Energy Loss of Cosmic-Ray Mu-Mesons in Sodium Iodide Crystals*

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(Received June 20, 1952)

A scintillation counter telescope, employing three NaI(Tl) crystals in coincidence, has been used to study the energy losses experienced by mu-mesons in traversing one of the crystals. The magnitude of the light flash in NaI(Tl) has been assumed to be proportional to the energy lost in the crystal by the meson. The observed pulse distribution is compared with one calculated on the basis of the Bethe-Bloch theory without the density correction and also with the Halpern-Hall theory which includes the effect of the dielectric shielding. Each calculated curve employs a fold of the Landau straggling theory with the cosmic-ray mu-meson spectrum. Comparison of the data with the two theories definitely favors the Halpern-Hall theory both in respect to absolute value of the most probable energy loss as well as the shape of the distribution.

INTRODUCTION

THE energy loss suffered by charged particles in passing through absorbing layers has been the subject of many calculations since the original classical treatment of Bohr.[1] Quantum-mechanical calculations were first made by Bethe[2] for hydrogen, and by extension, to other materials. Bloch[3] has also calculated the mean energy loss using the methods of quantum mechanics and showed how Bohr's classical result could be reconciled with Bethe's formula. Bloch also extended the energy loss formulas to materials of any atomic number by employing the Thomas-Fermi atomic model in a particularly successful and practically useful way. Further significant ideas and calculations were contributed by Williams[4] who showed, as Bohr previously had done for the classical case, how modifications of the energy loss formula had to be made in order to include the effects of relativity.

The particular expression for the mean energy loss now commonly employed is taken from the work of Bethe and Bloch and is termed the Bethe-Bloch formula. The practical way of using the formula is described in the well-known paper of Rossi and Greisen.[5] The validity of the formula has been established for gases in and near the minimum of the energy loss curve although the accuracy with which the formula has been checked in the region above the minimum is not high. Further, there is some disagreement as to whether the Bethe-Bloch result applies to μ-meson losses.[6]

Additional contributions to the theory of energy loss have been made in connection with the passage of charged particles through dense media, i.e., liquid or solids. In such cases the dielectric behavior of the material introduces a shielding effect in the region of relativistic energies of the primary particle, and produces a reduction of energy loss. The original suggestion of a dielectric shielding effect is due to Swann.[7] The problem was first treated by Fermi[8] who proposed a simple theory embodying a dispersion model of the atom based on a single resonant frequency.

The Fermi model is perhaps a bit too simple since, in actuality, electrons in all shells corresponding to many "frequencies" are involved in the shielding. The calculation has been improved by Halpern and Hall[9] who take the multifrequency model as a basis of the theory. The Fermi model and the Halpern-Hall theory give identical results in the region of very high particle energies. The two theories give similar results in all regions provided a suitable "effective" dielectric constant can be used in the Fermi formulation. The weakness of the Fermi theory is that one must essentially guess a value of the dielectric constant. The Halpern-Hall theory, in effect, calculates this value for a reasonable model of the atom. The calculated values differ in most cases only a little from the value unity.

Further calculations on the dielectric effects have been made by Wick[10] and by A. Bohr[11] and by Schoenberg.[12] A. Bohr discusses the connection between Čerenkov radiation, energy loss, and the dielectric shielding effects. Recently, the experimental reality of the ideas and calculations on dielectric shielding have been demonstrated through the work of Bowen and Roser[13] on a light material, anthracene. Earlier experiments on AgCl indicated similar results.[14]

The theoretical treatments discussed above are con-

* This work was partially supported by the joint program of the ONR and AEC.

[1] N. Bohr, Phil. Mag. **25**, 10 (1913); **30**, 581 (1915).

[2] H. Bethe. Ann. Physik **5**, 325 (1930); Z. Physik **76**, 293 (1932); Handbuch der Physik **24**, Part 1, 491 (1933).

[3] F. Bloch, Ann. Physik **16**, 285 (1933); Z. Physik **81**, 363 (1933).

[4] E. J. Williams, Proc. Roy. Soc. (London) **135**, 108 (1932); **139**, 163 (1933); **169**, 531 (1931); Phys. Rev. **58**, 292 (1940).

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[7] W. F. G. Swann, J. Franklin Inst. **226**, 598 (1938).

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[9] O. Halpern and H. Hall, Phys. Rev. **57**, 459 (1940); **73**, 477 (1948).

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[11] A. Bohr, Kgl. Danske Videnskab. Selskab Mat.-fys. Medd. **24**, No. 19 (1948).

[12] M. Schoenberg, Nuovo cimento **8**, No. 3 (1951); Nuovo cimento **9**, No. 4 (1952).

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[14] W. L. Whittemore and J. C. Street, Phys. Rev. **76**, 1786 (1949).

