π^0 Production Cross Section for 430-Mev Protons on H and Be*

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(Received June 23, 1952)

The differential cross section for the production of photons in the forward direction by the reaction (430-Mev $p+H\rightarrow p+p+\pi^0$; $\pi^0\rightarrow 2\gamma$) is measured as 0.20×10^{-27} cm²/sterad. The ratio of total to differential gamma cross section does not depend strongly on the symmetry of pion emission. Using an average calculated ratio, we find the pion-production cross section from hydrogen to be 0.45×10^{-27} cm².

The differential gamma-production cross section in the forward direction on Be is measured to be 4.35×10^{-27} cm²/sterad. We assume that in analogy with the case of H the ratio of total to differential photon cross section will not depend on symmetry of pion emission. Using the ratio calculated for isotropic emission we find $\sigma_{\pi^0}=8.7\times 10^{-27}$ cm².

I. INTRODUCTION

THE absolute cross section for neutral pion production by 340-Mev protons has been measured in the case of carbon by Crandall[1] to be 1.4×10^{-27} cm². The relative cross sections for light nuclei at the same energy have been measured by Hales, Hildebrand, Knable, and Moyer.[2] They have interpreted the yield for nuclei from hydrogen to oxygen as due only to the interaction of the incident protons with the neutrons in the nuclei. In the case of hydrogen they observed zero yield within the experimental error.

Near the threshold of pion production, one expects the neutral pion yield from the collision of two protons or two neutrons to be smaller than that from a proton-neutron collision provided that the neutral pion is pseudoscalar.[3] This difference should tend to disappear with increasing energy as the higher angular momentum states of the final nucleons begin to contribute to the reaction.

Accordingly, we have used the 450-Mev maximum energy protons accelerated by the 170-inch synchrocyclotron of the University of Chicago to examine the neutral pion yields from hydrogen and from beryllium. At the higher energy one might expect to find a yield from hydrogen large enough to measure and to obtain additional information on the nature of the neutral pion.

The absolute cross section has been measured both for beryllium and for hydrogen.[4] These elements were bombarded with protons in the cyclotron; the gamma-rays from decay of neutral pions formed in the target were observed in the forward direction. The resulting differential gamma-ray cross section was multiplied by the calculated ratio of total to differential cross section. For hydrogen, this ratio was found to have approximately the same value whether one assumed the pions to emerge isotropically or according to a $\cos^2\theta$ distribution in the barycentric system of the colliding nucleons. For beryllium, whose nucleons have an internal momentum distribution which complicates the calculation, this ratio was calculated only for the isotropic model. It was then assumed in analogy with the calculations on hydrogen that the ratio was little affected by the type of angular distribution.

In the case of beryllium, the absolute cross section was measured in two independent ways. In the first method, the energy spectrum of the gamma-ray beam was measured using a pair spectrometer and was integrated to obtain the differential cross section. In the second method, the collimated gamma-ray beam was passed through lead and the conversion electrons measured in a water Čerenkov telescope.[5] The two values so obtained for the cross section agree within the estimated errors.

For the measurement of the absolute cross section in hydrogen, only the second method was used. This was necessitated by the much lower intensity of gamma-rays from the hydrogen target. The first method is believed to be less reliable because of possible error in the extremely energy dependent efficiency calculated for the round pole pair spectrometer used here.

II. METHOD OF THE WATER ČERENKOV COUNTER

A. Hydrogen Cross Section

There is no sizeable external proton beam from the Chicago synchrocyclotron. On this account, it was convenient to bombard our targets with the internal beam of the cyclotron. To eliminate the large background which would accompany the bombardment of a hydrogenous compound, liquid hydrogen was used as the target material.

The cyclotron beam, during acceleration, gains radius only at a rate of about 0.001 inch per revolution. In order to get the protons of the beam into the liquid hydrogen past the container walls, it was necessary to deflect the beam by a much larger amount in one revolution around the cyclotron. This is done very conveniently by vertical scattering.

A platinum scattering target 0.020 inch thick was

* Research supported by the joint program of the ONR and AEC.

[1] W. E. Crandall, University of California Report UCRL-1637 (unpublished).

[2] Hales, Hildebrand, Knable, and Moyer, Phys. Rev. **85**, 373 (1952).

[3] K. Watson and K. Brueckner, Phys. Rev. **83**, 1 (1951).

[4] L. Marshall *et al.*, Phys. Rev. **87**, 220 (1952); J. Marshall *et al.*, Phys. Rev. **87**, 220 (1952).

[5] John Marshall, Phys. Rev. **86**, 685 (1952).

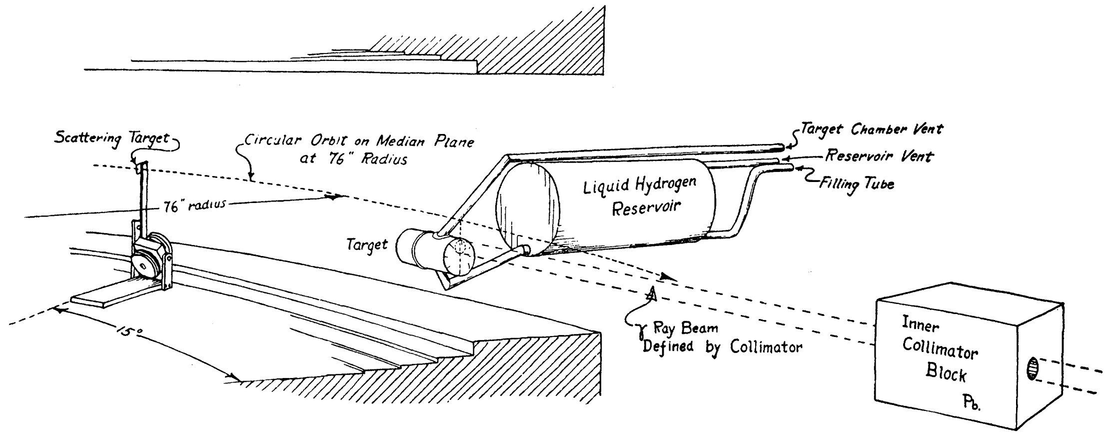

FIG. 1. The arrangement of the scattering target, the liquid hydrogen target, and the collimator within the cyclotron vacuum chamber.

placed at a 76-inch radius in the median plane of the cyclotron. It could be inserted or removed by a flip coil controlled externally, Fig. 1. The vertical oscillations of the beam have a frequency at this radius which is approximately one-quarter of the cyclotron frequency. One effect of the scatterer is to make the amplitude of the vertical oscillations much larger. The beam that passes through the scatterer has larger vertical oscillations than before, and has a phase of vertical oscillation such that there is a node at the scatterer, and consequently a loop one revolution later. In other words, the beam is blown up vertically with a maximum displacement about one revolution after hitting the scatterer.

