the calculated transmissions for $k\Theta=0.122$ ev, and is in agreement with the experimental points.

### VII. CONCLUSIONS

The experimental results show that over a wide range of crystal parameters the Einstein model of a crystal gives sufficiently accurate information about neutron scattering to be of use where the fine details of the scattering are not required. It can give no information about actual energy transfers smaller than $k\Theta$, or angular variations of the order of the separation of Debye-Scherrer rings.

The agreement of the gas model with the Einstein model over a wide region of conditions shows how little influence the binding has on the general scattering pattern; energy and momentum conservation between the neutron and individual nuclei are, in most cases, the over-riding factors. The gas model fails when Bragg scattering, i.e., interference, is important or when the Einstein temperature is large compared with either the temperature or the half-width of the energy distribution.

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# Nuclear Reactions in the Stars. I. Proton-Proton Chain

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The rates are calculated for the main reactions making up the "proton-proton chain," whose net effect is the conversion of four hydrogen atoms into one helium atom. These calculations are carried out for temperatures and densities corresponding to central conditions in main sequence stars.

The mean reaction rate for the beta-decay conversion of two protons into one deuteron is calculated accurately, using the latest data on the two-nucleon system and on beta-decay. It is shown that, under normal stellar conditions, the reaction chain is completed by the radiative capture of a proton by a deuteron and by the collision between two of the resultant $He^3$ nuclei to form one $He^4$ nucleus and two protons. Values are given for the rate of energy production and for the concentrations of deuterium and $He^3$ at various temperatures.

### I. INTRODUCTION

DETAILED calculations on nuclear reactions in main-sequence stars were presented over a decade ago in two papers by Bethe[1] and by Bethe and Critchfield,[2] hereafter referred to as B and BC, respectively. The present paper is largely a continuation of these papers and, wherever possible, the same notation will be used.

The main aim of the present paper is to revise the work of B and BC on the reaction rates for the chain of reactions starting with the combination of two protons in main-sequence stars, in the light of the most recent experimental information on nuclear reaction rates and $Q$-values. No detailed calculations on the stellar hydrodynamics of the problem, using any specific stellar model, are carried out. The results are presented largely in the form of reaction rates as a function of temperature, density, and relative abundance of the nuclear species. In later papers, the carbon cycle and reactions in stars which have exhausted their hydrogen supply will be discussed.

Many of the reactions we shall discuss are of the thermonuclear type, i.e., an exothermic reaction undergone by two nuclear species with a Maxwellian velocity distribution colliding with each other. Under stellar conditions, the mean thermal energy $kT$ (of the order of 1 kev for the sun) is extremely small compared with the kinetic energies at which the Coulomb-barrier is negligible (of the order of a few Mev). Consequently, most of the reactions are undergone by nuclei in a fairly narrow energy-region in the tail of the Maxwell distribution (10 to 50 kev for the sun), which we shall call the "stellar energy-region," $E_{st}$. An approximate formula for $p$, the number of reactions per second per nucleus of type 2, is then obtained (see for instance B) which can be written in the form

$$p=(4\rho x_1 N_0\Gamma a_0 R_0^2/3^{5/2}\hbar)\times[(A_1+A_2)^{5/3}/A_1^2A_2Z_1Z_2]\tau^2 e^{(\lambda-\tau)}, \quad (1)$$

where

$$\tau=3\left[\frac{\pi^2e^4M_0}{2\hbar^2kT}\frac{Z_1^2Z_2^2A_1A_2}{(A_1+A_2)}\right]^{\frac{1}{3}} \quad (2)$$

and

$$\lambda=4[R_0A_1A_2Z_1Z_2/a_0(A_1+A_2)^{\frac{1}{3}}]^{\frac{1}{4}}. \quad (3)$$

In (1) $x_1$, $x_2$ are the concentrations (by weight), $A_1$, $A_2$ the atomic weights and $Z_1$, $Z_2$ the atomic charges of the two reacting nuclear species. $N_0$ is Avogadro's number, $M_0$ one atomic mass unit, $\rho$ the density in g cm$^{-3}$, and $\Gamma$

[1] H. A. Bethe, Phys. Rev. **55**, 434 (1939); B.
[2] H. A. Bethe and C. L. Critchfield, Phys. Rev. **54**, 248 (1938); BC.

the "reaction width," i.e., $\Gamma/\hbar$ is the reaction probability (for energy $E_{st}$) in sec$^{-1}$ after penetration. $R_0$ is the "nuclear radius per nucleon,"[3]

$$R_0=(1.45\pm0.20)\times10^{-13}\text{ cm}, \tag{4}$$

and $a_0$ is the "Bohr radius" for one amu,[4]

$$a_0=\hbar^2/M_0e^2=2.904\times10^{-12}\text{ cm}. \tag{5}$$

The considerations leading to (1) are valid only if the compound nucleus formed in the collision of particles 1 and 2 has no resonance levels for excitation energies corresponding to the neighborhood of the stellar energy-region, $E_{st}$.

