Atomic Masses from Palladium through Xenon*

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The atomic masses of 42 stable nuclides in the region from palladium through xenon have been measured with a double focusing mass spectrometer. Combined with nuclear reaction mass differences the data provide a table of 74 mass values between mass numbers 102 and 136. A fit of the Wigner semi-empirical mass formula to these values reveals an irregularity at the magic number of 50 protons similar in nature to those previously reported from this laboratory at 20 and 28 protons. Also evident is an inability of the formula to account for the binding energy variations between isobaric nuclei. An empirical modification of the formula is observed to correct largely for this latter effect.

THIS paper presents the results of a portion of an extensive program of mass measurement undertaken with the double focusing mass spectrometer developed at the University of Minnesota. Atomic masses for 47 nuclides have been previously reported.[1-3] 42 additional atomic mass values of stable isotopes containing approximately 50 protons are given here. The treatment of data in the most recent report[1] has been paralleled to facilitate a consideration of all data from this laboratory. A brief description of the apparatus and measurement procedure has been previously given.[2,3]

EXPERIMENTAL RESULTS

Table I summarizes the mass doublets measured and the mass differences obtained. The atomic masses derived from these data for the heavy nuclide of each doublet are also presented in the last column of Table I in the form of mass defects (atomic mass minus mass number). It is possible to obtain mass values for many of these isotopes by combining previously reported nuclear reaction and mass spectrographic measurements. Due to the uncertainty of the source of possible error in values from combined data, only mass defects previously determined in terms of the mass standards O^{16}, H^1, and C^{12} by the work of any single laboratory are given for comparison purposes. In the computation of these mass defects, the masses of H^1 and C^{12} determined in this laboratory have been arbitrarily employed except in the case of the time-of-flight measurements. There the method of adjusting the reported results for comparison purposes was not obvious and was believed of no concern in view of the magnitude of the errors quoted.

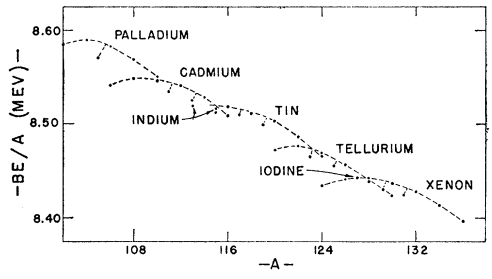

FIG. 1. Mass spectrometric values for average binding energy per nucleon.

Much of this work was done concurrently with that last reported.[1] As in the previous work, a "run" consists of 10 or more consecutive tracings of the mass doublet. Different runs were in most cases scattered over a period of several months. Recorded hydrocarbon ion currents have been corrected for the inherent presence of an unresolved component containing a C^{13} atom instead of one C^{12} and one H^1 atom. An upper limit was placed on the possibility of a systematic error proportional to mass by measuring with every day's set of data a known mass difference, i.e., the difference between two hydrocarbon fragments differing in mass by one hydrogen atom. Seventy-one such measurements over a period of nine months gave an average value of 1.0078. The deviation of this value from the assumed correct value of 1.008146 is approximately 1/3000. No correlation was observed, however, between such hydrogen mass measurements and the deviations of associated mass doublets from their average values. Therefore, no correction was made to the data for this error. The errors quoted for the doublets in Table I are a combination of the statistical probable error and a possible error of 1/3000.

In Table II, mass differences from the results of this paper are compared with mass differences determined from the energy balance of nuclear reactions. The agreement is considered good in most cases. Further evidence of data consistency can be seen in the plot of average binding energy per nucleon presented in Fig. 1. Probable errors are of the size of the plotted points. Connecting lines have been drawn for the isotopes of even Z elements to aid in identification and emphasize regularity. The systematic pattern evident in this figure encourages confidence in the reliability of these results.

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TABLE I. Experimental mass doublets and derived[a] atomic masses expressed as mass defects ($M-A$).