The radial oscillations at this radius have a frequency that is only slightly smaller than the cyclotron frequency. Protons striking the scatterer are refocused radially about 375 degrees later. It was shown by an exploratory experiment that about 6 percent of the beam was deflected into an area of one cm^2 placed about three cm below the median plane.

The hydrogen target, constructed of copper, was a cylinder four inches long and two inches in diameter with 0.020-inch wall thickness except for the ends which were etched to make them 0.007 inch thick. It was mounted with its axis two inches below the median plane and parallel to the direction of the beam. It was placed fifteen degrees after the scattering target so that the scattered beam was focused into a vertical line through the target 375 degrees after scattering. Liquid hydrogen was admitted to the target from a reservoir (Fig. 1) by gravity feed whenever the target vent tube was open to the atmosphere. When the vent tube was closed the liquid hydrogen was pushed out of the target by its own vapor pressure. Tubes from the reservoir to the outside of the cyclotron allowed it to be refilled. The target was designed and its construction was supervised for the most part by Professors E. Long and E. Fermi who originally had intended to do experiments of this type.

Most of the cyclotron beam has vertical oscillation amplitude smaller than $\frac{3}{4}$ inch. A significant fraction, however, has larger amplitude and might strike the walls of the hydrogen target and cause an unpleasant background if it were not removed. For this purpose a clipper was provided above the median plane to remove protons of vertical amplitude more than $\frac{3}{4}$ inch.

Gamma-radiation from neutral pion decay in the target emerging in the forward direction passed through a hole in the cyclotron shield into the experimental area where it was detected. The gamma-rays were collimated by a two-element collimator. The first element was a $\frac{5}{8}$-inch round hole in a lead block eight inches long placed just outside the 76-inch orbit in the cyclotron chamber. The second element was a $\frac{3}{8}$-inch hole in another eight-inch lead block placed in the experimental area about 40 feet from the target (Fig. 2).

The second element of the collimator determined the solid angle subtended at the target by the detecting apparatus. The first element determined the effective area of the target. This area was chosen to be as close as possible to the top wall of the hydrogen target without allowing gamma-radiation from the wall to pass through the collimator.

The first lead block was mounted on a track so that the $\frac{5}{8}$-inch hole could be moved horizontally across the face of the target from outside the cyclotron. This motion was effected by means of a screw so that the position of the block was quite accurately reproducible.

Directly beyond the second lead block, a magnet was placed to sweep out of the beam any charged particles formed in the collimator. The final beam defined by this system was circular in shape and about $\frac{3}{8}$ inch in diameter. It consisted of high energy gamma-rays with a considerable contamination of neutrons from the end windows of the target.

The gamma-rays in the beam were converted partially to electron pairs by lead converters of various thicknesses. The electrons produced in the converter passed through a diphenyl acetylene scintillation crystal

