Energy Distribution of Slow Neutrons Scattered from Solids

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The sharp dependence on energy of the cadmium cross section for neutrons of energies near 0.35 ev has been used to investigate the energy distribution of 0.35-ev neutrons scattered through 90° by lead, aluminum, diamond, and graphite. The transmission of the scattered neutrons by cadmium filters was measured as a function of cadmium thickness and compared with cadmium transmissions calculated for an Einstein crystal model. In all cases agreement was obtained. For lead and aluminum at room temperature the simpler model of a gas of free atoms gave equally good agreement, a well-known behavior at high energies, but here applying when the temperature and neutron energy were only moderately greater than the Einstein temperature and much less than the binding energy. Theoretical formulas for the partial differential scattering cross sections for the two models are given in convenient form.

I. INTRODUCTION

WHEN slow neutrons are scattered by a crystal, energy is exchanged between the neutron and the vibrational states of the crystal. The scattered neutrons have an energy distribution which depends on the properties of the crystal, on its temperature, and on the angle of scattering. The only previous experimental work on this problem has been done by Egelstaff.[1] He determined the mean energy changes of very slow neutrons scattered by polycrystalline scatterers, but the large number of multiple scatterings precluded comparison with theory.

This paper describes an investigation of the energy distribution of neutrons scattered singly through 90° by polycrystalline diamond, graphite, aluminum, and lead. Before scattering, the neutrons had an energy of 0.35 ev. The work was originally undertaken to determine the general features of the energy distribution for use in correcting scattering measurements on resonant absorbers.[2] It has established the applicability of simple theoretical formulas and has proved to be a useful approach to a more detailed experimental study of the scattering by single crystals.

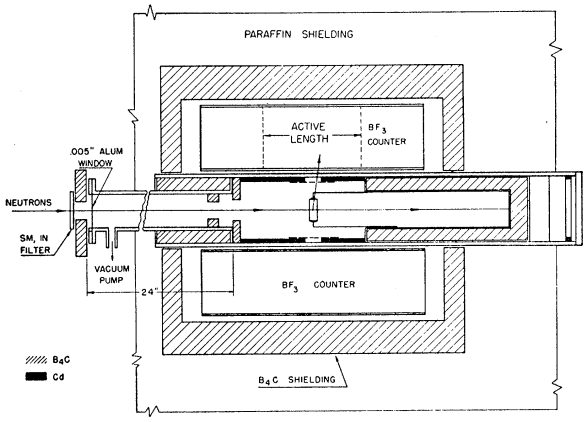

FIG. 1. The apparatus. In this figure the horizontal dimensions are compressed relative to the vertical in the ratio 2:1.

The theory of neutron scattering by crystals has been discussed by many authors in particular by Weinstock,[3] Finkelstein,[4] and Cassels.[5] Weinstock and Cassels took into account the normal modes of the crystal, but because of the complexity treated fully only those collisions in which a single phonon is transferred. Finkelstein discussed the scattering from an Einstein model of a crystal and included collisions with multiple phonon transfers. The neutron energy used in the present experiment is so high that a theory restricted to single phonon transfers is inadequate. On this account, the experimental results were first compared with predictions based on an extension of Finkelstein's work. As the agreement was satisfactory, only the Einstein model was considered in detail, the greater complexity associated with the more exact model not being warranted in the present case.

II. APPARATUS

The apparatus is shown in Fig. 1. It was mounted on the arm of the spectrometer previously described,[6] replacing the neutron counter and its shield. The NaCl crystal of the spectrometer was set to select neutrons of energy 0.35 ev, which were passed through filters of indium and samarium oxide to remove second-order and diffusely scattered thermal neutrons contaminating the beam. The resulting monoenergetic neutron beam was collimated by two $\frac{1}{2}$-inch diameter apertures in boron carbide blocks. The collimated beam then passed through a $\frac{5}{8}$-inch diameter aperture, which removed neutrons scattered by the collimating system, into an evacuated scattering chamber.