If $\Gamma$ is expressed in volts and $T$ in units of $10^6$ degrees, the expression for $p$ becomes, using DuMond and Cohen's[4] values for the atomic constants (for $\alpha^{-1}=137.036$),

$$p=2.59\times10^4\times\rho x_1\Gamma[(A_1+A_2)/A_1]\times(Z_1Z_2/A_1A_2)^{\frac{1}{3}}T^{-\frac{2}{3}}e^{(\lambda-\tau)}\text{ sec}^{-1}, \tag{1a}$$

$$\tau=42.48[Z_1^2Z_2^2A_1A_2/(A_1+A_2)]^{\frac{1}{3}}T^{-\frac{1}{3}}, \tag{2a}$$

$$\lambda=1.26(Z_1Z_2A_1A_2)^{\frac{1}{3}}(A_1+A_2)^{-\frac{1}{3}}. \tag{6}$$

In many cases, the cross sections $\sigma$ for such reactions have been measured in the laboratory as a function of energy for fairly low energies (100 kev and up). If the compound nucleus formed has no resonance levels in the region corresponding to these kinetic energies, then the cross section is approximately of the form

$$\sigma=(S/E)\exp(-2\pi e^2Z_1Z_2/\hbar v), \tag{7}$$

where $E$ and $v$ are the kinetic energy and velocity, respectively, of particle 1 (relative to particle 2) and $S$ is a constant (in units of ev barn). A simple formula [see B, Eq. (11)] then expresses $\Gamma$ (for the energy region $E$) in terms of the constant $S$. The experimental energies $E$ are usually larger than $E_{st}$. But, if $(E-E_{st})$ is small compared with the difference of $E_{st}$ and the energy corresponding to the nearest resonance level, then the values of the width $\Gamma$ will be approximately the same in the energy regions $E$ and $E_{st}$. Using Eq. (11) of B, we can eliminate $\Gamma$ and the poorly known $R_0$ from expression (1) for $p$, and we obtain an expression for $p$ in terms of the experimentally determined constant $S$,

$$p=(8\rho x_1N_0Sa_0/3^{5/2}\pi\hbar)(A_1^2Z_1Z_2)^{-1}\tau^2e^{-\tau}. \tag{8}$$

If $S$ is measured in units of ev barn, we have

$$p=434\rho x_1S(A_1^2Z_1Z_2)^{-1}\tau^2e^{-\tau}\text{ sec}^{-1}. \tag{9}$$

If there is one or more levels in or near the stellar energy-region, then the above formulas are not valid. The expression (9) is then usually a gross underestimate, since the nearest resonance level will usually give overwhelmingly the biggest contribution and destructive interference should be unimportant. If in particular there is a resonance level corresponding to a positive kinetic energy $E_r$ between zero and about 50 kev, then this level gives the biggest contribution and $p$ depends mainly on the ratio $(E_r/kT)$ and hardly on the width of the resonance level.

If $Q$ is the energy in Mev liberated in one reaction between a nucleus of type 1 and one of type 2, then the energy $\epsilon$ liberated per g per sec is

$$\epsilon=(N_0x_2/A_2)pQ, \tag{10}$$

or

$$\epsilon=4.19\times10^{20}(\rho x_1x_2SQ)(A_1^2A_2Z_1Z_2)^{-1}\tau^2\times e^{-\tau}\text{ erg g}^{-1}\text{ sec}^{-1}. \tag{11}$$

## II. THE PROTON-PROTON REACTION. ORBITAL MATRIX ELEMENT

For main sequence stars, which consist to a large extent of hydrogen, two chains of nuclear reactions were shown to be the main source of energy (see B), each of these chains having the net effect of converting four hydrogen atoms into one helium atom plus two neutrinos. The reaction governing the rate of one of these chains, the "proton-proton chain," is the initial and slowest reaction, the formation of a deuteron in the collision of two protons:

$$H^1+H^1\rightarrow D^2+e^++\nu+0.42\text{ Mev}. \tag{12}$$

Formulas for the reaction rate for this process were derived in BC. Our knowledge of the properties of the two-nucleon system and of beta-decay phenomena has improved so greatly, that a much more accurate re-evaluation of this reaction-rate is made possible. The formula for the reaction rate $p$ can be written in the form

$$p=16\pi\times3^{-5/2}gf(W)\,|M_{sp}|^2\gamma^{-3}\Lambda^2N_0A_{\rm H}^{-1}\rho x_{\rm H}\tau^2e^{-\tau}F_\tau. \tag{13}$$

Here $g$ is the Gamow-Teller part of the beta-decay constant, $f(W)$ is the well-known beta-decay $f$-function [see BC Eq. (8)], $x_{\rm H}$ is the concentration (by weight) of hydrogen, $F_\tau$ is a correction factor approximately equal to unity. $\Lambda$ is a dimensionless quantity which is proportional to the orbital part of the matrix element and is almost independent of energy, $M_{sp}$ is the spin part of the matrix element, and $\gamma^{-1}$ is the radius of the deuteron,

$$\gamma^{-1}=(4.314\pm0.003)\times10^{-13}\text{ cm}. \tag{14}$$

Equation (13) is the same as the Eq. (32) derived by BC, except for the factor[5] $|M_{sp}|^2$, which is omitted in the original derivation, and the correction factor $F_\tau$.

