Miss D. Rubenfeld for designing Fig. 4. Finally, the author would like to thank his wife for her encouragement.

Note added after completion of the paper: The content of this paper was presented at the Columbus meeting of the American Physical Society, March 22, before which time the details of the paper were worked out. At that meeting, the author obtained a copy of an unpublished paper by W. Martienssen, "Photochemische Vorgänge in Alkalihalogenidkristallen" [see Z. Physik **131**, 488 (1952)]. The suggestion is made in this paper that **x**-rays generate vacancies. Martienssen does not, however, examine in detail the consequences of this idea, nor does he suggest any detail regarding the mechanism of vacancy formation. The production of a strong α-band at 20°K in KBr reported in this paper (see also W. Martienssen, Naturwiss. **38**, 482 (1951)) has caused a slight revision of Sec. **IV**.

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Photoproduction of Mesons in Deuterium*

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The production of positive mesons by photons incident on deuterium is calculated in terms of an effective Hamiltonian containing one term which is independent of and another which depends upon the nucleon spin. The meson spectrum at a given angle to the incident photon beam is evaluated at high photon energies by the closure approximation. At low and intermediate energies the closure approximation is not made, but the neutron-neutron force in the final nucleon state is neglected. These spectra have been integrated over a bremsstrahlung spectrum. The total cross section is found at high photon energies and near the threshold for meson production. It is found that the meson spectrum for small angles is sensitive to the relative size of the spin dependent and spin independent terms.

I. INTRODUCTION

THE production of mesons by photons on deuterium is particularly sensitive to the details of the effective Hamiltonian describing the coupling among the photon, meson and nucleon fields. Its charge dependence is revealed by comparing the production of negative and positive mesons. Its spin dependence affects the variation of the total cross section as a function of the energy of the incident photons, the angular distribution and the energy spectrum of the mesons produced at a given angle. These last effects are a consequence of the Pauli exclusion principle[1] and are thus particularly important at small angles where the recoil neutrons have small relative momenta.

On the other hand, the deuteron is the simplest example of a target with structure. It may therefore be employed to test some of the approximate results given earlier for photo-meson production in nuclei.[2] There are, however, some significant differences from the case of heavy nuclei inasmuch as the mass of the residual nucleus is comparable to that of the particle absorbing the photon.

In the present paper we shall employ the same phenomonological treatment as that employed in reference 2, wherein it is assumed that the meson-photon interaction with a nucleus may be treated as a sum of the interactions with the individual nucleons. This clearly neglects cooperative higher order effects such as those given by exchange currents, and the scattering and absorption of the meson produced by one nucleon by another. These should be small in deuterium because of its relatively large structure and the relatively small nucleon scattering amplitude.[3] Once these assumptions are made, it is possible to affect all spin sums and reduce the calculation to quadratures. Further progress requires some statement on (1) the dependence of the effective Hamiltonian on nucleon momenta and (2) on the nature of the interaction of the two residual neutrons. We have omitted both possibilities for reasons of simplicity. The omission of the first of these may be of importance in computing the negative to positive meson production ratio. The second omission is invalid for final states in which the relative kinetic energy of the nucleons is small, i.e., near threshold, at the high energy end of the meson spectrum, or for mesons produced at small angles. A more precise calculation is now in progress.[4]

A similar treatment of this problem has been simul-

* This paper was presented to the American Physical Society. See Phys. Rev. **82**, 324 (1951).
[1] H. Feshbach and M. Lax, Phys. Rev. **76**, 134, 689 (1949).
[2] M. Lax and H. Feshbach, Phys. Rev. **81**, 761 (1951).
[3] G. Chew and H. Lewis, Phys. Rev. **84**, 779 (1951).
[4] Feshbach, Goldberger, and Villars, private communication.

taneously developed by Chew and Lewis,[3] in which, however, the attention was focused on the distribution of nucleon recoils whereas our primary interest here will be in the energy spectrum of the mesons at a given production angle. Other theoretical investigations have been made by Machida and Tamura[5] and Morpurgo.[6]