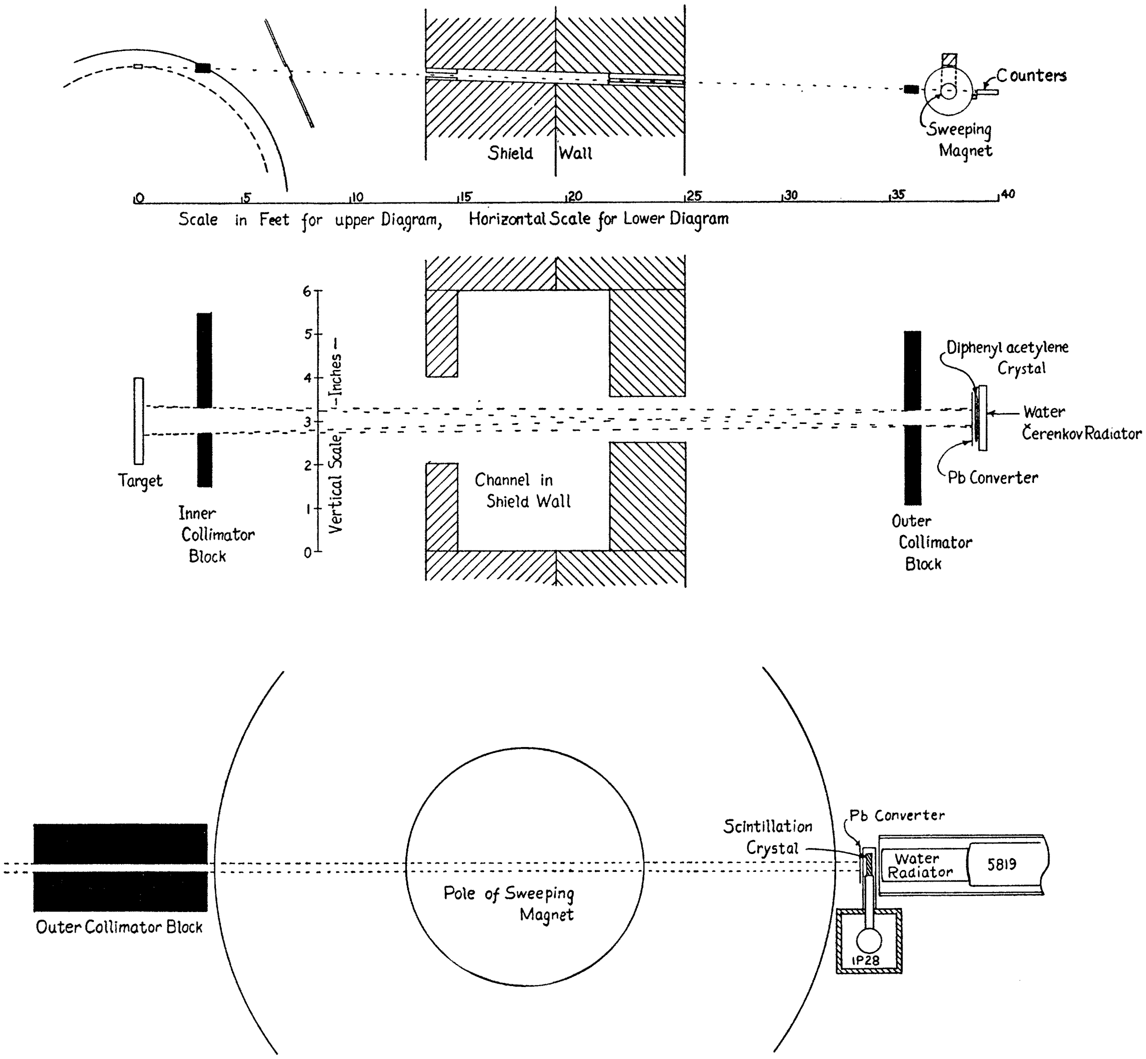

FIG. 2. Horizontal and vertical sections of the experimental layout and detail of the detector.

one inch square and $\frac{1}{4}$ inch thick, and then into a water Čerenkov radiator 1.5 inch in diameter and four inches long. The water of the Čerenkov radiator was contained in a glass tube the rear end of which was shaped so as to fit the cathode of a 5819 photomultiplier, and optical contact was provided by a thin layer of silicone vacuum grease (see Fig. 3). The photomultipliers which looked at the crystal and the Čerenkov radiator were connected through a 6BN6 fast coincidence circuit[6] to a scaling circuit.

The water Čerenkov counter was used in the detector telescope for a very important reason. Whereas a scintillator crystal produces light whenever it is traversed by ionizing particles, light is produced in water only by ionizing particles having velocity greater than 0.76 times the velocity of light. The telescope had to operate in a beam consisting of photons and neutrons up to 450 Mev, and had to observe only electrons from the photons. The knock-on protons from these neutrons have too small a velocity to produce Čerenkov radiation in water. The water Čerenkov counter then has the important characteristic of being sensitive only to electrons in this beam. It is true that mesons could be made having great enough velocity to be recorded; however, the number of these is insignificant.

The intensity of the cyclotron beam striking the scattering target was monitored through a separate collimator. Radiation passing through this collimator from the scattering target struck a paraffin converter. A scintillation crystal coincidence telescope counted particles from the converter.

The ratio of the counting rate of the detecting apparatus to that of the monitor was measured many times over, with and without liquid hydrogen in the target for two thicknesses of the lead converter with

[6] J. Fischer and J. Marshall, Rev. Sci. Instr. 23, 417 (1952).

and without the converter. The net counting rate {(H, Pb−H, no Pb)−(no H, Pb−no H, no Pb)} per monitor count and per unit thickness of lead converter was extrapolated to zero converter thickness (Fig. 4). This procedure was necessary to take account of electrons lost from the detecting apparatus through multiple scattering in the converter. From the extrapolated counting rate and the pair production cross section in lead, the total number of gamma-rays in the beam per monitor count was computed.

To obtain the pion production cross section, it was necessary to know the number of protons per monitor count striking the area of the hydrogen target seen by the collimator. This number was measured by the production of 15-hour radioactivity in aluminum foils by the reaction $Al^{27}(p,n3p)Na^{24}$. Two aluminum foils, each of 0.00025-inch thickness, were mounted on separate flip coils in such a way as to cover the exit window of the hydrogen target when they were raised. One of the foils was raised when and only when there was liquid hydrogen in the target. The other foil was raised when and only when the target was empty. Both foils were protected from stray protons by lead bricks when they were lowered. Also, a third foil was placed behind the lead to measure the background activity.

The time necessary to get data for one cross-sectional measurement was about three days. On the first day the vacuum chamber of the cyclotron was opened to allow the mounted foils and their flip coils to be put in place just behind the exit window of the liquid hydrogen target. The elements of the collimation system were carefully aligned. The area of the target seen by the detecting apparatus was determined as the area illuminated when a light was put in place of the detecting apparatus in the experimental area. The foils were raised into position covering the target window and the illuminated region was marked on them. This was the area passed through and activated by the same part of the proton beam which produced the pions whose decay was observed with the detecting apparatus. The N^{24} activity in this area of the foil therefore gave a measure of the integrated proton flux.

The tank was closed and evacuated. The second day was spent in measuring the water Čerenkov telescope counting rates for two thicknesses of lead converter, with and without liquid hydrogen in the target and with and without converter. At the same time the appropriate foil was raised so that one or the other foil

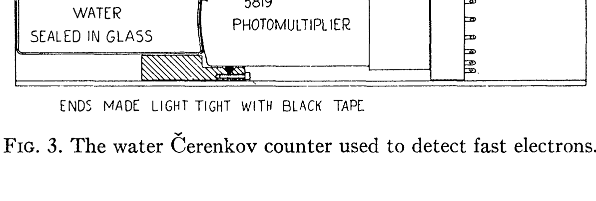

FIG. 3. The water Čerenkov counter used to detect fast electrons.

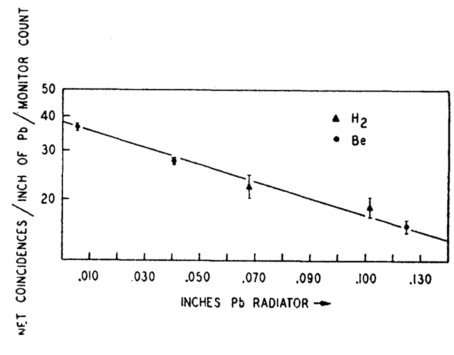

FIG. 4. Correction for thickness of lead converter.

always covered the end of the target when the detecting apparatus and the monitor were in operation. It was considered necessary to have one foil to measure the integrated flux through the empty target and a second foil for the flux through the target filled with liquid hydrogen, because the target filling might have affected the monitor, and the number of proton traversals of the target might also have been different with and without hydrogen. It turned out, however, that the ratio of foil activity to number of monitor counts was the same with and without hydrogen.