Doublet	No. of runs	ΔM (10^{-4} aMU)	Mass defect of heavy nuclide (10^{-4} aMU, all negative)
1. $C_4H_3 - \frac{1}{2}Pd^{102}$	4	710.6±0.4	625.0±0.9
2. $C_4H_4 - \frac{1}{2}Pd^{104}$	5	796.8±0.5	634.5±1.1
			636.3±4.2[b]
3. $C_8H_9 - Pd^{105}$	4	1656.5±1.4	616.0±1.5
4. $C_4H_5 - \frac{1}{2}Pd^{106}$	4	878.3±0.9	634.7±1.8
5. $C_8H_{10} - Pd^{106}$	4	1751.1±1.8	629.1±1.8
Weighted average of 4 and 5			632.0±1.9
6. $C_4H_6 - \frac{1}{2}Pd^{108}$	4	952.4±0.5	619.9±1.1
			631.0±3.3[b]
7. $C_4H_7 - \frac{1}{2}Pd^{110}$	4	1025.6±0.6	603.5±1.3
8. $C_4H_5 - \frac{1}{2}Cd^{106}$	5	861.8±0.7	601.6±1.4
9. $C_4H_6 - \frac{1}{2}Cd^{108}$	5	949.4±0.5	614.0±1.1
10. $C_4H_7 - \frac{1}{2}Cd^{110}$	5	1031.0±0.6	614.3±1.3
11. $C_8H_{15} - Cd^{111}$	5	2131.5±0.8	602.2±1.0
12. $C_4H_8 - \frac{1}{2}Cd^{112}$	5	1109.8±0.5	608.9±1.2
13. $C_8H_{16} - Cd^{112}$	4	2224.3±0.9	613.6±1.0
Weighted average of 12 and 13			611.5±1.7
			601.4±4.0[c]
14. $C_8H_{17} - Cd^{113}$	5	2286.1±0.9	593.9±1.1
15. $C_4H_9 - \frac{1}{2}Cd^{114}$	4	1186.6±0.7	599.5±1.5
16. $C_3H_5O - \frac{1}{2}Cd^{114}$	4	823.0±0.6	600.9±1.2
Weighted average of 15 and 16			600.3±0.9
17. $C_3H_6O - \frac{1}{2}Cd^{116}$	4	893.9±0.6	579.8±1.2
			578.5±4.2[c]
18. $C_8H_{17} - In^{113}$	5	2287.7±1.0	595.5±1.2
19. $C_9H_7 - In^{115}$	5	1512.0±1.0	596.0±1.1
20. $C_9H_7 - Sn^{115}$	5	1514.6±2.5	598.6±2.5
21. $C_3H_6O - \frac{1}{2}Sn^{116}$	4	907.8±0.9	607.5±1.8
22. $C_9H_8 - Sn^{116}$	3	1604.7±1.4	607.2±1.5
Weighted average of 21 and 22			607.3±1.1
			620.5±5.2[d]
23. $C_9H_9 - Sn^{117}$	5	1673.7±0.9	594.8±1.0
24. $C_3H_7O - \frac{1}{2}Sn^{118}$	4	986.3±1.3	601.6±2.6
25. $C_9H_{10} - Sn^{118}$	4	1762.9±1.9	602.5±1.9
Weighted average of 24 and 25			602.2±1.6
26. $C_9H_{11} - Sn^{119}$	6	1829.7±1.1	587.8±1.2
27. $C_5 - \frac{1}{2}Sn^{120}$	4	489.2±0.7	594.1±1.4
			612.5±4.2[d]
28. $C_5H - \frac{1}{2}Sn^{122}$	4	561.1±0.7	575.1±1.5
29. $C_5H_2 - \frac{1}{2}Sn^{124}$	5	630.5±0.5	551.0±1.1
30. $C_9H_{12} - Te^{120}$	4	1894.5±1.5	571.2±1.6
31. $C_5H - \frac{1}{2}Te^{122}$	5	563.9±0.4	580.7±0.8
32. $\frac{1}{2}Te^{123} - C_5H$	4	4444.9±2.0	563.2±3.9
33. $C_5H_2 - \frac{1}{2}Te^{124}$	6	641.1±0.5	572.2±1.1
34. $\frac{1}{2}Te^{125} - C_5H_2$	4	4368.0±1.6	554.0±3.1
35. $C_5H_3 - \frac{1}{2}Te^{126}$	6	715.6±0.3	558.0±0.7
36. $C_{10}H_8 - Te^{128}$	4	1570.9±1.2	535.1±1.3
37. $C_5H_5 - \frac{1}{2}Te^{130}$	6	856.7±0.4	514.7±1.0
			532 ±8[e]
38. $C_{10}H_7 - I^{127}$	6	1501.6±1.2	547.2± 1.3
			540 ±10[f]
39. $C_5H_2 - \frac{1}{2}Xe^{124}$	4	626.1±0.3	542.2± 0.7
40. $C_5H_3 - \frac{1}{2}Xe^{126}$	5	712.7±0.7	552.4± 1.4
41. $C_{10}H_8 - Xe^{128}$	5	1591.3±0.7	555.4± 0.9
42. $C_3H_7 - \frac{1}{3}Xe^{129}$	5	865.4±0.4	539.9± 1.5
			544 ±13[g]
			543.0± 2.6[h]
			545 ±15[f]
43. $C_5H_5 - \frac{1}{2}Xe^{130}$	7	874.3±0.4	549.9± 1.0
			550 ±20[f]
44. $CO_2 - \frac{1}{3}Xe^{131}$	5	3549.3±1.4	532.7± 4.2
			560 ±20[f]
45. $C_5H_6 - \frac{1}{2}Xe^{132}$	5	950.0±0.6	538.2± 1.3
46. $CO_2 - \frac{1}{3}Xe^{132}$	7	218.0±0.5	538.8± 1.4
Weighted average of 45 and 46			538.5± 1.0
			526.0± 6.5[h]
			550 ±20[f]
47. $C_5H_7 - \frac{1}{2}Xe^{134}$	6	1022.2±0.5	519.7± 1.2
			530 ±20[f]
48. $C_5H_8 - \frac{1}{2}Xe^{136}$	5	1091.5±0.4	495.4± 1.1

[a] Derived mass defects assume $H^1 = 1.008146$, $C^{12} = 12.003842$.
[b] Duckworth, Woodcock, and Preston, Phys. Rev. **78**, 479 (1950).
[c] H. Duckworth and R. Preston, Phys. Rev. **79**, 402 (1950).
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It is also of interest to note that the isotopes of tin have a greater average binding energy per nucleon than might have been expected by extrapolation from the other data of the figure.