The scattering chamber was surrounded by a ring of six BF_3 proportional counters which were shielded everywhere except for a short cylindrical opening surrounding the scatterer. This opening accepted neutrons

[1] P. A. Egelstaff, Nature **168**, 290 (1951).
[2] J. Tittman and C. Sheer, Phys. Rev. **83**, 746 (1951); Brockhouse, Hurst, and Bloom, Phys. Rev. **83**, 840 (1951).
[3] R. Weinstock, Phys. Rev. **65**, 1 (1944).
[4] R. J. Finkelstein, Phys. Rev. **72**, 907 (1947).
[5] J. M. Cassels, Proc. Roy. Soc. (London) **A208**, 527 (1951).
[6] Hurst, Pressesky, and Tunnicliffe, Rev. Sci. Instr. **21**, 705 (1950).

that had been scattered through 90°±30°. Cylindrical cadmium absorbers of various thicknesses could be placed over the opening. The counting rates with and without the cadmium absorber, corrected for background and variations in incident neutron flux, gave the transmission by the cadmium. As virtually no counts were produced by the beam in the absence of a scatterer, the background was taken to be the counting rate with the scatterer in position and the counters shielded from the scatterer by a thick cadmium absorber (~2 g/cm²). The neutron beam included a small third-order component (about 1.5 percent of energy 9×0.35 ev) for which all the cadmium absorbers were nearly transparent. This was included in the background. The total background was about 10 percent of the scattered intensity, and this set a limit upon the accuracy which could be obtained for thick absorbers with reasonable counting times. Corrections for fluctuations in the neutron flux from the reactor were found by means of a BF_3 counter which monitored the original undiffracted beam.

The specimens were held in position by a thin aluminum tube, no part of which was in the beam. All had about the same scattering power (≈12 percent) and occupied the same volume. The aluminum specimen was in the form of a cylinder $\frac{1}{2}$ inch long and $\frac{3}{4}$ inch in diameter. The lead specimen consisted of lead turnings contained between two lead foils in a thin aluminum case. The graphite specimen was made up of five individual graphite disks spaced over a length of $\frac{7}{16}$ inch. The diamond consisted of five layers of diamond powder held between thin aluminum foils at the same positions as the five graphite disks. These constructions avoided large multiple scattering effects which would have arisen in compact cylindrical specimens.

III. ANALYSIS AND CORRECTION OF THE MEASUREMENTS

Between neutron energies of 0.2 ev and 0.5 ev the cadmium cross section falls rapidly from over 6000 barns to less than 150 barns. The cadmium cross section is thus a sensitive indicator of the energy near 0.35 ev. For a cadmium foil having n atoms/cm², the transmission for neutrons having an energy distribution $F(E)$ is

$$T(n)=\int_0^\infty F(E)E^{-\frac{1}{2}}\exp\{-n\sigma(E)\}dE \Big/ \int_0^\infty F(E)E^{-\frac{1}{2}}dE, \quad (1)$$

where $\sigma(E)$ is the cross section as a function of neutron energy E. The factor $E^{-\frac{1}{2}}$ is introduced to allow for the energy dependence of the detector (assumed to be thin and obeying a $1/v$ law as is the case with the BF_3 counters).

Provided the neutron energies lie entirely in a region like that between 0.2 ev and 0.5 ev where the cross section is a single valued function of the energy, it may be useful to replace the energy by the cross section as the independent variable. Equation (1) then has the form of a Laplace transform of a function $f(\sigma)$ which is related to $F(E)$. In principle, $T(n)$ could then be inverted to obtain $f(\sigma)$ and thence $F(E)$, but unless the values of $T(n)$ were known over a wide range with high accuracy, this would hardly be justified. The alternative procedure of comparing measured values of $T(n)$ with values calculated from theoretically predicted energy distributions has been used in the present work.