The orbital matrix element $\Lambda$ is of the form

$$\Lambda=(\gamma^3/8\pi C_0^2)^{\frac{1}{2}}\int\psi_D^*\psi_p\,d^3r, \tag{15}$$

[3] H. Feshbach and V. F. Weisskopf, Phys. Rev. **76**, 1550 (1949).
[4] J. W. M. DuMond and E. R. Cohen, Phys. Rev. **82**, 556 (1951).
[5] G. Gamow and C. L. Critchfield, *Theory of Atomic Nucleus* (Oxford University Press, Oxford, 1949).

where $\psi_D$ is the normalized wave function for the deuteron ground state and $\psi_p$ is the $S$-part of the wave function for the two-proton system with a particular wave number $k$. Let $R$ be the "Bohr radius" for one proton,

$$R=\hbar^2/Me^2=2.881\times10^{-12}\text{ cm}, \tag{16}$$

and

$$\eta=e^2/\hbar v=(2kR)^{-1};\quad y=r/R. \tag{17}$$

$C_0$ in (15) is the Coulomb-barrier factor,

$$C_0=[2\pi\eta/(e^{2\pi\eta}-1)]^{\frac{1}{2}}. \tag{18}$$

The two-proton wave function $\psi_p$ is normalized per unit density at infinity. Outside of the range of nuclear forces, $\psi_p$ has the form

$$\bar{\psi}_p(y)=e^{i\delta}\sin\delta C_0^{-1}\{\Theta(y)+C_0^2\cot\delta(kR)y\Phi(y)\}/kr. \tag{19}$$

In (19) $\delta$ is the nuclear phase-shift constant for $S$-scattering of two protons with wave number $k$, $\Theta$ and $\Phi$ are functions of $k$ and $y$, defined by Yost, Wheeler, and Breit,[6] which are proportional to the irregular and regular Coulomb wave functions, respectively, and have the value unity for $y=0$.

The matrix element (13) has recently been evaluated very accurately by Frieman and Motz,[7] using accurate explicit wave functions $\psi_p$ and $\psi_D$ which are based on specific assumptions about the potential shape of nuclear forces with the constants chosen to fit low energy experimental data for the two-nucleon system. In such a calculation it is difficult to evaluate the inaccuracy in the final result due to the uncertainty in the potential shape and the experimental errors. For this reason, an approximate re-evaluation of $\Lambda$ seems worthwhile, using the theory of the effective range,[8–10] from which the effect of the present uncertainties in the theory of nuclear forces can be derived very simply.

The nuclear phase shift $\delta$ is of the form[8,11]

$$\pi\cot\delta/(e^{2\pi\eta}-1)=K(\eta), \tag{20}$$

where $K$ is a slowly varying function of $\eta$ (or $k$),

$$K(\eta)=(K_\infty+\tfrac{1}{2}r_{0S}Rk^2\cdots)-h(\eta), \tag{21}$$

and $h(\eta)$ is a function defined by Jackson and Blatt.[11] An analysis[11] of low energy proton-proton scattering data gives the following values for the two constants $K_\infty$ and $r_{0S}$:

$$r_{0S}=(2.65\pm0.07)\times10^{-13}\text{ cm}, \tag{22}$$

$$K_\infty=3.76\pm0.03. \tag{23}$$

We now make use of the fact that the range of nuclear forces is small compared with the deuteron radius, $\gamma^{-1}$, with the "Bohr radius" $R$ and with the wavelength $k^{-1}$. We write the matrix element $\Lambda$ in the form

$$\Lambda^2=F_r\Lambda_0^2, \tag{24}$$

where $\Lambda_0$ is the value obtained for expression (15) by replacing $\psi_D$ by its normalized asymptotic expression and $\psi_p$ by $\bar{\psi}_p$, Eq. (19). $\Lambda$ would be exactly equal to $\Lambda_0$ if the range of nuclear forces were zero, and hence the correction factor $F_r$ is only slightly different from unity.

At a temperature $T$ of about $15(\times10^6\,^\circ\text{K})$ (central temperature of the sun), the main contribution to the reaction comes from proton pairs of relative kinetic energy about 6 kev corresponding to $\eta\approx1.5$. At these energies both $\delta$ and $e^{-2\pi\eta}$ are of the order of magnitude of $10^{-4}$ and hence negligible compared with unity. Using this fact, considering only the $S$-part of $\psi_D$ and neglecting tensor forces completely, we get an accurate approximation to $\Lambda_0$, using (19) and (20),

$$\Lambda_0=(2R\gamma/K)J;\quad J=\int_0^\infty dre^{-\gamma r}\{\Theta+(Kr/R)\Phi\}. \tag{25}$$

Using the expansions for $\Theta$ and $\Phi$ for small values of $r$, (21), (22), and (23), the expression (25) can be evaluated for a particular value of $\eta$. For zero relative energy ($\eta=\infty$), one finds

$$J=1.40\gamma^{-1},\quad \Lambda_0^2=6.18\pm0.06, \tag{26}$$

the main uncertainty in the value of $\Lambda_0$ coming from the uncertainty in the experimental value for $K_\infty$. The expressions for $\Theta$, $\Phi$, and $K$ for energies in the stellar region differ only very slightly from those for zero relative energy: For $\eta=1.5$, one finds $h=0.04$, $K=3.73$, and $\Lambda_0^2=6.21$.