II. GENERAL CONSIDERATIONS

We, as well as Chew and Lewis, employ a notation similar to that developed in reference 2. If $\mathbf{D}$ is the initial deuteron momentum, $D_0=(4M^2+D^2)^{\frac{1}{2}}$ its total energy, and $\mathbf{D}'$ the momentum of the center of gravity of the residual nuclear system, the total cross section takes the form

$$d\sigma=(2\pi)^{-2}(1+(D/D_0))^{-1}\int\int|Q|^2 d\boldsymbol{\mu} d\mathbf{D}' d\mathbf{k}\,\delta(\epsilon+\mu_0+(k^2/M)+(D'^2/4M)-\nu_0-(D^2/4M)), \quad (1)$$

where $\boldsymbol{\mu}$ is the meson momentum, μ_0 its energy, k^2/M the final relative kinetic energy, ϵ is the deuteron binding energy, ν_0 is the incident photon energy. The δ-function is just the condition for energy conservation. The matrix element Q is taken between the initial and final nuclear states of the operator:

$$\begin{aligned} T&=T_1+T_2,\\ T_i&=(\boldsymbol{\sigma}_i\cdot\mathbf{K}+L)\exp[i(\boldsymbol{\nu}-\boldsymbol{\mu})\cdot\mathbf{x}_i]\tau_i^{+}. \end{aligned} \quad (2)$$

$\mathbf{K}$ and L will, in general, depend upon the momentum and energy of the three particles: photon, meson, and proton. This form, i.e., its representation as a sum of two terms involving each particle separately, neglects the possibility that T contain a term which depends, in a nonseparable fashion, on the coordinates of both particles, 1 and 2.

$|Q|^2$ must be summed over the final and averaged over the initial spin and isotopic spin states. The initial deuteron is a spin triple, an isotopic spin singlet:

$$|i\rangle=2^{-\frac{1}{2}}[p(1)n(2)-p(2)n(1)]^3\chi_m(u(\rho)/\rho)(2\pi)^{-\frac{3}{2}}\times\exp[i(\mathbf{D}\cdot\mathbf{R})], \quad (3)$$

where $\mathbf{R}=(\mathbf{x}_1+\mathbf{x}_2)/2$ is the center-of-mass coordinate, $\boldsymbol{\rho}=\mathbf{x}_1-\mathbf{x}_2$ the relative coordinate, and ${}^3\chi_m$ is the triplet spin function, m being the z component of the total spin.

The final state consists of two neutrons, an isotopic triplet, with relative momentum k. The Pauli principle requires separate consideration of the two final states, one that is symmetric and one that is antisymmetric in space:

$$|f_o\rangle=(2\pi)^{-\frac{3}{2}}n(1)n(2)^1\chi_0 u_{f,o}(\mathbf{k}\cdot\boldsymbol{\rho})\exp[i\mathbf{D}'\cdot\mathbf{R}], \quad (4a)$$

$$|f_e\rangle=(2\pi)^{-\frac{3}{2}}n(1)n(2)^3\chi_{m'} u_{f,e}(\mathbf{k}\cdot\boldsymbol{\rho})\exp[i\mathbf{D}'\cdot\mathbf{R}]. \quad (4b)$$

We now introduce (2), (4a), or (4b) into Q. The integration over R leads to requirement of conservation of momentum so that after integration over $d\mathbf{D}'$ in (1) one may make the replacement $\mathbf{D}'=\mathbf{D}+\boldsymbol{\nu}-\boldsymbol{\mu}$.