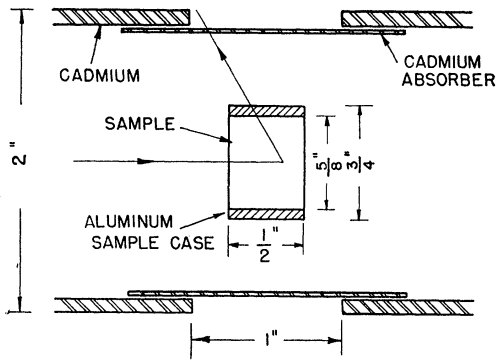

FIG. 2. Dimensions and arrangement of the scatterer and counter aperture.

The total cross section of cadmium has been measured[7] on the same spectrometer with the result

$$\sigma(E)=24.9(0.180/E)^{\frac{1}{2}}((E-0.180)^2+0.00319)^{-1}\times10^{-24}\ \text{cm}^2. \quad (2)$$

This has been used for $\sigma(E)$ in (1).

To insure that the energy setting of the scattering apparatus was in satisfactory agreement with that used in (2), the total cross section of cadmium for the incident neutrons was measured and found to be in almost exact agreement with the cross section predicted by (2) at 0.35 ev.

Before the measured transmissions of the cylindrical cadmium absorbers could be compared with (1), some small corrections were necessary. The formula for $T(n)$ supposes normal incidence of the neutrons on the absorber, but in the apparatus deviations from normal incidence were possible. The dimensions of aperture and specimen are given in Fig. 2. The correction for angular spread was found by a calculation of the effective transmission for an isotropic distribution of neutrons, taking into account the thicknesses of cadmium and counter traversed at each angle. The result of averaging over the specimen and aperture was that all geometrical effects could be accurately accounted for if the thickness of cadmium absorber used in (1) was taken as 1.055 times the measured thickness.

The measurements had to be corrected for a small scattering effect. A fraction of the neutrons which were

[7] B. N. Brockhouse (to be published).

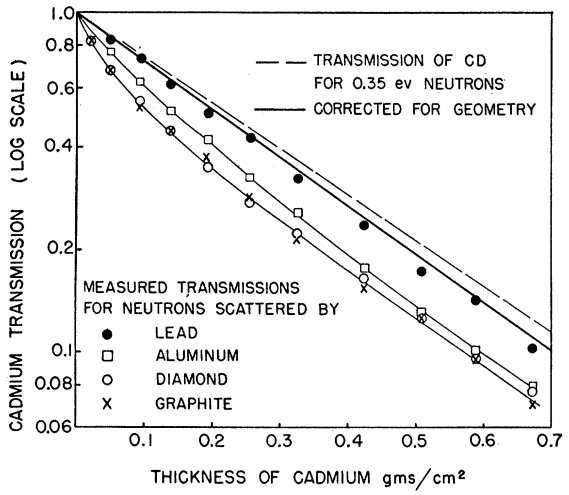

FIG. 3. The measured transmissions of the cadmium absorbers plotted against the thickness of the absorbers. The thick solid line shows the expected transmission for neutrons of unchanged energy.

finally counted was first scattered by the window of the scattering chamber and the walls of the counters. Those which were scattered back through the scattering chamber in such directions as to reach the counters traversed the cadmium absorber three times instead of once, possibly at a considerable inclination to the normal. A correction, which varied with the transmission but did not exceed $2\frac{1}{4}$ percent, was computed from the geometry and applied to the measurements.

The energy distribution for scattering through 90° was used in (1) in all cases. Use of the mean energy distribution for scattering over the range 60° to 120° made no significant difference in the results.