It was convenient to let the radioactivity in the cyclotron (and also incidentally the short-lived activities in the aluminum foils) decay overnight. On the third day the tank was opened for the removal of the foils. The activity distribution in the foils was mapped with a Geiger counter, and the critical areas of the two foils were cut out and measured under known conditions of solid angle and efficiency with an end window counter. The activity distribution is shown in Fig. 5. In this case it happened that the collimator was slightly displaced from the optimum radius relative to the radius of the scattering target.

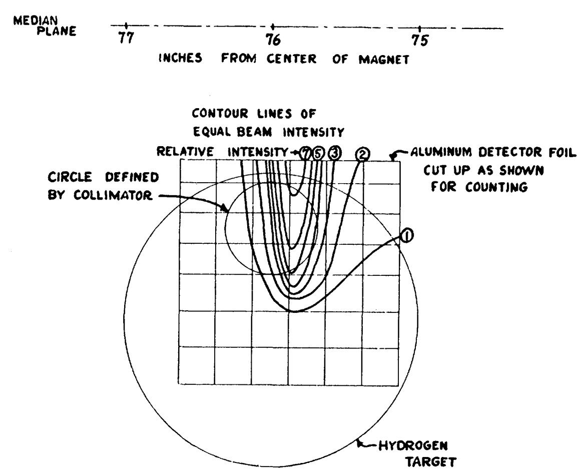

FIG. 5. The scattered-proton beam intensity distribution in the hydrogen target.

TABLE I. Total counts of the detecting apparatus for a 3-day run.

Thickness Pb converter inches	Coincidences per monitor count	Liquid hydrogen in target	Net coincidences per monitor count inch lead
0	16 coinc/6000 M	yes	
	11 coinc/6035 M	no	
0.068	805/26124	yes	0.223±0.024
	354/24079	no	
0.112 Pb	1159/28111	yes	0.188±0.015
	483/25122	no	

Net coincidences/monitor count/inch Pb extrapolated to zero thickness of lead =0.38±0.07.

The total counts of the detecting apparatus for the three-day run are given in Table I. From the determination of the cross section for beryllium where there was considerably more intensity, similar data proved to extrapolate to zero radiator thickness with the same slope as the hydrogen data, and with smaller statistical error. Therefore, we extrapolated the hydrogen data to zero thickness with more confidence than the hydrogen data alone would appear to allow. The resulting value is:

Net coincidences/monitor count/inch Pb, extrapolated to zero thickness of Pb=0.38±0.07.

This number is equal to the product:

Total photons at the converter per monitor count ×atoms/cm² for 1 in. Pb×σ (pair).

The value of σ (pair) was determined to be 32.7×10^{-24} cm² by averaging the experimental pair production cross section in Pb over the theoretical pion-decay gamma-ray spectrum for the forward direction. The spectrum was calculated assuming isotropic emission of the pions in the barycentric system. The pair production cross sections were read from a plot of cross section *vs* log energy obtained by extending a straight line drawn through the values of the cross section determined at 80 Mev by Lawson,[7] at 30 Mev by Lundby and Marshall,[8] and at 17.5 Mev by Walker.[9]

It follows that:

Total photons at the converter per monitor count $=0.38/[8.5\times10^{22}\times32.7\times10^{-24}]=0.13$.

The radiation subtended a solid angle of 5.60×10^{-7} steradian. The total number of photons per monitor count and per steradian is

$$\text{photons}/M\times\text{sterad}=0.137/5.60\times10^{-7}=2.45\times10^5.$$

From the activity of the foils (see Table II) one obtains the total number of protons per monitor count. These numbers are practically the same whether or not there is liquid hydrogen in the target, or in other words, the monitor is unaffected by the filling of the target. Therefore, we take an average value of 2.95×10^9 protons per monitor count.

The differential gamma-production cross section is obtained from the relation:

$$\frac{\text{protons}}{M}\times\frac{\text{atoms H}}{\text{cm}^2}\times\frac{d\sigma_\gamma}{d\Omega}=\frac{\text{photons}}{M\times\text{sterad}}.$$

For the forward direction one finds:

$$\frac{d\sigma_\gamma}{d\Omega}=\frac{2.45\times10^5}{(2.95\times10^9)(4.25\times10^{23})}=0.20\times10^{-27}\ \text{cm}^2/\text{sterad}.$$

An upward correction of 1 percent should be made in this value to allow for pion production in the cold gaseous hydrogen present in the nominally empty target chamber. This correction may be neglected.

The total cross section for pion production is obviously equal to one-half the total cross section for gamma-production. The latter quantity may be estimated as the product:

$$\left\{\left(\frac{d\sigma_\gamma}{d\Omega}\right)_{0°}\right\}_{\text{experimental}}\times\left\{\sigma_\gamma\Big/\left(\frac{d\sigma_\gamma}{d\Omega}\right)_{0°}\right\}_{\text{calculated}}$$

The latter quantity, the ratio of total to differential cross section in the lab system has been calculated for 450-Mev proton-proton collisions for two different distributions in the barycentric system of the two protons. For isotropic emission of the pions we find 4.7 for this ratio; for $\cos^2\theta$ emission of the pions we find 4.2. We conclude that for the purpose of calculating the total cross section it apparently makes little difference with what symmetry the pions are emitted.

Using a quantity midway between these numbers we obtain:

$$\sigma_{\pi^0}=\tfrac{1}{2}\sigma_\gamma=4.5\times\tfrac{1}{2}\times0.20\times10^{-27}=0.45\times10^{-27}\ \text{cm}^2.$$

There still remains to be discussed the question of whether or not the neutral pions, whose production cross section has been measured, were produced directly

TABLE II. Foil activity and number of protons per monitor count.

Monitor foils: The data given here relate to the area of the foils which measured the proton beam passing through the area of the target seen by the detector.