The matrix elements for the symmetric and antisymmetric cases become, respectively,

$$Q_e=2^{-\frac{1}{2}}\langle{}^1\chi_0|\mathbf{K}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)|{}^3\chi_m\rangle E, \quad (5a)$$

$$Q_o=i2^{-\frac{1}{2}}\langle{}^3\chi_{m'}|\mathbf{K}\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)+2L|{}^3\chi_m\rangle O, \quad (5b)$$

where

$$E=\int u_{f,e}^*(\mathbf{k}\cdot\boldsymbol{\rho})\cos(\mathbf{k}_0\cdot\boldsymbol{\rho})[u(\rho)/\rho]d\boldsymbol{\rho}, \quad (6a)$$

$$O=\int u_{f,o}^*(\mathbf{k}\cdot\boldsymbol{\rho})\sin(\mathbf{k}_0\cdot\boldsymbol{\rho})[u(\rho)/\rho]d\boldsymbol{\rho},$$

$$\mathbf{k}_0=\tfrac{1}{2}(\boldsymbol{\nu}-\boldsymbol{\mu}). \quad (6b)$$

One may already see from (5) that only $\mathbf{K}$ is effective in changing the spin state of the deuteron. Moreover, it is also clear from (6) that O approaches zero at threshold ($\mathbf{k}=0$), so that the presence or absence of a spin-flip term $\mathbf{K}$ will be strongly reflected in the energy dependence of Q.

To proceed further, (5a) and (5b) are squared, summed over final, and averaged over initial spin states with the result

$$|Q|^2=\tfrac{2}{3}|\mathbf{K}|^2E^2+[(4/3)|\mathbf{K}|^2+2|L|^2]O^2. \quad (7)$$

The cross section can according to (1) be written in the form

$$d\sigma=(2\pi)^{-2}(1+D/D_0)^{-1}|H|^2d\boldsymbol{\mu}, \quad (8)$$

where

$$|H|^2=\int|Q|^2d\mathbf{k}\,\delta,$$

and

$$\delta=\delta(\epsilon+\mu_0+(k^2/M)+(\mathbf{D}+\boldsymbol{\nu}-\boldsymbol{\mu})^2/4M-\nu_0-(D^2/4M)).$$

Since the final states u_f are either even or odd, it is conventional to carry the integration only over half of k space. We may extend the integration over all of k space providing the normalization of the final states is taken to be

$$\int u_{f,e}^*(\mathbf{k}\cdot\boldsymbol{\rho})u_{f,e}(\mathbf{k}'\cdot\boldsymbol{\rho})d\boldsymbol{\rho}=\tfrac{1}{2}[\delta(\mathbf{k}-\mathbf{k}')+\delta(\mathbf{k}+\mathbf{k}')];$$

$$\int u_{f,o}^*(\mathbf{k}\cdot\boldsymbol{\rho})u_{f,o}(\mathbf{k}'\cdot\boldsymbol{\rho})d\boldsymbol{\rho}=\tfrac{1}{2}[\delta(\mathbf{k}-\mathbf{k}')-\delta(\mathbf{k}+\mathbf{k}')].$$

(Conventional normalization would omit the factor $\frac{1}{2}$ on the right-hand side of these equations.)

III. DIFFERENTIAL SPECTRUM

High Photon Energies

At high photon energies we might expect (1) the deuteron cross section to approach the corresponding

[5] S. Machida and T. Tamura, Prog. Theoret. Phys. 6, 57 (1951).
[6] G. Morpurgo, Nuovo cimento 7, 855 (1950).

hydrogen cross section and (2) the struck proton to absorb all the recoil momentum. The second point is verified, in our calculations by the fact that the matrix elements E and O have a strong resonance near $k=k_0 =(\mathbf{v}-\boldsymbol{\mu})/2$. If we insert $\mathbf{k}=\mathbf{k}_0$, $\mathbf{D}=0$, $\epsilon=0$ into the energy conservation delta-function (8) we obtain the free proton conservation condition: $\delta_p=\delta(\mu_0+(\mathbf{v}-\boldsymbol{\mu})^2/2M-\nu_0)$. The delta-function can now be removed from the integral over k space in (8). The integrals over E^2 and O^2 can be performed exactly with the help of the closure properties,