The factor $F_r$, correcting for the finite range of nuclear forces, consists of the product of two terms:

$$F_r=(1-\rho_t'\gamma)^{-1}(1-DJ^{-1}). \tag{27}$$

The first term is due to the difference between the normalization integrals of $\psi_D$ and of its asymptotic expression, $\rho_t'$ being the effective range[10] $\rho_t(-\epsilon,-\epsilon)$ for the deuteron ground state. The second term takes care of the difference $D$ between $J$ as defined in (25) and the equivalent integral involving the correct wave functions normalized such that the asymptotic form of the integrand is equal to the integrand of $J$. In two previous papers[9,10] an integral $D$ was defined and evaluated (in connection with photomagnetic neutron-proton capture) which is identical with the expression $D$ occurring in (27), except that the singlet wave functions for the neutron-proton (instead of the proton-proton) system occur. The main contribution to $D$ comes from the range of integration less than the range of nuclear forces (0 to $3\times10^{-13}$ cm). In this region it was shown by Bethe[8] (see Fig. 5) that both the correct and asymptotic singlet wave functions are very similar

[6] Yost, Wheeler, and Breit, Phys. Rev. **49**, 174 (1937).
[7] E. Frieman and L. Motz, Phys. Rev. **83**, 202 (1951) and private communication.
[8] H. A. Bethe, Phys. Rev. **76**, 38 (1949).
[9] H. A. Bethe and C. L. Longmire, Phys. Rev. **77**, 647 (1950).
[10] E. E. Salpeter, Phys. Rev. **82**, 60 (1951).
[11] J. D. Jackson and J. M. Blatt, Revs. Modern Phys. **22**, 77 (1950).

for the $p$–$p$ and $n$–$p$ systems, except for the slight difference between the singlet ranges $r_{0s}$ for the two systems. We therefore get a fairly good approximation to $D$ by using the expression (20) of reference 10 for the various potential shapes, using the value (22) for $r_{0s}$ and the latest value[10] for the triplet range

$$\rho_t=(1.704\pm0.030)\times10^{-13}\text{ cm}. \tag{28}$$

Equation (27) then gives for $F_r$ a value of about 1.15 for a Yukawa (Hulthén), 1.13 for an exponential, and 1.12 for a square well shape for both the triplet and singlet potential. This gives, for these three potential shapes and for $\eta=\infty$,

$$\Lambda^2=7.11(\text{Y}),\quad 7.00(\text{E}),\quad 6.92(\text{S}). \tag{29}$$

Frieman and Motz,[7] using accurate wave functions for a specific potential shape find a value of 6.79 for $\Lambda^2$. The experimental errors in $\rho_t$ and $r_{0s}$ contribute probable errors of about $\pm3$ percent and $\pm1$ percent, respectively, to the factor $F_r$. From the numerical values of (29), we estimate a further probable error in $\Lambda^2$ due to the uncertainty in the potential shape of about $\pm3$ percent, the uncertainty in $\Lambda_0{}^2$ contributes another $\pm1$ percent. These four sources of error are also present in the calculations of Frieman and Motz. The numerical values in (29) contain an uncertainty of another 5 or 10 percent (due to the neglect of tensor forces and the approximation used for $D$), which is not present in the more accurate calculations. We therefore shall use the value for $\Lambda^2$ obtained by Frieman and Motz plus the small correction due to $\eta\approx1.5$ instead of infinity, calculated above (and with the four errors of 3.3, 1, and 1 percent estimated above). Since the sources of these errors are independent, we finally get

$$\Lambda^2=6.82(1\pm0.05). \tag{30}$$

## III. THE PROTON-PROTON REACTION. OTHER FACTORS

In the beta-decay factor $f(W)$, $W$ is the maximum beta-particle energy (including rest mass) in units of $mc^2$. Using a value[12] for the $H_2-D$ doublet of

$$H_2-D=(1.551\pm0.002)\text{mMU}, \tag{31}$$

we have

$$W_0=1.827(1\pm0.002), \tag{32}$$

where $W_0$ is the value of $W$ for zero relative kinetic energy of the two protons. The main contribution to the cross section comes from relative velocities.

$$v\approx(4\pi e^2kT/hM)^{\frac{1}{2}},$$

which corresponds to a relative kinetic energy of 5.9 kev for $T=15$. We evaluate $f(W)$ for a value of $W$ equal to $W_0$ plus this "most probable" relative kinetic energy. Using the very accurate approximations for $f(W)$ given by Feenberg and Trigg,[13] we find

$$f(W)=0.145[1+0.054(T/15)^{\frac{3}{2}}](1\pm0.03). \tag{33}$$

The reaction involves a transition from a singlet $S$-state of the two-nucleon system and hence requires Gamow-Teller selection rules. If these selection rules are valid, then the factor $|M_{sp}|^2$ (which takes care of the summation over spin-states and of the fact that either proton can turn into a neutron) becomes equal[14] to $\frac{3}{2}$.