There are several cases where the mass difference between naturally occurring isobars is of particular interest. Cd^{113} and In^{113}, In^{115} and Sn^{115}, and Sb^{123} and Te^{123} are three of the few known examples of adjacent "stable" isobars. In^{115} has been recently reported to emit natural β-radiation of 0.63±0.03 Mev energy with a half-life of $6\pm2\times10^{14}$ yr.[4] The same experimenters detected no radiations from the pair at mass 113 but established minimum half-lives ranging from 1×10^{13} to 2×10^{15} yr for several possible modes of decay. The mass spectrometric results of $In^{115}-Sn^{115}=0.24\pm0.25$ Mev and $Cd^{113}-In^{113}=0.15\pm0.15$ Mev agree that In^{115} is heavier than Sn^{115} and that more energy is available for decay at 115 than at 113. They suggest that Cd^{113} is the possible radioactive member of the $Cd^{113}-In^{113}$ pair. Combined with reaction data, the results reported here indicate that Te^{123} is the possible radioactive member of the pair at $A=123$.

The possibility of double β-decay of the isotopes Sn^{124}, Te^{128}, Te^{130}, Pd^{108}, and Pd^{110} has also received experimental attention in the literature.[5–8] The conclusions of these experiments are not significantly altered by the specific values for the transition energies determined from the masses of this paper. In the one case where a mass spectrographic value for the available transition energy has been previously reported,[9] the value of 1.5±0.4 Mev obtained for the $Sn^{124}-Te^{124}$ energy difference compares favorably with the 1.97 ±0.13 Mev result of this paper.

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TABLE II. Comparison of mass spectrometric and nuclear reaction mass differences.

	From Table I	From nuclear reactions	Reactions used	Ref.
$Xe^{128}-I^{127}$	1-0.00083±16	1-0.00070±44	$I^{128}(\beta)Xe^{128}$	a
			$I^{127}(n,\gamma)I^{128}$	b
$Te^{128}-I^{127}$	1.00121±18	1.00061	$Te^{128}(n,2n)Te^{127}$	c
			$Te^{127}(\beta)I^{127}$	d
$I^{127}-Xe^{126}$	1.00052±19	1.00029±15	$I^{127}(\gamma,n)I^{126}$	e,f
			$I^{126}(\beta)Xe^{126}$	a
$Te^{125}-Te^{124}$	1.00183±33	1.00169±32	$Te^{125}(\gamma,n)Te^{124}$	g
$Te^{122}-Sn^{120}$	2.00134±16	2.00167±40	$Sb^{122}(\beta)Te^{122}$	a
			$Sb^{121}(d,p)Sb^{122}$	h
		[2.00126] [r]	[$Sb^{121}(n,\gamma)Sb^{122}$]	i
			$Sn^{121}(\beta)Sb^{121}$	j
			$Sn^{120}(d,p)Sn^{121}$	h
$Sn^{119}-Sn^{118}$	1.00143±20	1.00199±16	$Sn^{119}(\gamma,n)Sn^{118}$	k
$Sn^{118}-Sn^{117}$	1-0.00074±18	1-0.00107±22	$Sn^{117}(d,p)Sn^{118}$	h
		[0.00103] [r]	[$Sn^{117}(n,\gamma)Sn^{118}$]	i
$Sn^{116}-In^{115}$	1-0.00113±16	1-0.00126±25	$In^{116}(\beta)Sn^{116}$	l
			$In^{115}(d,p)In^{116}$	h
$In^{115}-Sn^{115}$	0.00026±27	0.00068±3	$In^{115}(\beta)Sn^{115}$	m
		[0.00053±2] [r]	[$In^{115*}(\beta\gamma)Sn^{115}$]	n
$Cd^{114}-Cd^{113}$	1-0.00064±14	1-0.00035±65	$Cd^{113}(n,\gamma)Cd^{114}$	o
		[1.00147±22] [r]	[$Cd^{113}(n,\gamma)Cd^{114}$]	b
$Cd^{113}-Cd^{112}$	1.00176±20	1.00207±16	$Cd^{113}(\gamma,n)Cd^{112}$	k
$Cd^{110}-Cd^{108}$	2-0.00003±16	2-0.00048±94	$Ag^{110}(\beta)Cd^{110}$	a
			$Ag^{109}(n,\gamma)Ag^{110}$	b
			$Ag^{109}(\gamma,n)Ag^{108}$	p
			$Ag^{108}(\beta)Cd^{108}$	q
$Pd^{105}-Pd^{104}$	1.00185±19	1.00126±32	$Pd^{105}(\gamma,n)Pd^{104}$	g

[a] A. C. G. Mitchell, Revs. Modern Phys. **22**, 36 (1950).
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[r] Mass difference obtained using bracketed alternative reaction.