$$\int u_{f,e}^*(\mathbf{k}\cdot\boldsymbol{\varrho})u_{f,e}(\mathbf{k}\cdot\boldsymbol{\varrho}')d\mathbf{k}=\tfrac{1}{2}[\delta(\boldsymbol{\varrho}-\boldsymbol{\varrho}')+\delta(\boldsymbol{\varrho}+\boldsymbol{\varrho}')],$$

$$\int u_{f,o}^*(\mathbf{k}\cdot\boldsymbol{\varrho})u_{f,o}(\mathbf{k}\cdot\boldsymbol{\varrho}')d\mathbf{k}=\tfrac{1}{2}[\delta(\boldsymbol{\varrho}-\boldsymbol{\varrho}')-\delta(\boldsymbol{\varrho}+\boldsymbol{\varrho}')].$$

The resulting "closure" matrix element,

$$|H|^2=\{[|\mathbf{K}|^2+|L|^2]-[\tfrac{1}{3}|\mathbf{K}|^2+|L|^2]V\}\delta_p, \quad (9)$$

where

$$V=\int \cos(\mathbf{v}-\boldsymbol{\mu})\cdot\boldsymbol{\varrho}[u(\rho)/\rho]^2d\boldsymbol{\varrho}$$

has a first term exactly in agreement with the corresponding free proton matrix element. The second, "two-particle" term contains the interference integral V. If we use the approximate deuteron wave function given later in Eq. (15), V may be evaluated exactly and is given in Fig. 1. V decreases rapidly as $|\mathbf{v}-\boldsymbol{\mu}|$ increases so that for a given ν the two-particle term can be important only in the small angle region. It is, therefore, in this region only wherein the spin dependence of the effective Hamiltonian plays a role.

It may be emphasized that the closure results obtained here do not involve any specific assumptions concerning the final neutron-neutron state. However, the resulting closure cross section may be expected to be an overestimate, since it includes contributions from final states that are not energetically permissible. It should be accurate for high photon energies.

Low and Intermediate Photon Energies

In order to obtain an estimate of the cross section at intermediate or low energies, we must abandon the closure approximation. To simplify our calculations, however, we shall neglect the neutron-neutron interaction and use final states of the form:

$$\begin{aligned} u_{f,e}(\mathbf{k}\cdot\boldsymbol{\varrho})&=(2\pi)^{-\frac{3}{2}}\cos(\mathbf{k}\cdot\boldsymbol{\varrho}),\\ u_{f,o}(\mathbf{k}\cdot\boldsymbol{\varrho})&=(2\pi)^{-\frac{3}{2}}\sin(\mathbf{k}\cdot\boldsymbol{\varrho}).\end{aligned} \quad (10)$$

This approximation affects most seriously the high energy end of the meson spectrum since there the neutrons part with a low velocity. Its effects on the total cross section is not likely to be serious except in

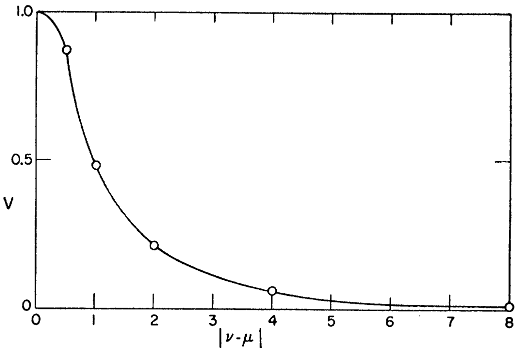

FIG. 1. The function $V(|\mathbf{v}-\boldsymbol{\mu}|)$.

the immediate neighborhood of threshold. (This lack of sensitiveness of the cross section to the choice of final state arises because a partial closure sum must be performed. A complete closure is entirely independent of the choice of final state.)