IV. THEORY

The crystal model chosen for the calculation of the energy distribution function $F(E)$ is that due to Einstein. In the Einstein approximation for a simple crystal containing only one type of atom, each atom is regarded as bound in a harmonic potential well. The wells are identical and are not affected by the motions of the atoms. The inelastic scattering cross section per atom is simply the cross section for a single oscillator[4] averaged over the thermal excitation according to the Boltzmann distribution. The elastic scattering includes interference effects and is made up of terms arising from diffraction and a disorder term, owing to differences in the thermal excitation of the oscillators, which may correspond to small energy losses in a real crystal. At high energies, the elastic scattering from a polycrystalline specimen produces many Debye-Scherrer diffraction rings. If the detector is not too restricted in angle, it will include a number of such rings. Under these conditions, the scattering per atom approaches the scattering from a single oscillator. Because some thirty or forty Debye rings were accepted by the detector in the present work, interference effects in the elastic scattering have been ignored.

The energy levels of an oscillator are separated by equal energy intervals $k\Theta$, Θ being the Einstein temperature of the crystal and k Boltzmann's constant. Consequently, the neutron gains or loses energy only in amounts $\gamma k\Theta$, where γ is an integer or zero. Finkelstein[4] has given the cross section for neutron scattering by an oscillator initially in state n which undergoes a transition to a state $n+\gamma$ as a result of the collision. The neutron loses energy of amount $\gamma k\Theta$. For comparison with the present experiment, it is necessary to calculate the cross section for loss of energy $\gamma k\Theta$ by the neutron irrespective of the value of n. That is, Finkelstein's expression for the cross section must be averaged according to the probability of thermal excitation of the initial states. If the crystal is at a temperature T, this probability is $\{1-\exp(-\Theta/T)\}\exp(-n\Theta/T)$. The average for any value of γ has been found by a method due to Pope,[8] with the result that the differential cross section for scattering of a neutron of initial energy E through an angle φ with loss of energy $\gamma k\Theta$ is

$$\sigma_\gamma(\varphi)=(\sigma_B/4\pi)(1-\gamma/\epsilon)^{\frac{1}{2}}I_{|\gamma|}(q^2/\sinh(\Theta/2T))\times\exp\{-q^2\coth(\Theta/2T)+\gamma\Theta/2T\}. \quad (3)$$

In this formula, σ_B is the bound scattering cross section of the atoms, i.e., $\sigma_B=\sigma_{\text{free}}/(\text{reduced mass of the neutron relative to the atom})^2$,

$$\epsilon=E/k\Theta,$$

$$q^2=(1/2Ak\Theta)\,|\text{vector change in momentum}\,|^2,$$

$$=(m/A)\{2\epsilon-\gamma-2[\epsilon(\epsilon-\gamma)]^{\frac{1}{2}}\cos\varphi\},$$

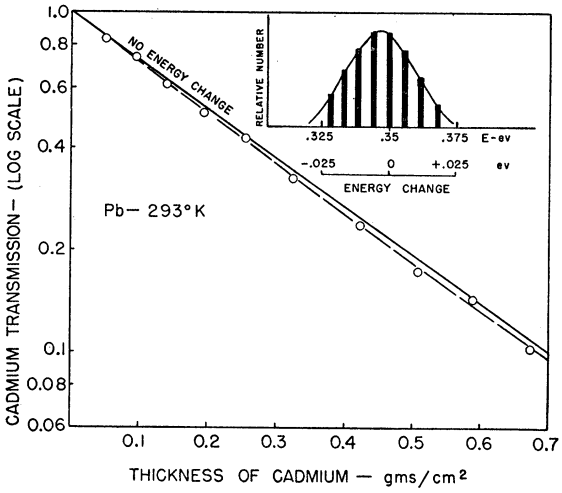

FIG. 4. Results for a lead scatterer. The insert shows the theoretical energy distribution of initially monoenergetic (0.35 ev) neutrons after scattering from a gas (solid line), and an Einstein crystal with $k\Theta=0.00565$ ev (line diagram). In the main diagram the experimental points are plotted together with the transmissions calculated for neutrons with unchanged energy (thick solid line) and with the energy distribution produced by scattering from the Einstein crystal (broken line).