COMPARISON WITH THE WIGNER FORMULA

Comparisons of experimentally determined masses with those predicted by existing semi-empirical mass formulas can reveal inadequacies in the hypotheses upon which the formulas are based. Of particular interest is the possibility of finding irregularities associated with nuclear shell structure. The Bohr-Wheeler and Wigner mass formulas have both been used in investigations of this nature.[1,2,10,11] In the interest of correlating results in this mass region with those at lower masses previously reported from this laboratory, the Wigner formula will be considered in the following development.

Wigner[12] has derived expressions for the Coulomb energy (CE), kinetic energy (KE), and potential energy (PE) components of the binding energy of a nucleus. With the insertion of a nuclear radius constant of 1.45×10^{-13} cm derived from mirror nuclei, these quantities become

$$\mathrm{CE}=0.635Z(Z-1)/A^{\frac{1}{3}}\ \text{milli-mass units}, \quad (1)$$

$$\mathrm{KE}=14.640A+32.53(T^2+\delta/2)/A\ \text{mMU}, \quad (2)$$

$$\mathrm{PE}=-\tfrac{1}{2}A(A-1)L'-\Xi L\ \text{mMU}. \quad (3)$$

For a given nucleus of mass number A, Z=number of protons, N=number of neutrons, $T=(N-Z)/2$="isotopic spin," and

$$\delta=\begin{cases}0 \text{ for even } N, \text{ even } Z;\\ 1 \text{ for odd } N, \text{ even } Z;\\ 1 \text{ for even } N, \text{ odd } Z;\\ 2 \text{ for odd } N, \text{ odd } Z.\end{cases}$$

L' is the average potential energy per pair of nucleons from forces independent of coupling symmetry. The magnitude of L is the average potential energy per pair of nucleons from forces dependent on symmetry of coupling. The sign of L changes with symmetry, so the contribution of these forces to the total potential energy is L times the difference, Ξ, in number of symmetrically and antisymmetrically coupled pairs of particles. In the mass region of interest here, Ξ is a negative number and can be computed from the formula

$$\Xi=2A-A^2/8-T(T+4)/2-3\delta/4. \quad (4)$$

The total binding energy of a nucleus is given by

$$\mathrm{BE}=\mathrm{CE}+\mathrm{KE}+\mathrm{PE}, \quad (5)$$

where BE and PE are both negative quantities.

The binding energy is determined experimentally by a knowledge of the atomic mass M,

$$\mathrm{BE}=M-(ZM_h+NM_n), \quad (6)$$

where M_h is the mass of the hydrogen atom and M_n is the mass of the neutron. The present status of the

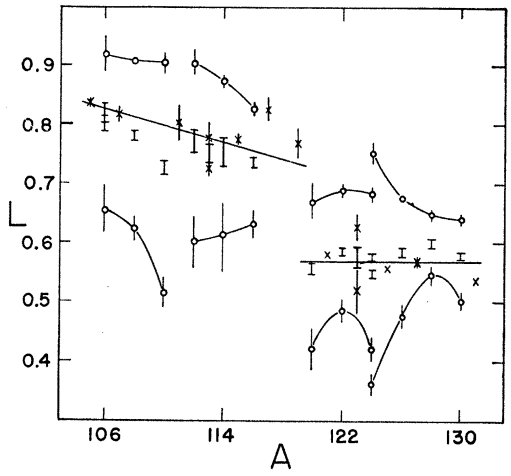

FIG. 2. L-values from unmodified Wigner formula. Lines have been drawn to emphasize pattern in the data. The isobaric pairs entering into given L-value determinations are classified by the symbols: O = adjacent even A isobars; × = adjacent odd A isobars; I = isobars differing by $Z=2$. For the points on the curves above the straight lines, the odd-odd nucleus has one less proton than the even-even nucleus. For points on the curves below the straight lines, the odd-odd nucleus has one more proton than the even-even nucleus.

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TABLE III. Data used in comparing experimental mass values with Wigner formula.