Foil No.	Weight mg	Number of Al atoms	Initial[a] number of Na^{24} atoms	Total number[b] of protons	Total number monitor counts (M)	Number protons per monitor count
1	8.95	2.00×10^{20}	5.35×10^8	2.47×10^{14}	82,416	2.99×10^9
	(with hydrogen in the target)					
2	8.45	1.89×10^{20}	4.84×10^8	2.37×10^{14}	81,410	2.92×10^9
	(no hydrogen in the target)					

[a] Corrected for solid angle of beta-counter, gamma-content, and absorption in foil, air, and window.
[b] No. of protons = No. of Na^{24} atoms/No. of Al atoms $\times\sigma(Al^{27}p,3pn)Na^{24}$. $\sigma(Al^{27}p,3pn\ Na^{24})$ at 450 Mev $=10.8\times10^{-27}$ cm² [Luis Marquez, Phys. Rev. **86**, 405 (1952)].

[7] J. L. Lawson, Phys. Rev. **75**, 433 (1949).
[8] A. Lundby and L. Marshall, Phys. Rev. (to be published).
[9] R. L. Walker, Phys. Rev. **76**, 1440 (1949).

in proton-proton collisions. Instead, it might be argued that the neutral pions are formed by charge exchange when negative pions collide with hydrogen atoms in the target. This background we call the "Panofsky effect."[10] Such negative pions are not produced in the hydrogen itself because of charge conservation, but they could conceivably come from the copper walls of the target container or from a more distant source such as the scattering target, the dee, or the poles of the cyclotron. In the case of negative pion production from a local source, the intensity of the gamma-radiation should be increased by increasing the amount of material in the target walls. To test this hypothesis, a sheet of copper $\frac{1}{32}$-inch thick was laid on top of the target itself. In the middle of a run, in which the usual measurements were made with and without liquid hydrogen, this sheet was pulled off the target from outside the cyclotron, and the measurement was continued with no other change in conditions. On the one hand the copper sheet occupied a position of considerably greater proton intensity; on the other hand the copper sheet was thicker than the copper wall of the target. If the observed photons resulted from Panofsky effect, their intensity should have been increased several times over in presence of the copper sheet. No difference in gamma-intensity was observed. This was taken as evidence against Panofsky effect from a local source of negative pions.

For the second hypothesis, namely that the observed effect could be due to negative pions arriving at the target from some relatively distant point, the intensity of gamma-rays should be almost constant over the width of the target. A test was made by observing the variation of gamma-intensity at the detector while the first element of the collimator was moved across the face of the target.

The observed intensities (hydrogen minus no hydrogen, with and without lead) are plotted against collimator position in Fig. 6. Over half the width of the target, the data suffered from a large background caused by our having forgotten to turn on the sweep magnet field. Nevertheless when the observed radial distribution (Fig. 5) of proton current in the target is combined with the width of the collimator slit, which was made especially narrow for this measurement, a dependence of intensity on radial position of the slit is computed which agrees quite well with the observed variation. This is taken to indicate that the neutral pions originate in proton-proton collisions.

B. Beryllium Cross Section

Since the Čerenkov telescope method accepts the entire gamma-spectrum simultaneously, and since it can make use of a rather thick converter, it is applicable to low intensity phenomena. In fact, the method was worked out after it became apparent that the gamma-rays from hydrogen were too few to be readily measurable with the pair spectrometer. The cross section of beryllium had already been measured with the pair spectrometer, but it was decided to measure it also with the Čerenkov telescope as a check on the validity of the method as applied to hydrogen.

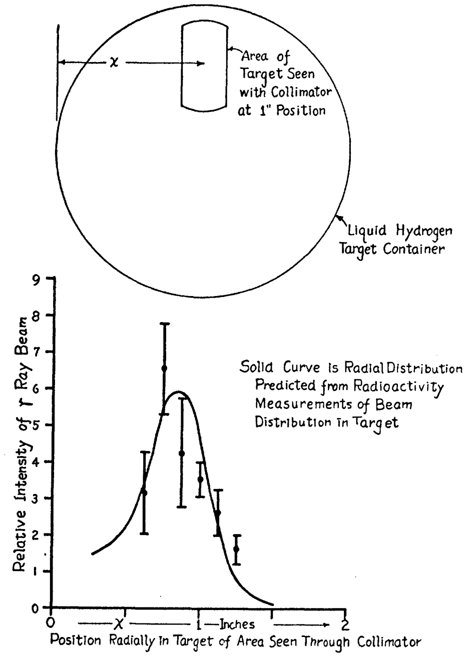

FIG. 6. This survey of the target showed that the gamma-rays observed in this experiment came from within the target.

The target was a cylinder of beryllium 1.73 cm long, 2.22 cm in diameter, and weighing 12.7 g. It was put in the median plane of the cyclotron with its inner face at 76 inches radius. It was mounted on a probe with its axis along a radius of the cyclotron. A copper rod $\frac{3}{16}$ inch in diameter was silver soldered to the beryllium both to support the target in the beam and to provide a measured heat leak to an Aquadag-coated sheet copper thermal radiator. The two junctions of a thermocouple were connected to the target and to the radiator. Hence, the temperature difference of the junctions measured the power delivered to the target by the proton beam. From this power it is possible to compute the total distance per second traveled by protons in the target.

The collimation in this case was similar to that for the hydrogen except that the inner lead collimator block was removed. The detecting apparatus and its geometry were the same. The counting monitor was arranged as before so that it counted radiation coming through a separate channel with the difference that in this case the radiation came from the beryllium target.

The counting data obtained at a power level of 2.22 watts in the target is given in Table III in the form of detector counts for 10^4 monitor counts. The background rate obtained with the beryllium target removed from the beam was negligible. The value of the net counting rate (Pb — no Pb) for 10^4 monitor counts and per inch

[10] Panofsky, Aamodt, and Hadley, Phys. Rev. 81, 565 (1951).

TABLE III. Counting data obtained at a power level of 2.22 watts.