A lower limit for $g$, the Gamow-Teller beta-decay constant used in Eq. (13), can be obtained from experimental data on those super allowed beta-decays which can only proceed by means of Gamow-Teller selection rules. For such a decay[14]

$$g=\ln2/[f(W)t_{\frac{1}{2}}|M|^2|M_{sp}|^2], \tag{34}$$

where $t_{\frac{1}{2}}$ is the half-life in seconds, and $M$ is the orbital matrix element, which is equal to or slightly less than unity. The most suitable decay of this type is that of $He^6$ into $Li^6$, which is a process quite analogous to our $p$–$p$ reaction.[15] Using for the energy end point the latest experimental value[16] of $(3.50\pm0.05)$ Mev and for the half-life[14] $(0.823\pm0.013)$ sec, we get a value of $(1270\pm90)$ sec$^{-1}$ for $[ft|M_{sp}|^2]$. Since the factor $|M|^2$ cannot exceed unity, we get from Eq. (34) and the upper limit of this value the inequality

$$g>5.1\times10^{-4}\text{ sec}^{-1}. \tag{35a}$$

The beta-decay of the neutron, on the other hand, provides an *upper* limit for $g$. For this decay the orbital factor $|M|^2$ is identically equal to unity, the energy end point[12] is $(0.782\pm0.001)$ Mev, the half-life[17] is $(12.8\pm2.5)$ min. If no Fermi type of beta-interaction existed, this would give a value of $(930\pm180)$ sec for $[ft|M|^2|M_{sp}|^2]$. Since part of the neutron decay probably proceeds by means of Fermi selection rules, Eq. (34) and the lower limit of this value furnish an inequality for the Gamow-Teller decay constant,

$$g<9.4\times10^{-4}\text{ sec}^{-1}. \tag{35b}$$

In addition to these two inequalities, actual estimates for $g$ can be obtained[18] from the decay of $H^3$ and other beta-transitions between mirror nuclei. We therefore take

$$g=7.5(1+\Delta)\times10^{-4}\text{ sec}. \tag{36}$$

The error factor $\Delta$ lies between about $\pm0.20$ at the moment, and it is hoped that further advances in beta-decay investigations will furnish a better value for it.

[12] Li, Whaling, Fowler, and Lauritsen, Phys. Rev. **83**, 512 (1951).

[13] E. Feenberg and G. Trigg, Revs. Modern Phys. **22**, 399 (1950).

[14] Using the notation of C. S. Wu, Revs. Modern Phys. **22**, 386 (1950).

[15] The decay of $He^6$ can be considered qualitatively as the transition of two neutrons in a singlet state to a deuteron in a triplet state, all in the field of an $\alpha$-particle. The value of $|M_{sp}|^2$ is $\frac{3}{2}$, the same as for the $p-p$ reaction. (See reference 14.)

[16] C. S. Wu, Phys. Rev. (to be published).

[17] J. M. Robson, Phys. Rev. **83**, 349 (1951).

[18] G. L. Trigg, Phys. Rev. **86**, 506 (1952).

The correction factor $F_\tau$, which occurs in (13) [also (1), (4), (8), and (9) should be multiplied by $F_\tau$] is of the form

$$F_\tau=(\tau/\pi)^{\frac{1}{2}}e^\tau\int_0^\infty dx\,\exp\{-(\tau/3)[(2/x)+x^2]\}. \quad (37)$$

The quantity $\tau$, defined in (2), is greater than 10 for all cases of interest and is about 13.7 for the proton–proton reaction at a temperature of $15\times10^6$ °K. On putting $x=(1+y)$, Eq. (37) becomes

$$F_\tau=(\tau/\pi)^{\frac{1}{2}}\int_{-1}^\infty dy\,\exp(-\tau y^2)\times\{(1+y)\,\exp[2\tau y^2/3(1+y)]\}.$$

We replace the lower limit of integration by $(-\infty)$, which introduces only errors of the order of magnitude of $e^{-\tau}(<10^{-4})$, expand the expression inside the curly bracket in positive powers of $y$ and integrate. This gives

$$F_\tau=1+(5/12\tau)-(35/288\tau^2)+\cdots. \quad (38)$$

For a temperature of $T=15$, the numerical value of $F_\tau$ for the proton–proton reaction is 1.030.

In the formulas derived above, the screening due to the electron charge cloud surrounding a proton has been neglected. But for a relative kinetic energy of about 6 kev, the classical turning point corresponds to a distance between the two protons of only about $2.5\times10^{-11}$ cm. The electron charge cloud, on the other hand, is spread out over distances as large as the mean distance between neighboring nuclei, which is about $2\times10^{-9}$ cm for a density $\rho$ of about 100 g cm$^{-3}$. Under these circumstances the correction to the reaction rate due to screening is less than 1 in $10^4$ and hence will be neglected.