Nuclide	Reaction	Mev	−Mass defect (mMU)	−PE (mMU)	−Ξ
Pd^{102}			62.5±0.1	2723.1	1119
Pd^{104}			63.4±0.1	2772.7	1174
Rh^{105}	β^-	0.570[a]	61.0±0.2	2788.1	1212
Pd^{105}			61.6±0.2	2795.6	1203
Rh^{106}	β^-	3.55[b]	59.4±0.3	2811.6	1242
Pd^{106}			63.2±0.2	2821.8	1231
Ag^{106}	(p,n)	−3.8[c]	60.0±0.2	2826.3	1224
Cd^{106}			60.2±0.1	2834.6	1215
Ag^{107}	(d,p)	4.78[d]	61.3±0.2	2852.0	1252
Cd^{107}	β^+	0.32[e]	59.7±0.2	2858.6	1244
Pd^{108}			62.0±0.1	2870.3	1290
Ag^{108}	β^-	1.49[f]	59.8±0.1	2875.3	1282
Cd^{108}			61.4±0.1	2884.4	1272
Ag^{109}	(γ,n)	9.3[g]	60.6±0.5	2900.7	1311
Pd^{110}			60.4±0.1	2918.9	1351
Ag^{110}	β^-	2.86[b]	58.4±0.2	2923.6	1342
Cd^{110}			61.4±0.1	2933.5	1331
Ag^{111}	β^-	1.06[h]	59.1±0.1	2949.1	1372
Cd^{111}			60.2±0.1	2957.2	1362
Cd^{112}			61.2±0.2	2982.8	1392
In^{112}	β^+	1.7[i]	58.2±0.4	2987.6	1384
Sn^{112}	β^-	1.0[i]	59.3±0.5	2996.7	1374
Ag^{113}	β^-	2.1[j]	57.1±0.2	2997.6	1435
Cd^{113}			59.4±0.1	3006.1	1424
In^{113}			59.6±0.1	3013.4	1414
Cd^{114}			60.0±0.1	3031.7	1455
In^{114}	(γ,n)	−9.5[d]	58.4±0.6	3037.2	1446
Sn^{114}	β^-	2.05[k]	60.6±0.6	3046.8	1435
Cd^{115}	β^-	1.80[l]	57.7±0.2	3054.7	1488
In^{115}			59.6±0.1	3063.2	1477
Sn^{115}			59.9±0.2	3070.9	1467
Cd^{116}			58.0±0.1	3080.1	1520
In^{116}	β^-	2.95[m]	57.7±0.2	3086.4	1510
Sn^{116}			60.7±0.1	3096.4	1498
In^{117}	β^-	1.73[n]	57.6±0.1	3111.4	1542
Sn^{117}			59.5±0.1	3120.5	1531
Sn^{118}			60.2±0.2	3145.7	1563
In^{119}	β^-	2.7[o]	55.9±0.3	3160.2	1609
Sn^{119}			58.8±0.1	3169.4	1597
Sn^{120}			59.4±0.1	3195.1	1630
Sb^{120}	(γ,n)	−9.25[p]	56.5±0.3	3199.4	1620
Te^{120}			57.1±0.2	3207.4	1608
Sn^{121}	(d,p)	4.0[d]	57.1±0.4	3218.2	1665
Sb^{121}	(n,γ)	7.02[q]	57.4±0.2	3225.2	1653
Sn^{122}			57.5±0.2	3243.9	1699
Sb^{122}	β^-	1.94[b]	56.0±0.1	3249.0	1688
Te^{122}			58.1±0.1	3258.0	1675
Sn^{123}	(γ,n)	−8.50[r]	55.0±0.2	3267.0	1735
Sb^{123}	(γ,n)	−9.3[s]	57.0±0.3	3275.1	1722
Te^{123}			56.3±0.4	3281.3	1710
Sn^{124}			55.1±0.1	3292.6	1770
Sb^{124}	β^-	2.973[b]	54.0±0.2	3297.6	1758
Te^{124}			57.2±0.1	3307.2	1744
I^{124}	β^+	3.83[b]	53.1±0.2	3310.8	1734
Xe^{124}			54.2±0.1	3319.8	1722
Sb^{125}	β^-	0.768[b]	54.6±0.3	3323.5	1793
Te^{125}			55.4±0.3	3330.7	1780
Te^{126}			55.8±0.1	3356.2	1815
I^{126}	β^-	1.268[b]	53.9±0.2	3361.5	1804
Xe^{126}			55.2±0.1	3370.2	1791
Te^{127}	β^-	0.70[t]	54.0±0.2	3379.9	1852
I^{127}			54.7±0.1	3387.3	1839
Te^{128}			53.5±0.1	3404.8	1888
I^{128}	β^-	2.02[b]	53.4±0.1	3411.3	1876
Xe^{128}			55.5±0.1	3420.4	1862
Xe^{129}			54.0±0.2	3444.1	1899
Te^{130}			51.5±0.1	3454.0	1963
I^{130}	β^-	2.975[b]	51.8±0.2	3460.5	1950
Xe^{130}			55.0±0.1	3470.1	1935
I^{131}	β^-	0.968[b]	52.2±0.4	3486.3	1987
Xe^{131}			53.3±0.4	3493.8	1973
Xe^{132}			53.8±0.1	3519.6	2010
Xe^{134}			52.0±0.1	3568.8	2087
Xe^{136}			49.5±0.1	3617.7	2166

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theory of nuclear forces does not permit a computation of binding energy from Eq. (5) to compare with Eq. (6). It is of interest, however, to combine the experimental binding energy with the computed Coulomb and kinetic energies and examine the behavior of the functions L and L' in the resulting value for the potential energy.

For this purpose, Table III has been prepared containing the masses of 74 stable and radioactive isotopes between mass numbers 102 and 136. Nuclear reaction mass differences have been combined with the masses reported in this paper. The type of reaction and reaction energy used in computing the mass defects are also given for the nuclei concerned. The Bureau of Standards Circular *Nuclear Data* provided an invaluable aid in locating this information. Although the question might legitimately be raised as to whether or not certain individual values are the "best" values from existing data, the table as a whole presents a much more extensive and accurate compilation of masses than previously possible in this mass region. With 1.008146 and 1.008987 for the hydrogen atom and neutron masses, respectively, Eqs. (5) and (6) combine to give

$$-\text{PE}=\text{mass defect}+8.146Z+8.987N+\text{CE}+\text{KE mMU}. \quad (7)$$

The potential energy values obtained from this formula are listed in Table III. The value of Ξ, which is an integer from its definition as a count of couplings, is given in the final column.