[8] N. K. Pope, Can. J. Phys. (to be published).

where m = mass of neutron, A = mass of atom, and $I_{|\gamma|}(x)$ = modified Bessel function of first kind $= i^{-|\gamma|}J_{|\gamma|}(ix)$. With the introduction of variables $\beta=\frac{1}{2}\Theta/T$ and $x=q^2/\sinh(\frac{1}{2}\Theta/T)$, the cross section may be written

$$\sigma_\gamma(\varphi)=(\sigma_B/4\pi)(1-\gamma/\epsilon)^{\frac{1}{2}}I_{|\gamma|}(x)\exp(-x\cosh\beta+\gamma\beta). \quad (4)$$

Before the application of this formula to the experimental results is discussed, another model of the scatterer will be considered. During calculations on the Einstein crystal model, it was noticed that some of the cross sections and energy losses were almost equal to the corresponding values for free atoms at rest. To improve the free atom model, the free atoms were assumed to form a gas with a Maxwellian distribution of velocities corresponding to the temperature T. The gas gives a closer parallel to the crystal at a temperature T than would an atom at rest; in particular, the neutron can gain energy from the gas. The differential cross section for scattering by a gas of atoms (isotropic scattering in the center-of-mass system) was obtained by suitable modification of formulas given by Spiers.[9] For comparison with the Einstein crystal the energies are written in units of $k\Theta$, although Θ has no significance in the gas. The differential cross section for scattering of a neutron through an angle φ with loss of energy $\gamma k\Theta$ is, per unit interval in γ,

$$\sigma_\gamma(\varphi)(\text{gas})=(\sigma_B/4\pi)[(1-\gamma/\epsilon)(\beta/2\pi)]^{\frac{1}{2}}q^{-1} \times\exp\{-\tfrac{1}{2}\beta(\gamma/q-q)^2\}. \quad (5)$$

The gas cross section is a continuous function of γ with a maximum very close to the point where the energy loss is the same as that due to recoil of an atom

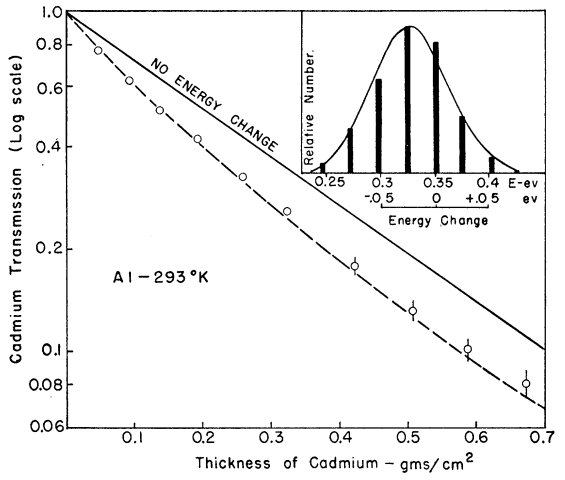

FIG. 5. Results for an aluminum scatterer. The insert shows the theoretical energy distribution of initially monoenergetic (0.35 ev) neutrons after scattering from a gas (solid line), and an Einstein crystal with $k\Theta=0.0259$ ev (line diagram). In the main diagram the experimental points are plotted together with the transmissions calculated for neutrons with unchanged energy (thick solid line), and with the energy distribution produced by scattering from the Einstein crystal (broken line).

[9] J. A. Spiers, National Research Council (Canada) Report CRT-417 (April, 1949) (N.R.C. No. 1940) (unpublished).