Thickness of Pb converter, inches	Coincidences per 10^4 monitor counts
0	1315±25
0.0055	3325±41
0.041	12,575±65
0.125	21,410±84

of lead, extrapolated to zero thickness of converter, was 3.8×10^5. The number of lead atoms per cm³ is 3.35×10^{22}. One finds from these numbers and from the lead pair production used above:

$$\frac{\text{Total photons at the converter}}{10^4M}=\frac{3.8\times10^5/10^4M}{32.7\times10^{-24}\times2.54\times3.35\times10^{22}}=1.37\times10^5.$$

The converter subtended a solid angle of 5.60×10^{-7} sterad at the target. Therefore

$$\left\{\frac{\text{Total photons}/10^4M}{\text{sterad}}\right\}=\frac{1.37\times10^5}{5.60\times10^{-7}}=2.45\times10^{11}.$$

The time for 10^4 monitor counts was 2.9 minutes at this intensity, 2.22 watts. It is estimated that 18 percent of the heat in the target is produced from stars; then 82 percent is ionization energy lost in proton traversals of the target. We obtain for the total ionization energy dissipated in the target in the time taken for 10^4 monitor counts

$$10^4M=\frac{2.22\times10^7\times2.9}{1.6\times10^{-6}}\times0.82\times60\text{ Mev}$$
$$=1.98\times10^{15}\text{ Mev}.$$

Assuming the energy loss of a 430-Mev proton in beryllium to be 4.44 Mev per cm, one finds for the proton path length in the target corresponding to 10^4 monitor counts

$$1.98\times10^{15}/4.44=4.57\times10^{14}\text{ proton cm}.$$

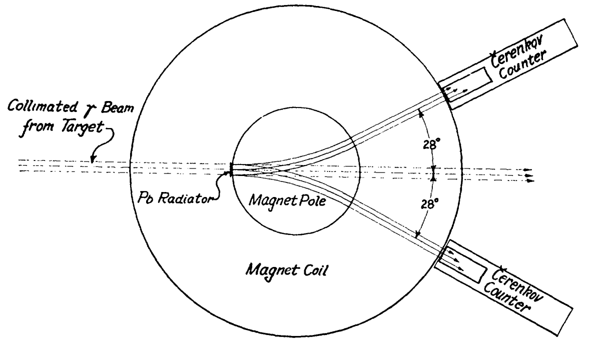

FIG. 7. The pair spectrometer.

Now the differential cross section follows directly from the relationship

$$\left(\frac{d\sigma_\gamma}{d\Omega}\right)_{\text{Be}}=\frac{\text{photons/sterad}}{\text{proton cm}}\times\frac{1}{(\text{atom Be/cm}^3)}$$
$$=\frac{2.45\times10^{11}}{4.57\times10^{14}}\times\frac{1}{1.23\times10^{23}}$$
$$=4.35\times10^{-27}\text{ cm}^2/\text{sterad}.$$

To convert the differential gamma-ray cross section to the total pion production cross section one needs to know the factor $\sigma_\gamma/(d\sigma_\gamma/d\Omega)_{0°}$ for a Teller-McMillan nucleus. For the case of isotropic emission of the pion from the barycentric system of the colliding nucleons

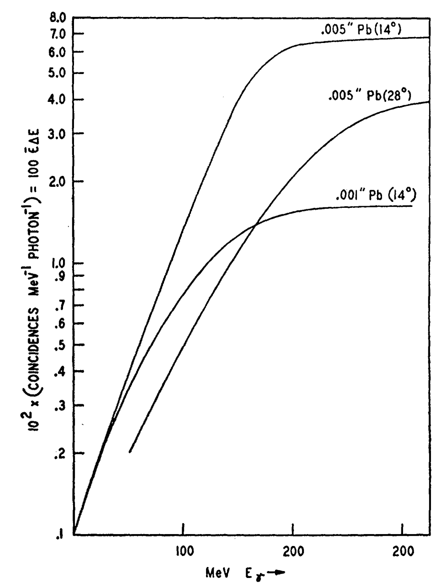

FIG. 8. The calculated variation of counting efficiency with gamma-ray energy.

we find a factor 4.0. We have not made the similar calculation for the case of the $\cos^2\theta$ pion emission from a Fermi gas nucleus. Instead, the argument may be proposed that as in the case of hydrogen this factor will not be changed much by the assumption of $\cos^2\theta$ pion emission, because the effect of a momentum distribution of the nucleons in the nucleus is to smooth out any directional features.

With the assumption then that the total cross section is 4 times the differential cross section in the forward direction for the case of beryllium regardless of the symmetry with which the pions are emitted, we find

$$\sigma_{\pi^0}\text{ for Be}=\tfrac{1}{2}\times4\times4.35\times10^{-27}\text{ cm}^2=8.7\times10^{-27}\text{ cm}^2.$$

III. METHOD OF THE PAIR SPECTROMETER

The pair spectrometer (Fig. 7), a round pole magnet of 11 inches diameter with a 2-in. gap, was placed in the experimental area about 40 feet from the beryllium target already discussed in the previous section. The gamma-ray beam in the forward direction from the target came into the experimental area through a hole in the cyclotron shield. Electron contamination was removed with two magnetron magnets.

The beam was collimated with two diaphragms of lead bricks, a horizontal collimation $\frac{1}{2}$-inch high at the port of the cyclotron, and the other a $\frac{5}{8}$-inch diameter round collimation in the experimental area. A photograph of the beam made about 10 feet past the pair spectrometer during a measurement showed the beam cross section to be approximately a circle one inch in diameter.

The collimated beam went between the poles and along a diameter of the pair spectrometer magnet after passing through a lead converter on the circumference of the magnet. The detecting counters were placed symmetrically at an angle θ with respect to the beam so that only electron-positron pairs whose components had approximately equal energy came to them.

Because of the fact that the beam in the forward direction contained a great many high energy neutrons as well as the gamma-rays to be measured, we again used water Čerenkov counters, especially suitable for their sensitivity to electrons and not to protons from this beam. The two counters were connected through a 6BN6 coincidence circuit.