Substituting all the quantities calculated in this section into Eq. (13), we obtain for the mean reaction rate per proton

$$p=1.73(1+\Delta\pm0.06)\times10^{-16}\rho x_{\rm H}\tau^2e^{-\tau}\times[1+(5/12\tau)+\cdots][1+0.054(T/15)^{\frac{2}{3}}]\ \text{sec}^{-1}. \quad (39)$$

The largest source of error in this expression is still the 20 percent error $\Delta$ in the beta-decay constant.[19]

For temperatures in the neighborhood of 15 million degrees, we obtain the following approximation for $p$,

$$p=3.9_4(1+\Delta\pm0.06)\times10^{-18}\times(\rho x_{\rm H}/100)(T/15)^{3.96}\ \text{sec}^{-1}. \quad (40)$$

## IV. REACTIONS INVOLVING DEUTERONS

Once deuterons are formed by the proton–proton reaction, they are quickly converted into $He^3$ by the reaction

$$H^1+D^2\rightarrow He^3+\gamma+5.5\ \text{Mev}. \quad (41)$$

We first consider the contribution to this reaction from protons of zero orbital angular momentum, which contributes practically the whole reaction rate at the low stellar energies.

A recent measurement of the capture cross section of thermal neutrons by deuterium[20] gives

$$\sigma=(0.57\pm0.01)(2200\ \text{m sec}^{-1}/v)\times10^{-27}\ \text{cm}^2. \quad (42)$$

$H^3$ and $He^3$ are mirror nuclei which are extremely similar except for the Coulomb repulsion. We therefore assume that the cross section for reaction (41) for low energy $S$-state protons is the same as Eq. (42), except for the Coulomb-barrier penetration factor $C_0^2$, Eq. (18).* This gives a cross section of the form of Eq. (7) with

$$S=9.0\times10^{-2}\ \text{ev barn}.^{21} \quad (43)$$

Substituting into Eq. (9), we find for the reaction rate $p$,

$$p=39(\rho x_{\rm H})\tau^2e^{-\tau}\ \text{sec}^{-1}. \quad (44)$$

At temperatures $T$ near 15 (in $10^6$ °K) this can be written as

$$p=0.25(\rho x_{\rm H}/100)(T/15)^{4.3}\ \text{sec}^{-1}. \quad (44a)$$

The biggest uncertainty in the numerical values in Eqs. (44) and (44a) comes from the assumption that the matrix elements for neutron and proton capture are the same except for the Coulomb barrier. This uncertainty in $p$ should not be more than about $\pm40$ percent.

The cross section for reaction (41) has been measured directly[22] for proton energies $E$ between 400 kev and 1.5 Mev. The observed angular distribution indicates that most of the cross section at these energies is contributed by $P$-state protons. By means of the well-known Coulomb barrier penetration formula[23] for orbital angular momentum unity, the energy dependence of the partial cross section for reaction (41) for $P$-state protons can be calculated at all energies. For energies of a hundred kev or more, this cross section rises sharply with energy. In the 1 Mev region, this theoretical energy dependence was found to agree very well with the measured one,[22] confirming the fact that the bulk of the reaction comes from $P$-state protons in this energy region. For much lower energies, however, the $P$-state cross section becomes smaller than that for the $S$-state. At energies of about 20 kev or less the energy dependence of the $P$-state cross section is of the form of Eq. (7) with $S$ about $2\times10^{-3}$ ev barn, which is negligible compared with Eq. (43). We therefore neglect any $P$-state corrections to Eqs. (44) and (44a).

The mean life for reaction (41) is of the order of

[19] The expression (39) is much larger than the value given originally by BC. This is mainly due to the fact that too low a value for $g$ was used and the factor $|M_{sp}|^2$ omitted in BC.

[20] Kaplan, Ringo, and Wilzbach, Phys. Rev. **87**, 785 (1952).

* *Note added in proof.*—As was kindly pointed out by Dr. W. A. Fowler, Eqs. (43) and (44) have to be multiplied by an additional factor resulting from the finite size of the nuclei involved. This factor was estimated to be about $(1.3\pm0.3)$. The numbers in the second lines of Tables II and III should be divided by this factor.

[21] Corresponding to a width $\Gamma$ of about 0.25 ev.

[22] Fowler, Lauritsen, and Tollestrup, Phys. Rev. **76**, 1767 (1949).

[23] N. F. Mott and H. S. Massey, *The Theory of Atomic Collisions* (Clarendon Press, Oxford, 1949), second edition, p. 54.

seconds or minutes for normal stellar conditions (see Table III). Dynamic equilibrium between the concentrations $x_D$ of deuterium and $x_H$ of hydrogen will therefore be established very quickly; the ratio $(x_D/x_H)$ is then equal to the ratio of the mean lifetimes of reactions (12) and (41) (formation and destruction, respectively, of deuterons) multiplied by the ratio of atomic weights. Because of the high speed of reaction (41), the concentration $x_D$ is extremely low and values for $x_D$ are given in Table II. No spectroscopic measurements of $x_D$ in stellar atmospheres are available as yet, but terrestrial deuterium concentrations are enormously larger than those given in Table II. This fact may suggest that the material from which the earth was formed could not have come from the interior of a main sequence star (like our sun).

