 20$–30 or $Z\approx 2$ as the value above which nuclear absorption from the $2p$ state prevails over the emission of π^-K_α x-rays.

Actually, the pseudoscalar character of the π-meson field and the apparent importance of gradient coupling for determining the salient features of the π-meson-nucleon interaction[5] enable one to establish a close connection between w_{abs} and the cross section σ for nuclear absorption from the continuum. Thus, if we write for the interaction Hamiltonian[6]

$$H=(f/\mu)\Sigma_i\tau^i(\sigma^i\cdot\nabla)\phi, \tag{1}$$

then[7]

$$w_{\text{abs}}=(\hbar^2/q\mu)\sigma|\nabla\phi(0)|^2\mathcal{R}, \tag{2}$$

where σ^i, τ^i are the spin and isotopic spin operators of the ith nucleon, $\phi(0)$ is the Coulomb wave function of the π^- meson evaluated at the nucleus, q the π^- momentum in the continuum, and $\mathcal{R}$ the ratio of the square of the nuclear matrix elements from the bound state and the continuum. It would certainly seem like a good approximation to set $\mathcal{R}=1$ when the kinetic energy of the π^- meson is small compared to its rest mass. If we use Eq. (2), insert 3 mb[8] for the cross section for the reaction $\pi^++D\rightarrow P+P$ at 25 Mev and assume charge symmetry, we obtain $w_{\text{abs}}=0.06w_{2p\rightarrow 1s}$. For carbon, the lowest energy at which the absorption cross section has been measured is 62 Mev[9] with the result $\sigma=150$ mb; inserting this value into Eq. (2) yields $w_{\text{abs}}\approx 11w_{2p\rightarrow 1s}$, which prediction is much less reliable than the deuteron prediction. For nuclei other than deuterium and carbon, nuclear absorption cross sections are not available for π-meson energies below 85 Mev.[10]

One would expect a monotonic increase in the ratio $w_{\text{abs}}/w_{2p\rightarrow 1s}$, roughly as Z^2 beyond deuterium, unless special selection rules associated with Eq. (1) operate for a given nucleus. Such selection rules could lead to strong fluctuations in the ratio $w_{\text{abs}}/w_{2p\rightarrow 1s}$ and have already been shown to exist[11] for H^3, He^3, and He^4. It can also be shown that if Wigner's supermultiplet theory (based on the spin and charge independence of nuclear forces) were rigorously true, the $\tau^i(\sigma^i\cdot\nabla)$ operator would give a strictly zero contribution for all nuclei belonging to the supermultiplet (0, 0, 0). This follows because the $\tau^i(\sigma^i\cdot\nabla)$ operator does not act on the space coordinates of the nucleons and therefore only allows transitions to final states whose space part belongs to the same irreducible representation of the group of permutations, namely, to final states belonging to the same supermultiplet (0, 0, 0); however, the conservation laws of spin and isotopic spin are violated. The case of O^{16} is especially interesting since the same selection rule obtains on the basis of the shell model assuming charge-independent and central (but not necessarily spin-independent) nuclear forces; the resulting $(1s)^4(1p)^{12}$ configuration yields an O^{16} wave function which accidently belongs to the supermultiplet (0, 0, 0). It is, of course, very unlikely that any of these selection rules will hold absolutely, both because the π-meson-nucleon interaction must possess an s component (in addition to the p component, see reference 6) and because "shell structure" effects ought to lose their potency for processes involving such high energy transfers as π^- absorption.[12] Nevertheless, it is conceivable that fluctuations in the ratio $w_{\text{abs}}/w_{2p\rightarrow 1s}$ can be observed, and it would be interesting to study π-mesic x-ray yields for "magic number" nuclei and their near neighbors. Since He^4 raises difficult experimental problems and since, for $Z\gtrsim 10$, nuclear absorption from the $3d$ level should rapidly dominate over the $3d\rightarrow 2p$ optical transition, O^{16} is the only suitable candidate for this type of experiment. Preliminary measurements of the π-mesic x-ray yields from C and O performed at this laboratory[13] support the theoretical interpretation presented in this note and indicate that "shell structure" does indeed play a role in the absorption of slow π^- mesons.

We are indebted to Camac *et al.*[11] for informing us of their results prior to publication.

* This work was assisted by the AEC and the French Direction of Mines.

[1] Harrison, Keuffel, and Reynolds, Phys. Rev. **83**, 680 (1951).
[2] J. A. Wheeler, private communication to Dr. J. B. Platt.
[3] S. Tamor and R. Marshak, Phys. Rev. **80**, 766 (1951); Brueckner, Serber, and Watson, Phys. Rev. **81**, 575 (1951) and **84**, 258 (1951).
[4] Comment of H. A. Bethe, Proceedings of the Rochester Conference on Meson Physics, January 11–12, 1952, unpublished.
[5] Durbin, Loar, and Steinberger, Phys. Rev. **84**, 581 (1951).
[6] Equation (1) represents the dominant term (there is an s-coupling term of order μ/M where μ and M are the π-meson and nucleon masses, respectively) in the $PS(PV)$ theory; however, it is not excluded that some higher order treatment of the $PS(PS)$ theory will similarly lead to the dominance of the p-coupling type interaction.
[7] Brueckner, Serber and Watson, reference 3.
[8] Clark, Roberts, and Wilson, Phys. Rev. **83**, 649 (1951); also reference 5.
[9] Byfield, Kessler, and Lederman, Phys. Rev. **84**, 17 (1952); A. M. Shapiro's measurement at 48 Mev [Phys. Rev. **84**, 1063 (1951)] does not distinguish between star production and inelastic scattering.
[10] Chedester, Isaacs, Sachs, and Steinberger, Phys. Rev. **82**, 958 (1951); Bernardini, Booth, and Lederman [Phys. Rev. **83**, 1075 (1951)] have results at energies as low as 30 Mev but for photographic emulsion.
[11] Messiah, Caianiello, and Basri (H^3), Phys. Rev. **83**, 652 (1951); A. M. L. Messiah (He^3), Phys. Rev. **87**, 639 (1952); A. G. Petschek (He^4), private communication.
[12] "Shell structure" effects have been considered for μ^- absorption by Ca and Pb [J. M. Kennedy, Phys. Rev. **86**, 616 (1952)].
[13] Camac, McGuire, Platt, and Schulte, Phys. Rev. **88**, 134 (1952); the K_α lines from C and O should be 100, 175 kev, respectively, with widths of the order of 2 to 5 (due to nuclear absorption from the K shell).