From Eq. (3) one can see that potential energy difference between two isobars (nuclei of same A) designated a and b, is independent of L', and that one can obtain a value for the function L from the relation

$$L=-(\text{PE}_a-\text{PE}_b)/(\Xi_a-\Xi_b). \quad (8)$$

Values for L' can then be obtained by returning to Eq. (3). Figure 2 is a plot of experimental points for the function L determined by applying Eq. (8) to the isobars

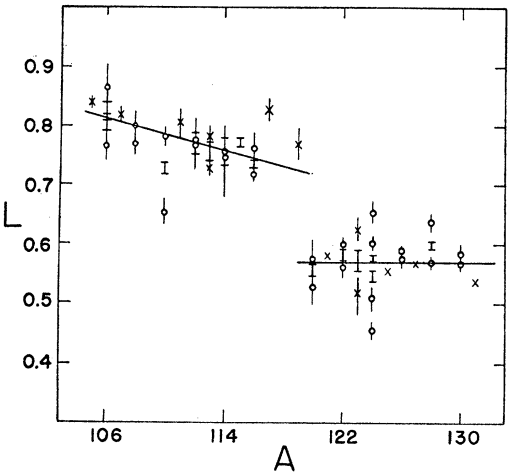

FIG. 3. L values from modified Wigner formula. The notation is identical to that of Fig. 2.

of Table III. As previously observed by Frisch,[13] there is evidence of systematic eccentricities in the data. Lines have been drawn to make this fact more evident. Straight lines have been drawn through two central groups of points. These L values were obtained from adjacent odd A isobaric nuclei and from isobaric pairs differing by $Z=2$. The upper series of curves contains adjacent even A isobars where the odd-odd nucleus has one less proton than the even-even member of the pair. These are even A isobars between the elements I—Xe, Sb—Te, In—Sn, and Ag—Cd. The remaining adjacent even A L values, those where the odd-odd nucleus contains one more proton than the even-even nucleus, fall in the lower curves. These points come from Te—I, Sn—Sb, Cd—In, and Pd—Ag.

The conclusion of interest evident in Fig. 2 is the fact that a simple line function for L will not fit the data. More specifically, there is a definite discontinuity in the L values between mass numbers 119 and 120 associated with the magic number of 50 protons. Also, the Wigner formula does not correctly predict the potential energy difference between adjacent even A isobars. Concerning the magic number discontinuity, all L values below $A=119$ in Fig. 2 are determined from isobars involving isotopes with $Z=50$ (tin) or less. All L values above $A=120$ are determined from isobars involving isotopes with $Z=50$ or more. Unfortunately, the masses of the antimony isotopes below Sb^{120} are not accurately known. Hence it is not at present possible to observe an overlap of these two regions, e.g., the difference in L determined from $Sn^{119}-In^{119}$ and $Sb^{119}-Sn^{119}$. However, one might infer from the absence of a discontinuity in the curve for tin in Fig. 1 that the observed break in Fig. 2 is associated with 50 protons. It should also be noted that this discontinuity in L is in the same direction as those observed at 20 and 28 protons in previous results from this laboratory.

The two extreme L values at a given even A contain the same odd-odd nucleus. Hence if the mass formula correctly accounted for the potential energy associated with an odd-odd configuration, the effect in Fig. 2 would be to collapse the extreme L values into the central group. The discontinuity at 50 protons would still remain. A simple empirical alteration of the Wigner formula will appreciably improve its account of the potential energy differences between isobaric nuclei. The revision occurs in formulas (2) and (4) which become

$$\mathrm{KE}'=14.640A+32.53(T^2+2+3\delta/2)/A \text{ mMU}, \quad (9)$$

$$\Xi'=2A-A^2/8-T(T+4)/2-2-5\delta/4. \quad (10)$$

A revised plot for L is given in Fig. 3. The method of arriving at this revision is presented below.

Wigner's assumptions lead to the conclusion that the greatest nuclear binding occurs when as many states as possible are occupied by four nucleons and the remaining nucleons are paired off, two to a state. It was observed, however, that the type of correction necessary to account for the observed potential energy differences between isobaric nuclei was in the nature of an increased

TABLE IV. An example of nuclear wave function modification which will contribute towards an improved fit of the Wigner formula to mass data.

Assumed by Wigner			Empirical change
n			n
$n\,\bar{n}$			$n\,\bar{n}$
$n\,\bar{n}$			$n\,\bar{n}$
$n\,\bar{n}\,p$			$n\,\bar{n}\,p$
$n\,\bar{n}\,p\,\bar{p}$			$n\,\bar{n}\,p$
$n\,\bar{n}\,p\,\bar{p}$			$n\,\bar{n}\,p\,\bar{p}$
	$\frac{1}{2}(Z-1)$	$\frac{1}{2}(N-1)$	
$n\,\bar{n}\,p\,\bar{p}$			$n\,\bar{n}\,p\,\bar{p}$

TABLE V. A summary of the effect of breaking up one possible pair of particles on Wigner formula components for heavy nuclei.