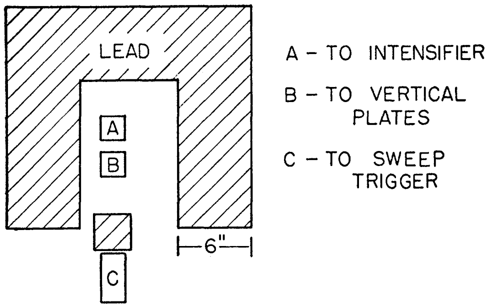

FIG. 1. Schematic drawing of counter telescope and shielding arrangement.

cerned with mean energy losses. As Bohr[1] early pointed out in his second paper, we usually measure the most probable energy loss rather than the mean; the two values are significantly different. If, for example, the energy loss of individual particles can be studied, one may expect to find a most probable energy loss and a straggling of the individual losses around this value. The most probable energy loss will be the same as the mean only in those cases of a symmetrical straggling curve, e.g., for low energy alpha-particles. On the other hand, for large energy transfers, when the straggling curve is unsymmetrical, the most probable value will differ from the mean value. The Rutherford-Bohr relation giving the energy transfer in collision of a heavy particle with an atomic electron implies that the straggling curve will not be symmetrical. This relation states that the probability for a given energy to be transferred to an electron depends inversely on the square of the energy imparted to the electron. This type of probability function produces a long and unsymmetrical tail on the high energy side of the straggling curve and is due to projected electrons of high energy which are conventionally called δ-rays.

This situation was known to Bohr and Williams and calculations were made by the latter on the form of the straggling curve.[15] A discussion of straggling including the work of Williams is given by N. Bohr[15] in his comprehensive treatment of atomic energy loss problems.

It remained for Landau[16] to calculate the straggling function in closed form and to bring to it the renewed attention it has recently received. Landau has given both an expression for the most probable energy loss and the shape of a universal straggling curve when the energy loss is small compared with the particle's energy. The universal Landau curve may be applied to particular materials, thicknesses of sample, particular energy, etc. Symon[17] has extended the theory to intermediate cases where the energy loss may be a large fraction of the total energy.

Recently, Jauch[18] has calculated the effect of a radiative correction to the energy loss formula. The experimental information now available is unable to distinguish small virtual or real radiative effects.

It is generally appreciated that cosmic-ray μ-mesons at sea level offer an attractive source of particles for testing the various proposed energy loss theories. The facts that μ-mesons are poor radiators and that their nuclear interactions are small are important points in their favor in such applications. In present day high energy physics many situations are encountered in which energetic charged particles are not stopped by conventional thicknesses of absorbers. We are forced, at best, to sample a particle's behavior as it passes through an absorbing slab of matter. Hence, we feel that it is of some importance to study the exact laws of interaction, energy loss, and straggling. Bowen and Roser have made a beginning in this direction by investigating anthracene absorbers (carbon) at specific meson energies. We have chosen to study NaI(Tl) because of its availability, high Z, high density and good proportionality between energy loss and light pulse output.[19] This publication presents a summary of our work to date.

APPARATUS

A sketch of the experimental layout is shown in Fig. 1. A, B, and C represent three NaI(Tl) crystals arranged in the form of a μ-meson telescope. Each of the three crystals is viewed by an appropriate photomultiplier. A C7157 photomultiplier is used with crystal B; 5819 photomultipliers are used with the other crystals. The photosurface of the C7157 is large enough so that the whole area of contact of the crystal B is viewed directly through a close-fitting light pipe. Crystals A and B are approximately two-inch cubes. The length of crystal B along the telescope axis is 2.17 inches. Crystal C is 2 in.$\times$2 in.$\times$4 in. Above and on the sides of the three-crystal-telescope are layers of

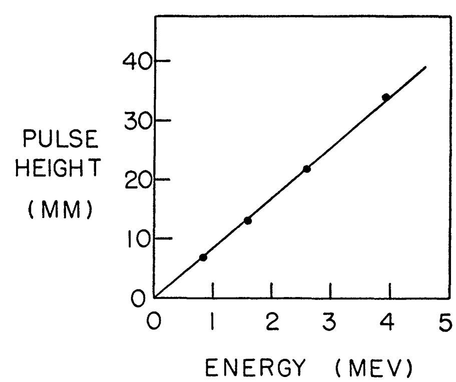

FIG. 2. The linearity curve used in this experiment.

[15] N. Bohr, Kgl. Danske Videnskab. Selskab Mat.-fys Medd. **18**, No. 8 (1948). (See pp. 50 ff.)
[16] L. Landau, J. Phys. (U.S.S.R.) **8**, 201 (1944).
[17] K. R. Symon, (unpublished) Harvard University thesis (1948).
[18] J. M. Jauch, Phys. Rev. **85**, 950 (1952).
[19] R. Hofstadter, Phys. Rev. **75**, 796 (1949); R. Hofstadter and J. A. McIntyre, Phys. Rev. **80**, 631 (1950).

lead six inches thick. The lead is used to separate, in a conventional way, the hard from the soft component. Between crystals B and C a three-inch layer of lead prevents mesons with energy less than 120 Mev from tripping the counter telescope. Connections are indicated schematically on the side of the figure and show that the pulses in the middle crystal become visible on the oscilloscope tube face when and only when a triple scintillation coincidence of all crystals occurs; the oscilloscope face is used as a coincidence device. A triple coincidence implies that each of the three crystals has been traversed, presumably by a μ-meson (see below). Biases are applied to the discriminator circuits of crystals A and C. Experiments with different biases above a minimum showed no changes in the pattern of pulses in crystal B.