There are a few reactions involving deuterons which might compete with reaction (41), but these were all found to be much slower. Collisions between two deuterons are negligible under stellar conditions, because of the extremely small concentration $x_D$. Collisions between a deuteron and an alpha-particle are unimportant because of the high Coulomb barrier and small gamma-ray width $\Gamma$. The reaction between one deuteron and one $He^3$ nucleus is somewhat more important, because of a resonance[24,25] near the stellar energy region. Using the experimental cross sections[24,25] and the equilibrium concentrations of $He^3$ (calculated in the next section), this reaction was found to be slower than reaction (41) by a factor of at least 50 at temperatures of $5\times10^6$ °K or more.

## V. REACTIONS INVOLVING $He^3$

Since $Li^4$ is unstable to particle disintegration, no direct $(p,\gamma)$ reaction on $He^3$ is possible. There are, however, a number of competing reactions which use up $He^3$. Of these the reaction

$$He^3+He^3\rightarrow He^4+2H^1+12.8\text{ Mev}, \qquad (45)$$

first suggested by Schatzman[26] and by Fowler and Lauritsen,[27] turns out to be the fastest and most important. This reaction has been detected experimentally,[28] but no accurate cross-sectional measurements are available as yet. But cross sections for the similar reaction involving two $H^3$ nuclei have been measured[29] for energies of 100 kev and up. For energies between 100 kev and 500 kev, the experimental cross section is of the form of Eq. (7) with

$$S=(3.2\pm1.5)\times10^5\text{ ev barn}. \qquad (46)$$

$H^3$ and $He^3$ being very similar, we assume that the cross sections for reaction (45) and for the reaction involving two tritons are the same except for the different Coulomb barrier factors. This gives for reaction (45) a value for $S$ of four times (ratio of the values for $Z^2$) the expression (46), if the nuclear radii are assumed to be zero. The finite size of the nuclei involved introduces an additional correction factor, which cannot yet be calculated accurately. This factor was estimated to be between two and eight (but may even lie slightly outside these limits) and a value of four was adopted for it. Using Eq. (9), the rate $p$ of reaction (45) becomes

$$p=6\times10^7(\rho x_3)\tau^2e^{-\tau}\text{ sec}^{-1}, \qquad (47)$$

where $x_3$ is the concentration (by mass) of $He^3$. The numerical factor in Eq. (47) should not be in error by a factor of more than five.

When dynamical equilibrium has been set up, the concentration $x_3$ adjusts itself such that the number of $He^3$ nuclei created per $cm^3$ per sec by means of reactions (12) and (41) equals the number destroyed by means of reaction (45). The ratio $(x_3/x_H)$ can then be computed using Eqs. (39) and (47). Values computed for $x_3$ in this manner are given in Table II for different temperatures. For temperatures $T$ about 10 to 15 (central portions of the sun), $x_3$ is very low, which agrees with the fact that $He^3$ has not yet been detected with certainty in solar spectra.[30] But at temperatures $T$ of the order of 5 or lower (extremely cool stars), the equilibrium concentration of $He^3$ is quite appreciable and it would require a very long time to build up this concentration (starting from pure hydrogen).

A list of reactions involving $He^3$ which compete with reaction (45) is given in Table I, together with the estimated mean reaction times at two temperatures which respresent lower and upper limits to central temperatures of main sequence stars. Although the stated estimates for the reaction times for all reactions except (45) could be in error by a factor of as much as

TABLE I. Mean life of $He^3$ corresponding to various competing reactions for $\rho x_H=100$ and for two temperatures $T$ (in $10^6$ °K).

| Reaction | Mean life (years) $T=5$ | $T=30$ |
|---|---|---|
| $He^3+He^3\rightarrow He^4+2H^1$ | $6\times10^{10}$ | 240 |
| $D^2+He^3\rightarrow He^4+H^1$ | $7\times10^{12}$ | $3\times10^5$ |
| $He^4+He^3\rightarrow Be^7+\gamma$ | $1\times10^{18}$ | $8\times10^8$ |
| $H^1+He^3\rightarrow He^4+e^++\nu$ | $5\times10^{14}$ | $2\times10^8$ |
| $e^-+He^3\rightarrow H^3+\nu$ | $2\times10^{23}$ | $3\times10^4$ |

TABLE II. Concentrations (by mass) of deuterium, $x_D$, and of $He^3$, $x_3$, relative to that of hydrogen for various temperatures $T$ (in $10^6$ °K).

| $T$ | 5 | 8 | 15 | 30 |
|---|---|---|---|---|
| $x_D/x_H$ | $5.6\times10^{-17}$ | $4.1\times10^{-17}$ | $3.0\times10^{-17}$ | $2.3\times10^{-17}$ |
| $2x_3/x_H$ | $0.2_3$ | $0.01_5$ | $8\times10^{-5}$ | $1.8\times10^{-6}$ |

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[29] H. M. Agnew *et al.*, Phys. Rev. **84**, 862 (1951).
[30] J. L. Greenstein, Astrophys. J. **113**, 531 (1951).