Type of nucleus N	Z	Change	$\Xi'-\Xi$	$f(P)'$
even	even	$\bar{n}\to n$, $\bar{p}\to p$	-2	$(T^2)+2$
odd	odd	$\bar{n}\to n$, $\bar{p}\to p$	-3	$(T^2+1)+4$
odd	even	$\bar{n}\to n$	-3	$(T^2+\frac{1}{2})+4$
		$\bar{p}\to p$	-2	$(T^2+\frac{1}{2})+2$
even	odd	$\bar{n}\to n$	-2	$(T^2+\frac{1}{2})+2$
		$\bar{p}\to p$	-3	$(T^2+\frac{1}{2})+4$

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TABLE VI. Difference in 10^{-4} aMU between a fitted Wigner formula and experimental masses. The Wigner functions L and L' are obtained in each case by a least squares fit of the nuclides indicated by heavy type. (a) Unmodified Wigner formula fitted to nuclides containing 50 or less protons; (b) unmodified Wigner formula fitted to nuclides containing 50 or more protons; (c) empirically modified Wigner formula fitted to nuclides containing 50 or less protons; (d) empirically modified Wigner formula fitted to nuclides containing 50 or more protons.

(a)

Neutrons	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
82										−58
80										−57
78								−28	−44	−56
									−51	−62
76								−34		−56
								−33	−50	−63
74						**10**	−16	−29	−48	−55
						1	−25	−34	−54	
72						**7**	−10	−28		−56
						−2	−21	−34	−62	
70					**−5**	**6**	−19	−25		−57
						−1	−25			
68				**4**	**−8**	**1**		−32		
				−3	**−8**	**1**				
66			**−3**	**3**	**−2**	**1**				
				−3	**−10**	**−1**				
64		**14**	**−5**	**2**	**−7**	**2**				
			−12	**−3**	**−12**					
62		**9**	**−2**	**1**		**1**				
	−2		**−6**							
60	**−3**	**8**	**1**	**4**						
		−3	**−3**	**1**						
58		**8**		**5**						
56		**5**								
	45	46	47	48	49	50	51	52	53	54

Protons

(b)

Neutrons	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
82										**35**
80										**22**
78								**5**	**9**	**13**
									−3	**4**
76								**−4**		**7**
								−7	**−8**	**−3**
74						**−1**	**−8**	**−4**	**−7**	**4**
						−10	**−18**	**−10**	**−14**	
72						**−2**	**−2**	**−2**		**0**
						−9	**−13**	**−8**	**−23**	
70					−28	**1**	**−8**	**2**		**−1**
						−4	**−14**			
68				−26	−24	**2**		**−2**		
				−31	−21	**5**				
66			−42	−16	−8	**10**				
				−18	−13	**11**				
64		−27	−30	−7	−2	**21**				
			−32	−7	−4					
62		−14	−12	5		**30**				
	−35		−9							
60	−24	4	7	25						
		0	10	27						
58		22		41						
56		39								
	45	46	47	48	49	50	51	52	53	54

Protons

(c)

Neutrons	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
82										−54
80										−53
78								−28	−36	−52
									−38	−53
76								−33		−54
								−27	−37	−54
74						**5**	−11	−30	−42	−53
						3	−14	−29	−41	
72						**4**	−6	−28		−54
						−2	−10	−29	−50	
70					**−3**	**2**	−14	−25		−56
						2	−14			
68				**−1**	**−6**	**−3**		−33		
				−1	**1**	**4**				
66			**−3**	**−1**	**1**	**−2**				
				−1	**−1**	**3**				
64		**7**	**−4**	**−3**	**−4**	**−2**				
			−4	**−2**	**−3**					
62		**2**	**−1**	**−4**		**−3**				
	6		**2**							
60	**−3**	**3**	**2**	**−1**						
		−2	**6**	**3**						
58		**2**		**1**						
56		**−1**								
	45	46	47	48	49	50	51	52	53	54

Protons

(d)

Neutrons	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
82										**31**
80										**19**
78								**2**	**10**	**10**
									4	**5**
76								**−8**		**3**
								−5	**−1**	**−1**
74						**−4**	**−6**	**−8**	**−6**	**−1**
						−9	**−10**	**−8**	**−8**	
72						**−6**	**−1**	**−6**		**−4**
						−7	**−6**	**−7**	**−17**	
70					−25	**−2**	**−6**	**−2**		**−6**
						−2	**−6**			
68				−30	−21	**−2**		**−5**		
				−28	−13	**7**				
66			−40	−20	−6	**6**				
				−17	−4	**14**				
64		−29	−27	−9	−1	**18**				
			−22	−4	4					
62		−17	−10	3		**27**				
	−25		−1							
60	−20	1	10	21						
		4	19	30						
58		20		38						
56		36								
	45	46	47	48	49	50	51	52	53	54

Protons

dependence on δ in Eqs. (2) and (4). This effect can be produced by breaking up one possible pair of nucleons. For example, with the following notation:

n = neutron with spin up, $\bar{n}$ = neutron with spin down,

p = proton with spin up, $\bar{p}$ = proton with spin down,

the suggested change in nuclear wave function for the case of breaking up a proton pair in an odd-odd nucleus may be symbolized as in Table IV. The result of breaking up one pair of particles in all the various configurations of interest in this mass region is summarized in Table V. Ξ' denotes the revised value of Ξ. $f(P)'$ denotes the revised value of the sum of the squares of the partition quantum numbers entering into the deriva-

tion of the kinetic energy term of the binding energy of a nucleus. The original value of this quantity is enclosed in parenthesis.