With the choice (10) E and O may be expressed in the form

$$\begin{aligned} 2E&=C(\mathbf{k}-\mathbf{k}_0)+C(\mathbf{k}+\mathbf{k}_0),\\ 2O&=C(\mathbf{k}-\mathbf{k}_0)-C(\mathbf{k}+\mathbf{k}_0),\end{aligned} \quad (11)$$

where $C(\mathbf{k})$ is the normalized deuteron wave function in momentum space:

$$C(\mathbf{k})=(2\pi)^{-\frac{3}{2}}\int \exp[i\mathbf{k}\cdot\boldsymbol{\varrho}][u(\rho)/\rho]d\boldsymbol{\varrho}. \quad (12)$$

Referring to (7) and (8), $|H|^2$ takes the form

$$|H|^2=(|\mathbf{K}|^2+|L|^2)\int|C(\mathbf{k}-\mathbf{k}_0)|^2\delta d\mathbf{k}$$

$$-(\tfrac{1}{3}|\mathbf{K}|^2+|L|^2)\int C(\mathbf{k}-\mathbf{k}_0)C(\mathbf{k}+\mathbf{k}_0)\delta d\mathbf{k}. \quad (13)$$

The deuteron wave function in ordinary space may be represented by

$$u(\rho)=\left[\frac{\alpha}{2\pi(1-\alpha\rho_1)}\right]^{\frac{1}{2}}[e^{-\alpha\rho}-e^{-\beta\rho}], \quad (14)$$

where $\alpha^2=M\epsilon$, ρ_1 = effective triplet scattering range, 1.74×10^{-13} cm, and β is given by

$$(3/\beta)\simeq\rho_1[1+(4/9)\alpha\rho_1].$$

The corresponding momentum-space wave function is

$$C(\mathbf{k})=\left[\frac{\alpha}{\pi^2(1-\alpha\rho_1)}\right]^{\frac{1}{2}}\frac{\beta^2-\alpha^2}{(\alpha^2+k^2)(\beta^2+k^2)}. \quad (15)$$

We may now determine the energy and angular distribution of the mesons. It is most convenient to

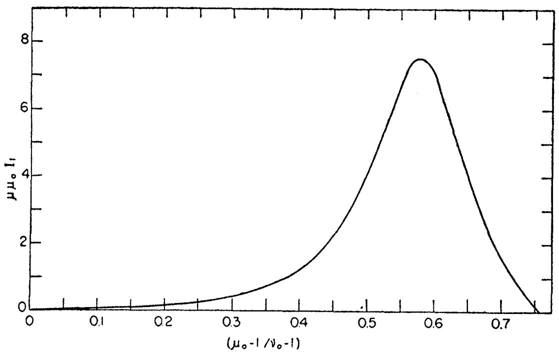

FIG. 2. The function $\mu\mu_0 I_1$ for $\vartheta=90°$ and $\nu=2$.

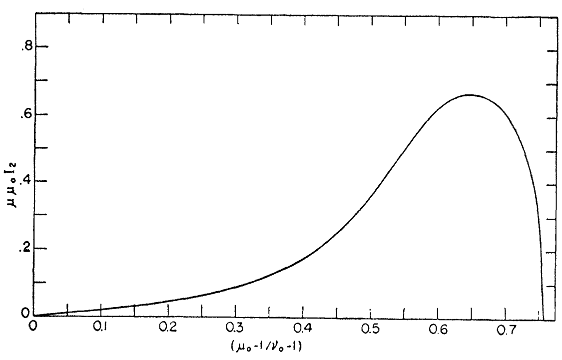

FIG. 3. The function $\mu\mu_0 I_2$ for $\vartheta=90°$ and $\nu=2$.

employ the laboratory system. Hence, from (1) and (3),

$$(d\sigma/d\Omega_\mu d\mu_0)=(2\pi)^{-2}\mu\mu_0[(|\mathbf{K}|^2+|L|^2)I_1 - (\tfrac{1}{3}|\mathbf{K}|^2+|L|^2)I_2], \quad (16a)$$

where

$$I_1=\int |C(\mathbf{k}-\mathbf{k}_0)|^2\delta(\epsilon+\mu_0+(k^2+k_0^2)/M-\nu_0)d\mathbf{k},$$
$$I_2=\int C(\mathbf{k}-\mathbf{k}_0)C(\mathbf{k}+\mathbf{k}_0)\delta(\epsilon+\mu_0 +(k^2+k_0^2)/M-\nu_0)d\mathbf{k}, \quad (16b)$$