The energy loss of the μ-meson in crystal B can be measured by the height of the light pulse.[19] Calibration of crystal B was achieved with the well-resolved pair and photolines of ThC″ (2.62 Mev) and C^{12*} (4.45 Mev) gamma-ray lines. The latter was obtained from a weak Po-Be source.

It will be assumed that the process of energy loss of mesons in NaI(Tl) is similar to that of electrons and therefore results in a linear relationship between energy loss and pulse height of the scintillation. Evidence obtained with electrons up to energies of $9mc^2$ (m = electron mass) completely supports such a linear relationship.[19] By using the stated pair and photolines a linearity curve could be constructed. Figure 2 shows such a linearity curve. For actual calibration, the sharp peak due to the ThC″ photoline was used.

The ordinate of Fig. 2, namely, the vertical deflection of the oscilloscope, was carefully examined for linearity with respect to magnitude of the input voltage pulse by using two independently calibrated artificial pulse sources. Up to deflections of 70 mm the deviation from linearity was less than 1 percent, which was about the experimental error in making measurements. To obtain 70 mm deflections on the Tektronix 511 AD oscilloscope, a single stage, push-pull supplementary amplifier was employed in addition to a Model 501 amplifier. The attenuator of the 501 amplifier was calibrated both with artificial pulses and with the photolines of known gamma-rays. Since the largest gamma-ray energy obtainable conveniently is 4.45 Mev, (the corresponding pair lines are at 3.43 and 3.94 Mev) and the range of cosmic-ray pulses centers about 27 Mev, an extrapolation of the photoline calibration had to be made. The actual factor used was 8.07. While we have made a careful attempt to avoid error in making this extrapolation to higher energies, the largest source of possible error in the experiment rests here. We believe that energies in the range 27 Mev can be measured to within 1.0 Mev or perhaps a bit less. The relative values of energy loss are not affected by an error in absolute calibration.

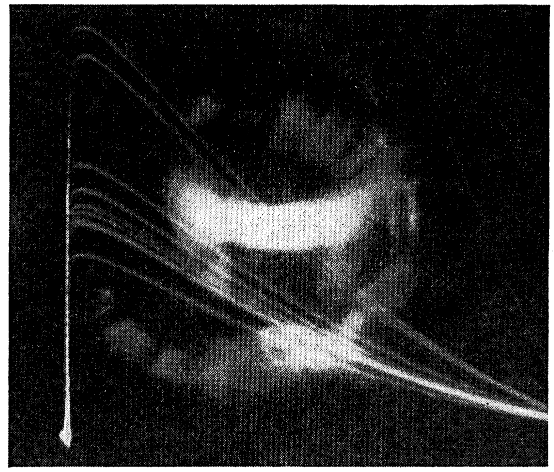

FIG. 3. A sample of exposed film showing representative traces. The tail of a very large pulse may be seen at the right. The irregular bright pattern in the center and the surrounding bright patch are due to glow from the filament of the oscilloscope tube.

In order to test whether different parts of the crystal cube B produced pulses of varying sizes, we have taken pictures of the pulse distribution produced by a collimated Co^{60} gamma-ray source. The gamma-beam was directed at different areas of the crystal, but in all cases the pulse distributions were found to be very similar. The sharp lines of the calibration photographs provide further confirmation of the uniformity of different parts of the crystal.

Single traces of oscilloscope pulses in the middle crystal were photographed by an f2.5 10-in. focal length lens on 3 in.×4 in. sheets of cut film. Each exposure lasted fifteen minutes and on the average eight traces appeared. The traces were measured directly against ruled millimeter paper and the pulse heights so obtained were grouped into seventy channels each one millimeter wide. Figure 3 shows a sample of an exposed film with several traces visible. Occasionally, more than one pulse of the same height was recorded during a fifteen-minute exposure. A little practice in observation soon showed that the greater opacity of a double trace could easily be distinguished from that of a single trace. Very rarely, a triple trace was observed. In addition to these types of multiple traces, two examples in about 3000 traces were observed in which a time coincidence occurred within the sweep duration of 100 microseconds.

In order to detect possible changes of pulse heights due to drift of photomultiplier voltage supply, amplifier gain, etc., approximately once every four hours a calibration point was taken with a ThC″ source. Over the eighty hours of running time (actual operating time ~200 hours) a gradual drift towards smaller pulses occurred. At the end of operating time the accumulated drift amounted to 3 percent. The data were corrected according to the two calibrations between which the data were sandwiched. Therefore, it is felt that no systematic error has occurred due to drift.

We have checked the observed triple coincidence counting rate against the known meson flux at sea level (0.82 meson/cm^2 per steradian in the vertical direction under six inches of lead[20]) in the following way. A full scale skeleton model was constructed of the three cross-sectional areas of the crystals in the telescope. With this model meson traces could be projected through all crystals. The effective solid angle accepted by the telescope for each 0.6 cm^2 area of crystal B could be measured by measuring areas in the model. Furthermore, the mean path length traversed by mesons in crystal B could also be found. This length exceeds the thickness of crystal on the average by only one part in two hundred, due to the slight obliquity of some meson paths. Using the solid angles measured on the model and the known meson flux quoted above, the expected meson counting rate was 8.3 counts per 15 minutes. Figure 4 shows a histograph of the observed counting rate. The mean observed value is 8.0 ± 0.5 count per 15 minutes. The agreement is good and indicates that the triple coincidences are indeed due to μ-mesons.