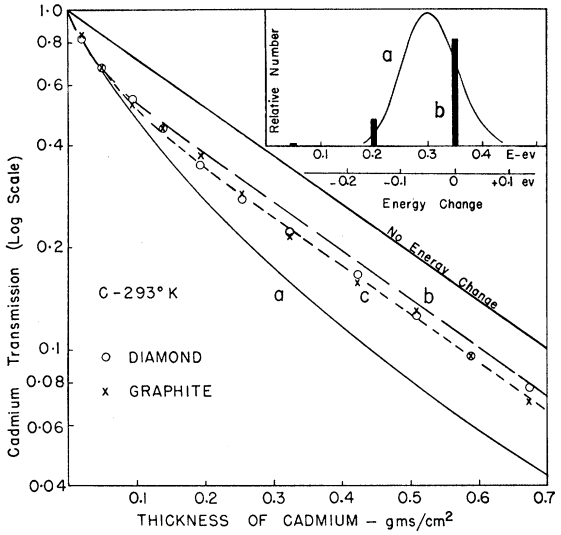

FIG. 6. Results for carbon scatterers. The insert shows the theoretical energy distributions of initially monoenergetic (0.35 ev) neutrons after scattering by a gas (solid line) and by an Einstein crystal with $k\Theta=0.150$ ev (line diagram). In the main diagram the experimental points for diamond and graphite are plotted together with the transmissions calculated for neutrons with unchanged energy (thick solid line) and with energy distributions produced by scattering from (a) the gas, (b) the Einstein crystal with $k\Theta=0.150$ ev, and (c) an Einstein crystal with $k\Theta=0.122$ ev.

initially at rest. The cross section decreases on either side of the maximum. If ΔE is the energy of the scattered neutron measured from the maximum of the cross section, the cross section is small for $(\Delta E)^2/(kTE) \gg 8(1-\cos\varphi)m/A$. The value of x in the region of large cross sections is roughly $(8m/A)(T/\Theta)(E/k\Theta)\sin^2(\varphi/2)$.

The crystal cross section exists only for integral values of γ and is appreciable in the same region as the gas cross section. For $x\gg1$, the two models are indistinguishable by experiments which cannot resolve energies separated by $k\Theta$. This criterion may be approximately expressed as

$$E/k\Theta\gg(A/8m)(\Theta/T)/\sin^2\tfrac{1}{2}\varphi \quad (6)$$

and applies at smaller energies than has previously been assumed.[10]

If $\Theta\gg T$, most of the oscillators will be in the ground state, for which the gas has no analog. Also energy exchange between the neutron and crystal will occur in multiples of $k\Theta$, which is large compared to the dominant exchanges with the gas. Clearly, one would not expect comparison of a gas with such a crystal to be fruitful. Carbon (diamond) is a case in point.

The comparison of the gas and crystal models shows that even when only a few low-lying levels are excited the atoms of the crystal act in scattering neutrons very much like a gas. The specific properties of the binding

[10] G. Placzek, in a recent paper [Phys. Rev. 86, 377 (1952)] has compared the scattering from a bound atom with that from a free atom having the same momentum distribution. He also finds that the generally accepted criterion $E/k\Theta\gg A/m$ is too restrictive.

have little bearing on the scattering provided the binding is not too tight. It is as if the motion in the oscillator levels was equivalent in Doppler effect to the motion of the gas atoms of corresponding energy. From a practical point of view, for application to problems requiring a knowledge of the energy distribution of scattered neutrons, e.g., correction of resonance scattering measurements,[2,7] incomplete knowledge of crystal parameters will have little effect provided the parameters are of suitable magnitude. Over a wider range of parameters than that for which the gas model gives an adequate description of the energy distribution, such quantities as the average energy loss and differential cross section are closely given by the simple model of a free atom at rest.

V. RESULTS

Measurements were made on specimens of lead, aluminum, diamond, and graphite. Figure 3 shows the results where the transmission of the cadmium absorbers is plotted against the thickness of cadmium in g/cm^2. The dashed line shows the transmission for neutrons of unchanged energy (0.35 ev) incident normally on the absorber; the thick solid line shows the expected transmission for neutrons of unchanged energy, after correction for the geometry of the apparatus.