A measure of the photon spectrum was obtained by observing the coincidence rate at various magnet fields while the geometry of counters and spectrometer was kept fixed. This was done for the gamma-spectrum from beryllium using two thicknesses of lead converter 5 mil and 1.75 mil, at two values of θ, 14° and 28°.

A round pole spectrometer has the unpleasant feature that its efficiency is a rapidly varying function of magnetic field and is a different function for each thickness of converter and for each counter position. The finite size of the converter and the Coulomb scattering in it must be taken into account. A second point is that because of the finite size of the counters, the efficiency depends on the shape of the gamma-spectrum itself. Consequently, the shape of the pion-decay gamma-spectrum[11] for beylium was calculated for the forward directed and was used to weight the probability of detection of a gamma-ray of given energy.

The calculated efficiency curves are shown in Fig. 8. The ordinate $\bar{\epsilon}\Delta E$ is essentially an average efficiency of detection times an effective energy width over which gamma-rays are detected for the given geometry and magnetic field. The conversion from observed counting rate to gamma-ray spectrum is accomplished by the relation

$$\frac{1}{\bar{\epsilon}\Delta E}\times\frac{\text{coincidences}}{\text{sterad collision}}=\frac{\text{photons}}{\text{Mev sterad collision}}.$$

The solid angle subtended at the target by the collimated beam at the converter was 3.7×10^{-6} steradian. The number of coincidence counts for 10^4 monitor counts was recorded as a function of magnetic field during the measurement. The beam intensity was calculated from the recorded voltage of the target thermocouple for each period of 10^4 monitor counts. The average value for the energy equivalent of 10^4 monitor counts was 11.9 watt minutes.

In the manner previously described, one corrects this number to remove the energy contribution of stars

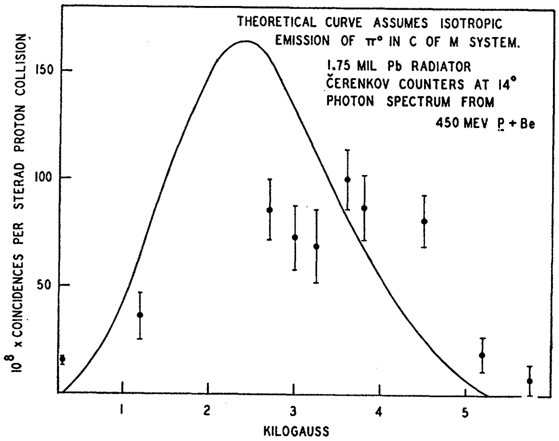

FIG. 9. The calculated and observed pair spectra from a 1.75-mil lead converter and with counters at 14°.

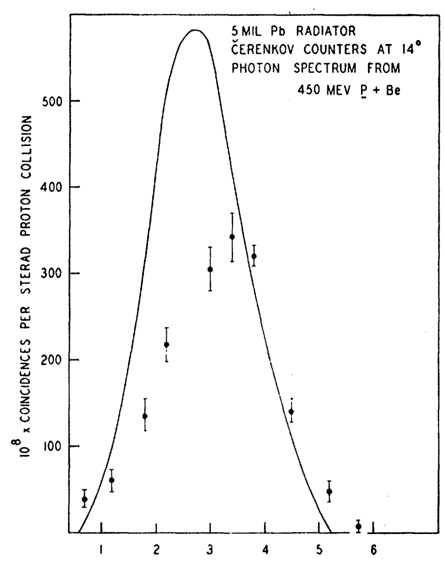

FIG. 10. The calculated and observed pair spectra from a 5-mil lead converter and with counters at 14°.

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[11] The differential pion production cross section used here was calculated by Orear, Rosenfeld, Schluter, and Teng assuming isotropic emission of the pions in the barycentric system of the colliding nucleons for a Fermi gas momentum distribution.

and converts it to 0.83×10^{15} proton cm path length traveled in the target for every 10^4 monitor counts.

Figures 9, 10, and 11 show the data observed for two angles and two thicknesses of radiator. The ordinate is coincidences per steradian proton collision which means that the observed coincidences per 10^4 monitor count have been corrected to unit proton path length and that also a total collision cross section of 220×10^{-24} cm^2 for protons on beryllium has been introduced for convenience. This factor cancels out in the final result.

The solid curves in these figures are obtained by multiplying the calculated spectrum described above by $\bar{\epsilon}\Delta E$ for the particular geometry and converter thickness. Only the shape of the solid curves has much significance. The amplitude is arbitrary.

In Fig. 12 the data have been converted to photons/steradian per collision by dividing it by $\bar{\epsilon}\Delta E$. One sees some tendency in all these figures for the experimental spectrum to contain somewhat more of the high energy component than the calculated spectrum. In the case of p–p collisions, the shape of the spectrum was calculated both for isotropic and for $\cos^2\theta$ production of the pions, and the latter curve was bulged more at high energies than the former. Although the $\cos^2\theta$ type of calculation has not been made for beryllium, one expected that similarly here there will be more of the high energy component. A possible explanation therefore of the excess of photons at high energy may be some $\cos^2\theta$ production of pions. An alternative factor which will have a similar effect is the p–p force acting on the final protons formed in the reaction

$$p+p\rightarrow\pi^0+p+p.$$

This effect has not been considered by us. The high points at low energies have large errors as a result of the very low spectrometer efficiency in this region and may or may not be significant.

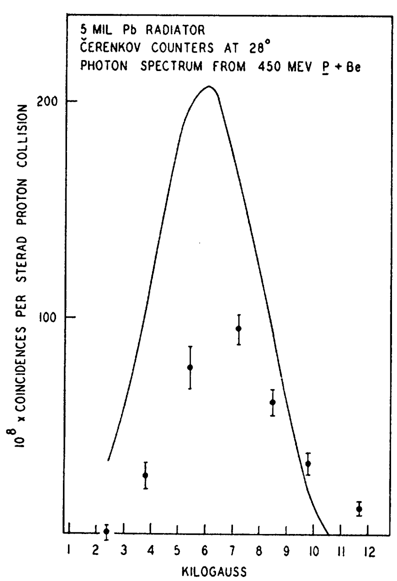

FIG. 11. The calculated and observed pair spectra from a 5-mil lead converter and with counters at 28°.