Preliminary experiments were carried out with a crystal of NaI(Tl), 0.53-in. thick, under conditions of better geometry but lower counting rate. In this case only a small extrapolation of pulse height is required to calibrate the crystal. The results of this earlier work are in agreement with those obtained with the 2.17-in. crystal B. However, the experiment was carried on for a shorter time and the statistical accuracy is considerably less than that obtained with the 2.17-in. crystal. For this reason the detailed results with the 0.53-in. crystal will not be reported.

It is to be noted that the μ-meson range is not measured in this experiment and, consequently, mesons of all energies exceeding a minimum contribute to the observed pulse spectrum. In this respect our experiment differs from that of Bowen and Roser[13] in which several distinct energy intervals were simultaneously studied. However, our results do not require correction for scattering in lead absorbers, as in the work of Bowen and Roser.

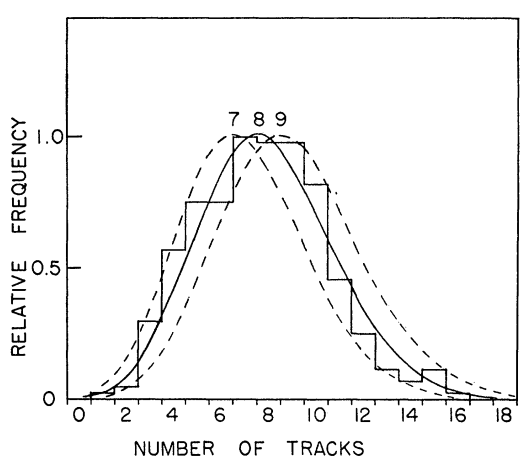

FIG. 4. Histograph of the observed counting rate and Poisson curves for mean counting rates of 7, 8, 9 counts per fifteen minutes.

[20] B. Rossi, Revs. Modern Phys. **20**, 537 (1948).

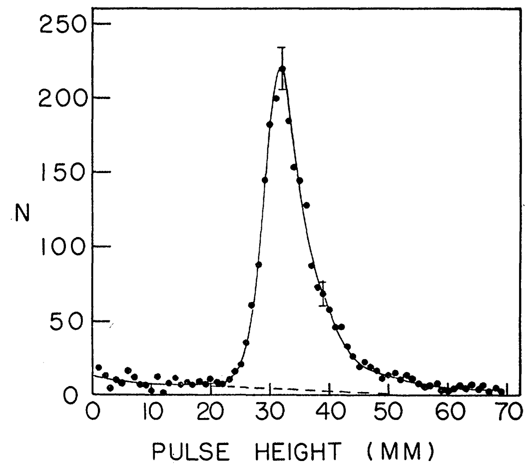

FIG. 5. The pulse distribution after 80 hours of running time. The ordinate represents the number of pulses in a one-millimeter channel.

RESULTS

The pulse distribution obtained after 80 hours of running time is shown in Fig. 5. The units of abscissas are actual millimeters of trace on the oscilloscope screen. When the data are translated into energy loss by means of the calibration curve, the experimental pulse distribution has the appearance of the solid line of Fig. 6. If it is assumed that the errors are purely statistical in nature, representative limits of error may be found. Such limits are shown at two points on the curve.

In Fig. 6 a small background effect has been subtracted from the data of Fig. 5. The dashed line in Fig. 5 represents the background effect, which is believed to be due to two causes: (1) showers which activate all crystals simultaneously are occasionally counted and (2) the middle crystal B was $\frac{3}{32}$ in. out of exact alignment with the other two crystals. The shower effect has been estimated by an independent experiment and is represented by the black circles in Fig. 7. These points were obtained in an 80-hour background run when the middle crystal was deliberately placed $2\frac{3}{32}$ in. out of line with the other crystals. It may be noted that there are virtually no pulses in the region of the peak of Figs. 5 or 6. This behavior is very satisfactory and is an additional check on the fact that single particles (μ-mesons) produce the observed triple coincidences.

In the actual experiment the alignment was not exact and, as stated above, the middle crystal was $\frac{3}{32}$ in. out of line. This can account for a small number of pulses due to μ-mesons passing through corners and edges of the middle crystal. These pulses will naturally also be small in magnitude.

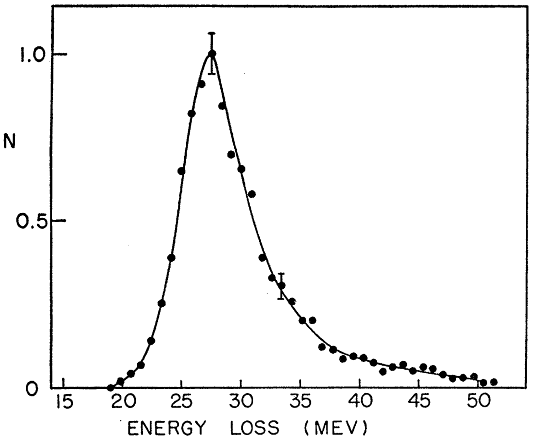

FIG. 6. The pulse distribution after calibration and correction for background pulses.