where $C(\mathbf{k})$ is given by (15). The integrals I_1 and I_2 may be evaluated exactly:

$$I_1=\frac{2M\alpha k_r}{\pi(1-\alpha\rho_1)}\left[\frac{1}{p_\alpha^4-4k_r^2k_0^2}+\frac{1}{p_\beta^4-4k_r^2k_0^2} -\frac{1}{2k_rk_0(\beta^2-\alpha^2)}\ln\frac{(p_\beta^2-2k_rk_0)(p_\alpha^2+2k_rk_0)}{(p_\beta^2+2k_rk_0)(p_\alpha^2-2k_rk_0)}\right],$$
$$I_2=\frac{M\alpha}{2\pi(1-\alpha\rho_1)}\frac{\beta^2-\alpha^2}{k_0(p_\alpha^2+p_\beta^2)}\left[\frac{1}{p_\alpha^2}\ln\frac{p_\alpha^2+2k_rk_0}{p_\alpha^2-2k_rk_0} -\frac{1}{p_\beta^2}\ln\frac{p_\beta^2+2k_rk_0}{p_\beta^2-2k_rk_0}\right], \quad (17)$$

where

$$k_r^2=M(\nu_0-\mu_0-\epsilon)-k_0^2,$$
$$p_\alpha^2=M(\nu_0-\mu_0-\epsilon)+\alpha^2=M(\nu_0-\mu_0), \quad (18)$$
$$p_\beta^2=M(\nu_0-\mu_0-\epsilon)+\beta^2=M(\nu_0-\mu_0)+\beta^2-\alpha^2.$$

Both I_1 and I_2 go to zero as $k_r=0$, at which μ_0 takes on its maximum value. This maximum value is attained when the maximum available relative energy k_0^2/M is completely converted into meson energy. At this limit,

$$I_1 \underset{k_r\to 0}{\longrightarrow} \frac{M\alpha}{\pi(1-\alpha\rho_1)}\frac{2(\beta^2-\alpha^2)^2}{p_\alpha^4 p_\beta^4}k_r, \quad I_2/I_1 \underset{k_r\to 0}{\longrightarrow} 1. \quad (19)$$

The fraction of the meson energy spectrum for which these limiting values are valid decreases with increasing ν. Examples[7] of the functions $\mu\mu_0 I_1$ and $\mu\mu_0 I_2$ are given in Fig. 2 and Fig. 3. In agreement with the estimate made in reference 2, $I_2 \ll I_1$, except near the maximum meson energy, near threshold, and for the smaller angles.

Thus at the larger angles and energies the differential spectrum is essentially proportional to $[|\mathbf{K}|^2+|L|^2]$. This combination may also be determined experimentally by measurement of photoproduction of mesons in hydrogen. However, in the hydrogen case this information is available at a given angle for a single meson energy only as given by the Compton relation

$$2\nu(\mu_0-\mu\cos\vartheta)=1+2M(\nu-\mu_0). \quad (20)$$

The experimental results for deuterium are therefore valuable because they extend our knowledge of the value of $|\mathbf{K}|^2+|L|^2$ at a given angle to a range of energies.

At the smaller angles where I_2 is appreciable, a different linear combination of $|\mathbf{K}|^2$ and $|L|^2$ is measured so that at the meson energy given by (20) two different linear combinations of $|\mathbf{K}|^2$ and $|L|^2$ are now available, one from hydrogen and the other from deuterium data. Thus, for meson energies and angles obeying the Compton relation $|\mathbf{K}|^2$ and $|L|^2$ may be determined separately.