TABLE III. Mean reaction times, $p^{-1}$, and the rate of energy production $\epsilon$ (in ergs $g^{-1}$ $sec^{-1}$) at various temperatures $T$ (in $10^6$ °K).

| Reaction | Mean reaction times* | $T=5$ | $T=8$ | $T=15$ | $T=30$ |
|---|---|---|---|---|---|
| $H^1+H^1\rightarrow D^2+e^++\nu$ | $(100/\rho x_H)p^{-1}$ | $1.7\times10^{12}$ yr | $1.3\times10^{11}$ yr | $8.1\times10^{9}$ yr | $7.6\times10^{8}$ yr |
| $H^1+D^2\rightarrow He^3+\gamma$ | $(100/\rho x_H)p^{-1}$ | 23 min | 80 sec | 4 sec | 0.3 sec |
| $He^3+He^3\rightarrow He^4+2H^1$ | $(200/\rho x_H)p^{-1}$ | $2.5\times10^{11}$ yr | $1.3\times10^{8}$ yr | $4\times10^{5}$ yr | $1.0\times10^{3}$ yr |
| Rate of energy production | $(100/\rho x_H^2)\epsilon$ | 0.24 | 3.1 | 50 | 530 |

* The "mean lives" of $H^1$ and $He^3$ are one-half of the mean reaction times for the first and third reaction respectively. $\rho$ is the density in $g/cm^3$, $x_H$ the concentration (by mass) of $H^1$.

ten or twenty, Table I shows that the competing reactions are probably much slower than reaction (45) for all temperatures $T$ between 5 and 30 (in $10^6$ °K) and almost certainly slower for $T$ between 10 and 20. The reaction rates were estimated as follows.

The rate of the reaction involving a deuteron was calculated from the experimental cross section[24,25] near resonance as described in the preceding section. The rate of the reaction involving $He^4$ was estimated as described by B, using an estimate for the $\gamma$-ray width of $\Gamma\sim0.1$ ev and an upper limit of $x_{He^4}\sim0.5$ in Eq. (1a).

The rate of the reaction involving capture of a proton and a beta-decay was estimated in analogy with the proton–proton reaction, (12). An approximate one-particle model was used, considering the $He^4$ nucleus as made up of one neutron moving in the field of a core of the same type as a $He^3$ nucleus. The matrix element for the transition of the incident proton to this neutron was then calculated, using the theory and constants of the effective range of nuclear forces.

Finally, the inverse beta-decay involving the absorption of an electron was calculated using the formulas of Gamow and Schönberg.[31] A value[12] of (18.5±0.2) kev for the energy end point and a half-life of $3.9\times10^8$ sec for the normal beta-decay of $H^3$ into $He^3$ was used and the electron gas was assumed to be nondegenerate. This nondegeneracy applies to the interior of all main sequence stars, but not to the interior of white dwarf stars. In fact, at densities of the order of $10^5$ g $cm^{-3}$ the Fermi energy is of the same order as the energy difference between $H^3$ and $He^3$ and the inverse beta-decay proceeds enormously faster than for a nondegenerate gas and would be faster than reaction (45) under these circumstances.

## VI. ENERGY PRODUCTION

The three reactions making up the proton–proton chain are given in Table III, together with their mean reaction times for four different temperatures corresponding to central conditions of different main sequence stars. The corresponding concentrations of deuterium and of $He^3$, under conditions of dynamic equilibrium, are given in Table II. The probable errors for the reaction times given in Table III are of the order of 40 percent for each of the first two reactions and of a factor of two or three for the third reaction.[32]

As pointed out before,[1,2] the net result of the proton–proton chain is the conversion of four hydrogen atoms (ionized) into one helium atom (ionized $He^4$) plus two neutrinos (the two positrons created quickly annihilating with two electrons of the ionized gas). Note that, since two $He^3$ nuclei are involved in reaction (45), two reactions of type (12) are required to form one $He^4$ nucleus. A fraction of about 0.6 of the energy release of reaction (12) escapes from the star in the form of kinetic energy of the neutrino. Allowing for this loss and using the known binding energy of an $\alpha$-particle,[12] one gets an energy release of 26.2 Mev for every $He^4$ nucleus formed [i.e., for two reactions of type (12)]. Substituting this into Eq. (10), one gets for the energy release $\epsilon$

$$\epsilon=1.26\times10^{19}(x_H p)\ \text{erg/g sec}, \tag{48}$$

where $x_H$ is the hydrogen concentration and $p$ is the reaction rate given by Eqs. (39) or (40). For temperatures near $15\times10^6$ °K, we get

$$\epsilon=49.6(1+\Delta\pm0.06)(\rho x_H^2/100)\times(T/15)^{3.96}\ \text{erg/g sec}. \tag{48a}$$

It should finally be emphasized that all the calculations of this paper were carried out under the assumption of a perfectly Maxwellian distribution of thermal energies, fixed and uniform chemical composition, density and temperature and complete dynamic equilibrium. The carbon nitrogen cycle and its relation to the proton–proton chain will be discussed in a later paper.

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[31] G. Gamow and M. Schönberg, Phys. Rev. **59**, 539 (1941).

[32] For the lower two temperatures in Table III the errors may be somewhat larger than those stated above.