If we assume that the shower contribution to the pulse distribution when the crystals are in line is about the same or possibly a little larger than the black circles measured in the out-of-line experiment, then the contribution of showers and small pulses ($\frac{3}{32}$ in. out-of-line effect) will account completely for the background. Our best estimate for this combined effect is shown by the dotted line in Fig. 5. Although the choice is a bit arbitrary, the correction is quite small and has no important influence on the final results.

We have considered the possible effect of protons in producing distortion of the meson pulse distribution, but from the work of Miller *et al.*[21] the number of protons at sea level is too small to be observed in this experiment.

COMPARISON WITH THEORY

In order to compare these results with theory we have had to compute various pulse distributions based on (a) the Landau straggling theory, (b) the cosmic-ray μ-meson spectrum, (c) the assumed nature of the energy loss curve, *viz.*, Bethe-Bloch or Halpern-Hall, etc. In each case the assumed theory (c) was folded together with (a) and (b) in numerical integrations.

The Landau formula[22] for the most probable energy loss Δ_0 for monoenergetic heavy charged particles of velocity v, ($\beta=v/c$) and charge e, has the form,

$$\Delta_0=\xi[\ln(\xi/\epsilon')+0.37], \tag{1}$$

where

$$\xi=(2\pi Ne^4\rho x/mv^2)(\sum Z/\sum A), \tag{2}$$

and

$$\ln\epsilon'=\ln\frac{(1-v^2/c^2)I^2}{2mv^2}+\frac{v^2}{c^2}. \tag{3}$$

In these formulas m is the mass of the electron, N is Avogadro's number, x is the thickness of the crystal in cm, ρ its density, Z and A the atomic number and mass number of the crystal material. $I=13.5Z$ ev has been used by Landau and others and is a constant which measures the average ionization energy of the atom in the Bethe-Bloch theory.

The mean energy loss α is expressed as

$$\alpha=\xi\ln(E_0/\epsilon'), \tag{4}$$

where E_0 is the maximum energy that a meson can transfer to an electron in a single collision, i.e., it is the maximum energy of a δ-ray. E_0 is given by a formula of Bhabha. The Bhabha formula may be approximated for incident meson energies less than 20 Bev by the quite accurate but more suggestive formula,[23]

$$E_0=2mc^2\beta^2/(1-\beta^2). \tag{5}$$

If we wish to calculate the mean energy loss of a meson, we must use expression (4), substituting for E_0 the value given in Eq. (5). However, if we wish to calculate the mean energy deposited in the crystal slab, we must use Eq. (4) but must now substitute for E_0 the maximum transferable energy E_1 of those δ-rays that can be completely absorbed in the crystal. For a sample of the size of crystal B, E_1 might be 10 or 20 Mev allowing for radiation losses. On the other hand, E_0 is quite different from E_1 and might be $E_0=100$ Mev for a 1-Bev meson and $E_0=300$ Mev for a 10-Bev meson. Thus the mean energy deposited in the crystal will be limited by the value E_1.

It has been shown by Bohr, the result being fairly intuitive, that we may use the mean value formula in computing the most probable value of energy loss if we substitute for E_0 in Eq. (4) that upper bound of δ-ray energy E_2, which occurs once on the average as the meson traverses the crystal. Bohr has shown that $E_2=\xi$. By substituting $E_2=\xi$ for E_0 we shall obtain the Landau expression for Δ_0 except for the constant 0.37ξ

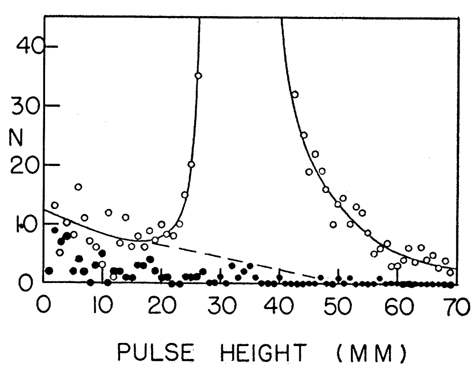

FIG. 7. The lower part of the observed pulse distribution is given by the open circles. The full circles represent the results of an 80-hour run with the middle crystal offset by $2\frac{3}{32}$ in. from the telescope axis.

[21] Miller, Henderson, Potter, and Todd, Phys. Rev. 84, 981 (1951).

[22] The effect of using Symon's calculations in place of Landau's is negligible in this experiment. For example, even in the lowest bracket (225–500 Mev/c) the average fractional energy lost is not more than 10 percent.

[23] See reference 5, formulas (1.5) and (1.5a).

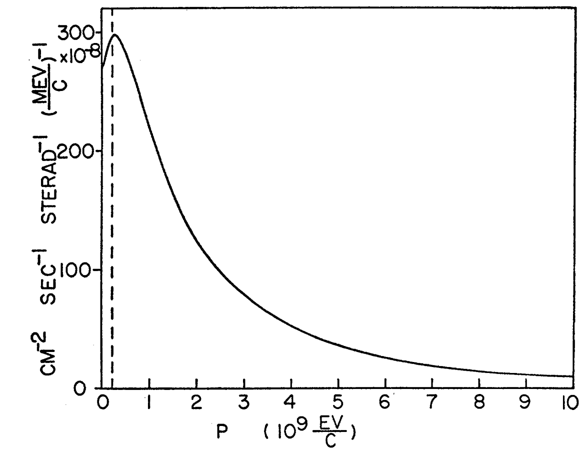

FIG. 8. The cosmic-ray spectrum when filtered by six inches of lead. The dashed line on the left cuts off all those mesons with momenta insufficient to pass through the three inch lead absorber and still give a pulse larger than the bias of crystal C (12 Mev).

which is generally small. Thus the exact Landau result may be made more readily understandable. In the case of crystal B, $\xi = 1.35$ Mev and falls within a range easily absorbed by the crystal.