Results for lead are shown as solid circles. Here the energy change is very small, as expected, because of the great mass of the lead atom. The order of the mean energy changes is indicated by the initial slopes of these curves—largest for carbon, next largest for aluminum, and smallest for lead in order of their atomic weights. The results show no significant difference between the two forms of carbon—diamond and graphite. The shapes of the curves for carbon and for aluminum are different indicating that their energy distributions have different forms.

VI. DISCUSSION

Lead

The results for lead are shown again in Fig. 4, together with the expected transmission for neutrons of unchanged energy. The insert shows (as vertical lines) the energy distribution calculated for an Einstein crystal according to Eq. (3). The Einstein temperature was taken to be 0.75 times the Debye temperature obtained from specific heat measurements,[11] and corresponded in this case to $k\Theta=0.00565$ ev.[11] The energy distribution for a monatomic gas, calculated from Eq. (5) is shown in the insert as a solid line. The two distributions are indistinguishable by experiments which cannot resolve energy changes smaller than $k\Theta$, corresponding to the fine structure of the Einstein crystal.

[11] N. F. Mott and H. Jones, *The Theory of the Properties of Metals and Alloys* (Oxford University Press, London, 1936), Chapter 1.

Using this common energy distribution, the transmissions have been calculated from Eqs. (1) and (2), and the theoretical transmission curve is shown in Fig. 4 as a dashed line. Good agreement is obtained, although in this case the agreement is chiefly evidence that the various corrections have been correctly applied.

In this and the following cases, the calculated energy distributions were modified slightly to take account of multiple scattering before computing the theoretical transmissions. This was done by assuming that a fraction $n\sigma_s/2$ of the scattered neutrons of each energy were rescattered with an appropriate energy distribution (σ_s is the scattering cross section of the atoms in the sample, n is the number of atoms per cm^2). Since $n\sigma_s$ has values of from 0.10 to 0.15, this correction was small.

Aluminum

The results for aluminum are shown in Fig. 5, again with the expected transmission for neutrons of unchanged energy. The insert shows the energy distribution for an Einstein crystal ($k\Theta=0.0259$ ev[11]) as vertical lines and that for a gas as a solid line. The two distributions are not exactly the same, since in this case the Einstein temperature was about the same as room temperature, i.e., $T/\Theta\simeq 1$. The cadmium transmissions computed from the distribution for the Einstein crystal are shown as a dashed line. The theoretical and measured transmissions are in agreement within the errors of this experiment.

Carbon

Figure 6 shows the results for diamond and graphite. In the insert the energy distribution for an Einstein crystal of carbon atoms ($k\Theta=0.150$ ev) is shown as vertical lines. This Einstein energy corresponds to a Debye temperature of 2340°K, the generally accepted value.[12] The energy distribution for a carbon gas at room temperature is shown as a solid line. The transmissions calculated from the distribution for a gas are shown as curve (a). This is not at all in agreement with the measurements as is to be expected because the temperature was much lower than the Einstein temperature, i.e., $T/\Theta\ll 1$. The calculated transmissions for the Einstein crystal [curve (b)] are in fair agreement with the measurements.

This agreement can be improved if, instead of defining the Einstein frequency as the simple mean of the Debye frequency distribution,[11] the Einstein temperature is chosen with particular reference to scattering experiments. If $k\Theta$ is taken to be 0.122 ev, the elastic cross section of diamond at room temperature is the same for the Einstein crystal[4] as for a Debye crystal[3] with a Debye temperature of 2340°K. Curve (c) shows

[12] This value is given in reference 11 and elsewhere. Other (somewhat lower) values are also to be found in the literature.

the calculated transmissions for $k\Theta=0.122$ ev, and is in agreement with the experimental points.

VII. CONCLUSIONS

The experimental results show that over a wide range of crystal parameters the Einstein model of a crystal gives sufficiently accurate information about neutron scattering to be of use where the fine details of the scattering are not required. It can give no information about actual energy transfers smaller than $k\Theta$, or angular variations of the order of the separation of Debye-Scherrer rings.