The values for $\Xi'-\Xi$ and $f(P)'$ in the case of odd-even and even-odd nuclei are observed in Table V to depend upon whether a neutron pair or a proton pair is broken up. The best account of experimental data was given by assuming an average of these two possibilities for each case, i.e., $\Xi'-\Xi=-2\frac{1}{2}$, $f(P)'-f(P)=3$. Equations (9) and (10) show how the kinetic and potential energy terms are affected by these changes.

The effect of the empirical modification can best be demonstrated by fitting both forms of the Wigner formula to observed binding energies. For this purpose, the function L has been approximated by a straight line on either side of the magic number discontinuity. Following the procedure of reference 1, it has been assumed that AL is constant for L values determined by isotopes of $Z=50$ or less. For L values determined by isotopes of $Z=50$ or more, L has been assumed constant. Since a large discontinuity is not observed in the binding energies, L' must change in a manner which largely compensates for the observed change in L. The values for L' obtained by substituting the average L values in the expression for potential energy, Eq. (3), can also be approximated closely in these limited mass regions by straight line functions. The empirical functions obtained for the cases of interest by a least square fit of straight lines to data are given by

1. $L=87.42/A$, $AL'=68.692+0.04129A$,
2. $L=0.56992$, $AL'=50.398+0.15760A$,
3. $L=86.48/A$, $AL'=68.709+0.03968A$,
4. $L=0.57162$, $AL'=50.457+0.15779A$,

where the cases referred to are 1. unmodified Wigner formula fitted to nuclides with $Z\leqq 50$; 2. unmodified Wigner formula fitted to $Z\geqq 50$; 3. modified Wigner formula fitted to $Z\leqq 50$; 4. modified Wigner formula fitted to $Z\geqq 50$.

Tables VI (a), (b), (c), and (d) give the differences between observed and computed binding energies in 10^{-4} aMU for these four cases. The entries in heavy type are the residuals for the fitted region. A positive value indicates that the nucleus has greater stability than predicted by the empirical formula. In each case the differences have been tabulated beyond the region of fit. In Table VI (a) the average deviation within the region of fit is 0.45 Mev. This is due primarily to a pattern of alternate high and low values evident along either rows or columns. Empirical formula revision reduces this average deviation to 0.27 Mev as observed in Table VI (c). The observed increase in error beyond the region of fit suggests that a change of slope in the binding energy surface is associated with the breaks in the functions L and L'. In Table VI (b) the pattern of alternate high and low values within the region of fit is again evident. However, it is superimposed upon a strong trend in the residuals which points out the inadequacy of the approximations introduced in fitting the Wigner formula to the data of this region. In Table VI (d) empirical formula revision has again reduced the local irregularities. The average deviation in this case is reduced only from 0.87 to 0.75 Mev due to the previously noted larger source of error in the fit.

The instances of deviation of individual points from the general pattern of results in Figs. 2 and 3, and Tables VI also merit examination. Deviations in excess of 10 percent of the average L values drawn in Fig. 3 occur at mass numbers 110, 117, 123, 124, and 128. The L value at $A=117$ is derived from a 1939 magnetic spectrometer measurement for $In^{117}(\beta)Sn^{117}$. At $A=110$ the upper point is derived from the well established $Ag^{110}(\beta)Cd^{110}$ decay. The lower L value at $A=128$ is also derived from a well-established β-decay. Errors in either or both of the stable isotopes at each of these mass numbers could result in the observed deviations. In these two cases the mass spectrometric measurements might well be checked in the interest of confirming the systematic pattern of results or establishing deviations from it. At $A=124$, the deviation appears well established since the extreme lower point is derived from $I^{124}(\beta^+)Te^{124}$. The tabulated mass of Sb^{123} involves (γ,n), β, and mass spectrometric measurements. An error in this value could cause the spread in L values at $A=123$.

In conclusion, the type of discontinuities previously observed between mass data and fitted Wigner formula at the magic numbers of 20 and 28 protons is equally evident at 50 protons. An empirical alteration of the formula was found to appreciably improve its agreement on either side of the discontinuity. The residuals in the least square fits still exceed the probable error in measurement. Prediction of mass values within a limited mass range by means of a locally fitted Wigner formula is rendered uncertain by some apparently well-established deviations. It should be noted again that this development has, in effect, assumed that the expressions for kinetic and Coulomb energy are correct and blamed the potential energy term for observed irregularities. This may have resulted in a distorted picture of the nature of the discrepancies. Nevertheless, systematic deviations from the formula are sufficiently well established to suggest that a profitable re-examination of its basic assumptions might be made.

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