Now the Landau theory may be used with and without a density correction. Without a density correction we shall refer to Δ_0 and resulting curves as the Bethe-Bloch values. With the density correction we shall refer to these as the Halpern-Hall values. The Halpern-Hall (and also Fermi) density correction essentially removes the factor $1/(1-\beta^2)$ from the logarithm of Eq. (1). The exact correction factor which we have used has been taken, however, from the published curves of Halpern-Hall[9] (their Fig. 1), and is applied to the most probable value Δ_0 directly as we would have applied it to the mean energy loss. After Δ_0 is calculated, the straggling curve is taken directly from Landau's universal curve. Individual straggling curves for selected momentum intervals are folded together

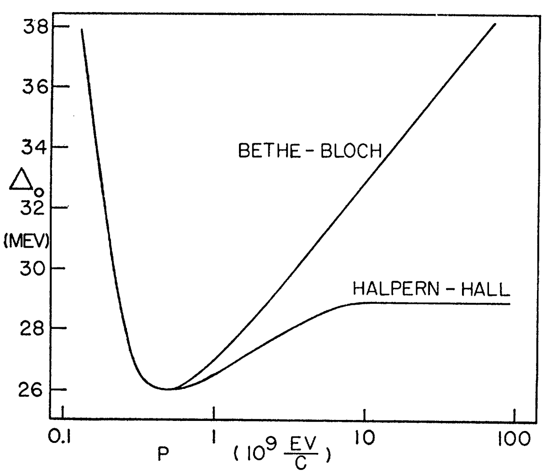

FIG. 9. A plot of the most probable energy loss Δ_0 against momentum for crystal B.

according to the weight of the corresponding momentum interval in the cosmic-ray spectrum. Nineteen momentum intervals were used.

In calculating the final theoretical curves, it is necessary to consider three further points: (A) the cosmic-ray spectrum is modified by the 6 in. of Pb above the counter telescope. The modified distribution is shown in Fig. 8. The final effect of the filtered distribution is not noticeably different from that of the original spectrum. (B) Some of the δ-rays may have high enough energies, and may be formed near the crystal surfaces so that they may emerge from the crystal before giving up all their energy. The experimental straggling curve may be distorted for this reason on the high energy side and may appear to be narrower than it really is. However, it may be seen from Figs. 5 and 6 that all the significant observations (those with better statistics) and comparisons with theory are made in the neighborhood of the most probable energy loss and just a few Mev larger. A rough evaluation of the solid angle available to those mesons traveling near the faces of the crystal which might produce δ-tracks of 5 Mev or larger in the proper directions to escape the crystal shows that this effect is of the order of about 5 percent at 5 Mev from the peak. This effect is therefore rather small in making the comparisons of theory with our present experiments. Further, the choice of ξ in the theory of Landau is consistent with these considerations. (C) Since NaI(Tl) has the two components Na and I,[24] the separate effects of each must be evaluated and combined. The combination has been effected by using a common[25] value of ξ for the compound material, NaI, in front of the logarithm of Eq. (1) and by adding

TABLE I. The momentum intervals of the modified mu-meson spectrum and the corresponding values of Δ_0.

Momentum interval (Mev/c)	Mean momentum (Mev/c)	Bethe-Bloch (Mev)	Halpern-Hall (Mev)	Percentage of modified mu-meson spectrum
225– 250	240	28.2	28.2	0.9
250– 500	375	26.1	26.05	9.5
500– 1000	750	26.5	26.05	16.3
1000– 1500	1250	27.8	26.9	12.2
1500– 2000	1750	28.4	27.3	9.2
2000– 3000	2500	29.3	27.6	13.0
3000– 4000	3500	30.2	28.15	8.5
4000– 5000	3400	30.8	28.35	5.6
5000– 6000	5500	31.4	28.58	4.0
6000– 7000	6500	31.9	28.7	2.8
7000– 8000	7500	32.3	28.8	2.1
8000–10,000	9000	32.7	28.8	3.1
10,000–12,500	11,250	33.3	28.9	2.8
12,500–16,000	14,000	34.0	28.9	2.5
16,000–20,000	18,000	34.2	28.9	1.8
20,000–30,000	25,000	35.9	28.9	2.5
30,000–40,000	35,000	36.4	28.9	1.3
40,000–60,000	50,000	37.3	28.9	1.3
60,000–80,000	70,000	38.2	28.9	0.6

[24] The percentage of thallium (<0.5 percent) is considered negligible.