The area within the dashed boundary in Fig. 12 which has been taken as a measure of the differential neutral pion production cross section is 1.38×10^{-2}. Therefore the differential cross section is 0.0138×220 mb$=3.0$ mb, except, however, that this figure must be increased by 12 percent to correct for the fact that the Born approximation pair production cross sections were used instead of the experimental cross sections for lead in the efficiency calculations. One obtains a value which within the probable errors of this measurement agrees with the Čerenkov telescope measurement previously described.

$$\left(\frac{d\sigma_\gamma}{d\Omega}\right)_{0°}\text{ in Be}=3.36\times10^{-27}\text{ cm}^2.$$

Using now the previously discussed factor of 4.0 to obtain the total gamma cross section we find

$$\sigma_{\pi^0}=\tfrac{1}{2}\times4.0\times3.36\times10^{-27}\text{ cm}^2=6.72\times10^{-27}\text{ cm}^2.$$

IV. ERRORS

Errors that are quite difficult to evaluate enter into a large fraction of the measurements described in this paper. One major possible source of error is for instance the measurement of the proton beam intensity. In the case of the hydrogen cross section measurement we use the cross section for production of Na^{24}. Marquez[12] estimates the error in this cross section to be about 10 percent. Some of the sources of error in Marquez' measurement are eliminated because we used the same counter that he used. In the case of beryllium we use the rather well known rate of energy loss by ionization and apply an 18 percent correction for energy introduced into the target by nuclear reactions caused by the protons. This correction may be wrong by a considerable amount. The thermocouple measurement of the power in the beryllium target depends on a knowledge of the heat conductance of the target stem. We have determined this by measuring the thermal relaxation time of the target assuming a value for the heat capacity of beryllium. Some error is certainly introduced here.

Then there are minor errors in collimator geometry which may contribute to the general inaccuracy of the final result. We have assumed the Čerenkov counter to be 100 percent efficient to high energy electrons passing through the water radiator. This assumption is born out by the existence of a reasonably good counting rate plateau when the photomultiplier voltage is varied. On the other hand, we have assumed that the crystal scintillator placed in front of the radiator and other necessary materials do not absorb any appreciable fraction of the electron pairs found in the converter. In order to calculate the efficiency function of the pair spectrometer,

[12] See Table II, reference b.

it was necessary first to assume a shape for the distribution of gamma-rays with energy. Some error is easily possible here.

Because of all these sources of error, we have assigned uncertainties to the final results which are considerably larger than the statistical errors.

V. SUMMARY OF RESULTS

We have measured the cross section for the production of neutral pions by the bombardment of hydrogen with protons of about 430-Mev kinetic energy to be

$$\sigma_{\pi^0}(p,p)=(0.45\pm0.15)\times10^{-27}\text{ cm}^2.$$

We have measured the cross section for the production of neutral pions by the bombardment of beryllium with 430-Mev protons to be

$$\sigma_{\pi^0}(p,\text{Be})=(8.7\pm2)\times10^{-27}\text{ cm}^2$$

by the use of the Čerenkov counter telescope method. With the pair spectrometer the cross section of beryllium was measured to be $(6.7\pm3)\times10^{-27}$ cm^2. These results depend on the measurement of the differential cross section for gamma-ray production in the forward direction alone. The total cross sections were arrived at by multiplying the differential cross sections by computed angular distribution factors as described elsewhere in this paper.

Let us assume that the yield of neutral pions is the same whether the incoming proton hits a free or a bound proton. Then the cross section of hydrogen is σ_p and the cross section of beryllium is $4\sigma_p+5\sigma_N$. At 430 Mev we find

$$(4\sigma_p+5\sigma_N)/\sigma_p=(8.7\pm2)/(0.45\pm0.15).$$

$$\sigma_N/\sigma_p=3\text{ with limits}\begin{cases}6.4\\1\end{cases}.$$

Similar treatment of the Berkeley measurements at 340 Mev gives the relation

$$\sigma_N/\sigma_p=13\text{ with limits}\begin{cases}102\\6.8\end{cases}.$$

A decrease in this ratio is expected to occur with increasing energy. In the case of the p–p collisions, as long as the energy available to the final protons is small so that they are created in an S state, conservation of total angular momentum and parity require that the meson can be emitted only in an S state. When higher angular momentum states are available to the two final protons, the meson can be created in a P state. The P state yield is expected to be greater than the S state yield in analogy with the creation of charged mesons.[3]

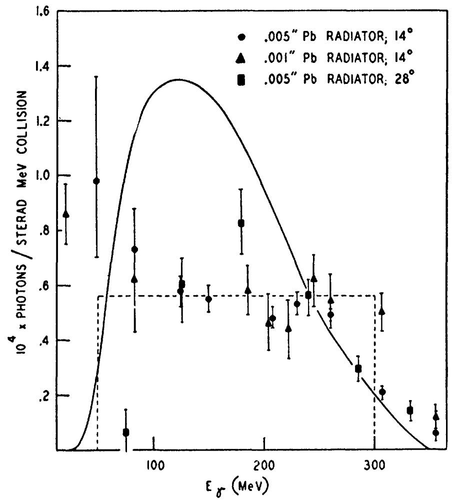

FIG. 12. The calculated and observed photon spectrum.

In the case of the beryllium nucleus, both p–p and p–n collisions occur. For p–n collisions the pion can be created in a P state even though both nucleons involved are constrained to an S state. Therefore at first the yield from p–p collisions, i.e., from hydrogen, should increase much faster with energy than the yield from p–n collisions, i.e., beryllium.

Assume now that at 430 Mev the cross section for pion production from a p–p collision in a nucleus is 0.45 mb, and that for a p–n collision is 3×0.45 mb$=1.3$ mb. We estimate the cross section for carbon at this energy as

$$6\times1.3+6\times0.45=10.5\text{ mb},$$

which agrees with an extrapolation of the curve for σ_{π^0}(carbon) from 180 to 340 Mev given by Crandall.[1]

We are indebted to Professor Earl Long for the liquid hydrogen.