The agreement of the gas model with the Einstein model over a wide region of conditions shows how little influence the binding has on the general scattering pattern; energy and momentum conservation between the neutron and individual nuclei are, in most cases, the over-riding factors. The gas model fails when Bragg scattering, i.e., interference, is important or when the Einstein temperature is large compared with either the temperature or the half-width of the energy distribution.

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Nuclear Reactions in the Stars. I. Proton-Proton Chain

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The rates are calculated for the main reactions making up the "proton-proton chain," whose net effect is the conversion of four hydrogen atoms into one helium atom. These calculations are carried out for temperatures and densities corresponding to central conditions in main sequence stars.

The mean reaction rate for the beta-decay conversion of two protons into one deuteron is calculated accurately, using the latest data on the two-nucleon system and on beta-decay. It is shown that, under normal stellar conditions, the reaction chain is completed by the radiative capture of a proton by a deuteron and by the collision between two of the resultant He^3 nuclei to form one He^4 nucleus and two protons. Values are given for the rate of energy production and for the concentrations of deuterium and He^3 at various temperatures.

I. INTRODUCTION

DETAILED calculations on nuclear reactions in main-sequence stars were presented over a decade ago in two papers by Bethe[1] and by Bethe and Critchfield,[2] hereafter referred to as B and BC, respectively. The present paper is largely a continuation of these papers and, wherever possible, the same notation will be used.

The main aim of the present paper is to revise the work of B and BC on the reaction rates for the chain of reactions starting with the combination of two protons in main-sequence stars, in the light of the most recent experimental information on nuclear reaction rates and Q-values. No detailed calculations on the stellar hydrodynamics of the problem, using any specific stellar model, are carried out. The results are presented largely in the form of reaction rates as a function of temperature, density, and relative abundance of the nuclear species. In later papers, the carbon cycle and reactions in stars which have exhausted their hydrogen supply will be discussed.

Many of the reactions we shall discuss are of the thermonuclear type, i.e., an exothermic reaction undergone by two nuclear species with a Maxwellian velocity distribution colliding with each other. Under stellar conditions, the mean thermal energy kT (of the order of 1 kev for the sun) is extremely small compared with the kinetic energies at which the Coulomb-barrier is negligible (of the order of a few Mev). Consequently, most of the reactions are undergone by nuclei in a fairly narrow energy-region in the tail of the Maxwell distribution (10 to 50 kev for the sun), which we shall call the "stellar energy-region," E_{st}. An approximate formula for p, the number of reactions per second per nucleus of type 2, is then obtained (see for instance B) which can be written in the form

$$p=(4\rho x_1 N_0\Gamma a_0 R_0^2/3^{5/2}\hbar)\times[(A_1+A_2)^{5/3}/A_1^2A_2Z_1Z_2]\tau^2 e^{(\lambda-\tau)}, \quad (1)$$

where

$$\tau=3\left[\frac{\pi^2e^4M_0}{2\hbar^2kT}\frac{Z_1^2Z_2^2A_1A_2}{(A_1+A_2)}\right]^{\frac{1}{3}} \quad (2)$$

and

$$\lambda=4[R_0A_1A_2Z_1Z_2/a_0(A_1+A_2)^{\frac{1}{3}}]^{\frac{1}{4}}. \quad (3)$$

In (1) x_1, x_2 are the concentrations (by weight), A_1, A_2 the atomic weights and Z_1, Z_2 the atomic charges of the two reacting nuclear species. N_0 is Avogadro's number, M_0 one atomic mass unit, ρ the density in g cm^{-3}, and Γ

[1] H. A. Bethe, Phys. Rev. **55**, 434 (1939); B.
[2] H. A. Bethe and C. L. Critchfield, Phys. Rev. **54**, 248 (1938); BC.