[25] We wish to thank Mr. T. Bowen for suggesting this procedure.

the two separate contributions for each material with the appropriate values of I in each case. The latter differ according to the values of the mean ionization potentials of sodium and iodine. The values of the mean ionization potentials were taken as $11.8Z=130$ ev for sodium and $9.7Z=514$ ev for iodine according to the recent work of Bakker and Segrè and Mather and Segrè.[26] When the combination is made in this way, the mean ionization potential for NaI is obtained as:

$$(\bar{I})^2 = I_{\mathrm{Na}}^{2(0.172)} I_{\mathrm{I}}^{2(0.828)}, \tag{6}$$

where 0.172 and 0.828 are the respective electronic compositions of Na and I, respectively. The calculation of Halpern-Hall, of course, has not been made specifically for NaI and a similar combination of effects has been made for the density correction. In the case of this correction, the difference between assuming pure

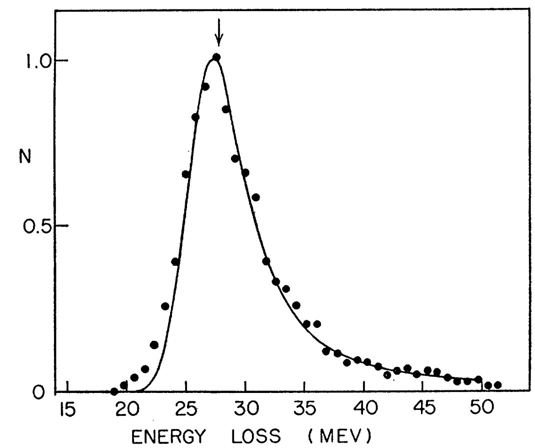

FIG. 10. The solid line gives the result of the calculated distribution based on the Halpern-Hall theory. The black circles are the experimental points. The theoretical curve actually shows a peak 0.4 Mev to the right (arrow position) but has been shifted to permit better comparison with experiment.

iodine and the actual compound is barely worth considering.

When the effects of (A), (B), and (C) above are taken into account, two theoretical curves are obtained according as one uses the Bethe-Bloch formula (uncorrected for the density effect) or that of Halpern-Hall. For further reference the experimental curves may also be compared with the Landau relation for a single energy in the neighborhood of the mean energy of the cosmic-ray spectrum. Table I and Fig. 9 show the various values of Δ_0 which have been calculated in the appropriate cases. The effect of making the various choices for comparison is shown in Figs. 10, 11, and 12. The energy loss is observed to show a saturation in Fig. 9 because of the density effect and because of the cut-off δ-ray energy in the neighborhood of ξ. When the contributions of all energy intervals in the cosmic-

[26] C. J. Bakker and E. Segrè, Phys. Rev. 81, 489 (1951); R. Mather and E. Segrè (to be published).

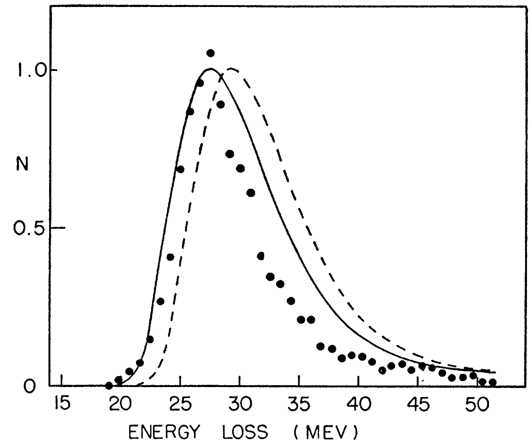

FIG. 11. The dashed curve gives the result of the calculated distribution based on the Bethe-Bloch theory without inclusion of a density correction. The solid line gives the same curve shifted by two Mev to permit better comparison with experiment.

ray spectrum are added together, the Halpern-Hall theory gives the solid curve of Fig. 10 and the Bethe-Bloch theory gives the dashed curve of Fig. 11. A single Landau curve for the meson energy 2.0 Bev (Halpern-Hall curve) is given by the solid line in Fig. 12.

Because of the small experimental uncertainty in the peak values (originating in the uncertainty of the extrapolation of the calibration data), the Bethe-Bloch and Halpern-Hall theoretical curves have been shifted to fit the experimental curve more closely. The shift required is 2 Mev for the Bethe-Bloch curve and 0.4 Mev for the Halpern-Hall theory. It is believed that a 2.0-Mev shift is outside our experimental error but a 0.4-Mev shift would be well inside our limits of precision. It is therefore observed that the fit with the Halpern-Hall theory is much better than with Bethe-Bloch both as regards the absolute value of the most probable energy loss as well as the shape of the straggling curve. In the latter case the Bethe-Bloch theory without the density effect shows a wide divergence

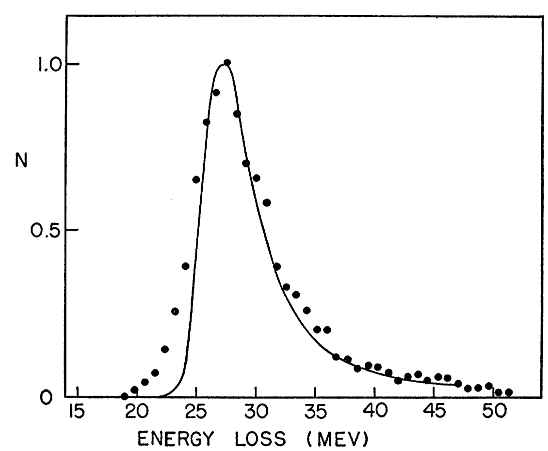

FIG. 12. The solid line gives the distribution calculated for a single meson energy of 2.0 Bev, using the Halpern-Hall theory.

from the experimental shape. The single Landau curve, Fig. 12, also fits less well than the Halpern-Hall theory although the difference here is not so striking. It seems quite interesting that, to a good approximation for these experiments, the entire cosmic-ray spectrum may be replaced by a single energy.

The fact that the peak of the Halpern-Hall curve agrees so well with the experimental curve may be taken as an indication that a small relativistic rise is present (see Table I). Thus our results are in agreement with those of Bowen and Roser who find evidence of saturation in the Halpern-Hall model of anthracene. Our experimental curve does, however, show a small divergence from the theoretical Halpern-Hall curve. The reason for this is not known at present.

CONCLUSION

It appears that the density or polarization effect is observed in NaI(Tl) as well as in anthracene (Bowen-Roser) and that a small relativistic rise probably exists in NaI(Tl). It would be desirable to improve the measurements of absolute energy-loss, the statistical accuracy of the data, and to use monoenergetic mesons, so that folding-in of the cosmic-ray spectrum will become unnecessary. We hope to do further work along all these directions. It is interesting that NaI(Tl) appears to be linear in the range of high meson energies in the cosmic rays, a result that might have been anticipated from previous experiments with other charged particles.

We wish to thank Dr. Sidney Drell for his very kind help in considering many of the theoretical aspects of this problem and also for his independent calculations of energy loss. We also wish to thank Dr. J. A. McIntyre for help in packaging the NaI(Tl) crystals. Finally we appreciate greatly the kindness of Dr. Burton Moyer, of the University of California, in lending us, from time to time, his Po-Be source.

PHYSICAL REVIEW VOLUME 88, NUMBER 3 NOVEMBER 1, 1952

Electron Diffraction from Small Crystals*

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(Received April 14, 1952)

Small crystals of silver and silver bromide were studied by electron diffraction. Small crystals of these materials are of interest because published experimental and theoretical considerations indicate changes in their structure which may occur. Since the diffraction theory which is applicable to large crystals cannot always be used in interpreting the photographs from materials of small size, computations were made for crystal structures and shapes which might be expected for silver and silver bromide. As a result of the computations, it was found that positions of diffraction peaks of substances having the sodium chloride structure depend on the crystal shape. It was also shown that the determination of particle size by the usual method of the Scherrer equation may give values which are much too small. Comparison of experimental and theoretical diffraction effects indicates that small silver particles have the same structure as the large particles, but that there is a real contraction of the lattice in the smaller particles which amounts to 2.7 percent for particles with a diameter of about 31A. The small silver bromide particles have the same structure as the large particles, and an apparent expansion of the silver bromide lattice of about 1.0 percent appears to be best explained not as an expansion but as a particle-shape effect which changes the position of the diffraction peak. The crystals appear to be plates bounded by (111) faces, just as in many photographic emulsions, although this is not the usual form of crystals having this structure.

INTRODUCTION

BECAUSE there are differences between the surface and the internal properties of crystals, it is of interest to compare the characteristics of large crystals with those of very small crystals in which the surface to volume ratio is relatively high. Scattering of electrons from small crystals is sufficiently strong to permit an electron-diffraction study to be made. Boswell[1] has recently published some results obtained by electron diffraction from small crystals of alkali halides, gold, and bismuth. He reports a decrease in lattice constant with decrease in particle size in the diameter range below 100A. The decrease is attributed to a surface-tension effect predicted by Lennard-Jones and Dent.[2]

The substances studied here are silver and silver bromide. Silver is of particular interest in view of Quarrell's[3] observation of many cubic close-packed metals which appeared to begin their growth with the hexagonal close-packed structure. A theoretical study of the lattice energies of the silver halides by Huggins[4] indicates a small difference in the energies of silver

* Communication No. 1488 from the Kodak Research Laboratories.
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[2] J. E. Lennard-Jones and B. M. Dent, Proc. Roy. Soc. (London) **A121**, 247 (1928).
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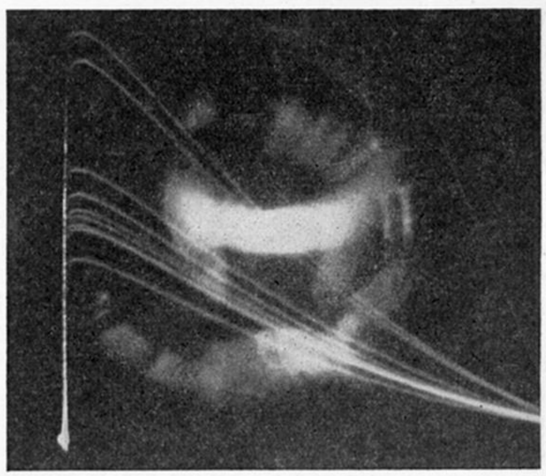

FIG. 3. A sample of exposed film showing representative traces. The tail of a very large pulse may be seen at the right. The irregular bright pattern in the center and the surrounding bright patch are due to glow from the filament of the oscilloscope tube.