These considerations are somewhat academic since the synchrotron does not provide monochromatic x-rays. A direct determination of $|\mathbf{K}|^2$ and $|L|^2$ for a given photon energy from experiment would require data obtained with the bremsstrahlung of electrons with several different energies. The available experimental data are not sufficiently accurate to justify such a direct analysis. We therefore adopt the more realistic approach in which a reasonable dependence of $|\mathbf{K}|^2$ and $|L|^2$ on the energy of the photon and meson is assumed permitting then the integration over a bremsstrahlung spectrum and therefore comparison with experiment.

A reasonable form for this dependence is suggested

[7] Tables of the functions $\mu\mu_0 I_1$ and $\mu\mu_0 I_2$ have been calculated for $\vartheta=26°$, 30°, 45°, 90°, 135° and for values of ν ranging from 1.1 to 2.4. These are available in hectographed form from H.F.

by the pseudoscalar meson theory in the nonrelativistic approximation, which gives

$$|\mathbf{K}|^2 \sim (1/\mu_0\nu_0)\left[1-\frac{\mu^2\sin^2\vartheta[1-(\nu_0-\mu_0)^2]}{2\nu_0^2(\nu-\mu\cos\vartheta)^2}\right]. \quad (21)$$

The dominant term in the energy range of interest is $(1/\mu_0\nu_0)$ which is simply a normalization factor. For the purpose of the integration over ν we may make use of the mean value theorem and replace $|\mathbf{K}|^2$ and $|L|^2$ as follows:

$$|\mathbf{K}|^2 \rightarrow \mathcal{K}^2/\mu_0\nu_0, \quad |L|^2 \rightarrow \mathcal{L}^2/\mu_0\nu_0. \quad (22)$$

We have inserted these forms into (16) and assumed a $(d\nu/\nu)$ bremsstrahlung spectrum with a maximum ν of 2.4, which is close to currently available synchrotron energies. These results are given in Fig. 4 and Fig. 5 for several different values of ϑ. The sensitivity of the results at small angles to the spin dependence of the effective Hamiltonian is maintained. Comparison of the small angle results with the hydrogen cross section or with the deuterium cross section for large values of the angle ϑ will determine the values of $\mathcal{K}^2$ and $\mathcal{L}^2$. Experimental evidence[8] indicates that $\mathcal{K}^2 \gg \mathcal{L}^2$, that is, the spin dependent term of the effective Hamiltonian is dominant.

Finally we turn to the examination of the spectrum for $\nu \gg \alpha$ and $\nu \gg \beta$, respectively. The results are most easily seen if we consider (17) in the limit $\beta \rightarrow \infty$; then

$$I_1=\frac{2M\alpha}{\pi}\frac{k_r}{[\alpha^2+(k_r-k_0)^2][\alpha^2+(k_r+k_0)^2]},$$
$$I_2=\frac{M\alpha}{2\pi}\frac{1}{k_0(k_r^2+k_0^2+\alpha^2)}\ln\frac{\alpha^2+(k_r+k_0)^2}{\alpha^2+(k_r-k_0)^2}. \quad (23)$$

For large ν, I_1 approaches the free proton value $\delta_p=\delta[\nu_0-\mu_0-(\nu-\mu)^2/2M]$ while $I_2\rightarrow 0$. Both of these behaviors are apparent from (23). Employing

$$\alpha/[\pi(x^2+\alpha^2)]\rightarrow\delta(x),$$

we see that

$$I_1\rightarrow(M/2k_r)[\delta(k_r-k_0)+\delta(k_r+k_0)] = \delta[(k_r^2-k_0^2)/M]=\delta_p,$$

which is the expected result if the deuteron binding energy is placed equal to zero. This derivation indicates that the free proton value is assumed when $|\mathbf{v}-\boldsymbol{\mu}|\gg\alpha$. This is, of course, true only in the limit $\beta\rightarrow\infty$. If β is kept finite, then the deuteron momentum wave function does not have as great an extension in momentum space so that the free proton value is assumed somewhat earlier.

IV. TOTAL CROSS SECTION

The total cross section will be evaluated in two energy regions (1) photon energies well above threshold and (2) photon energies near the threshold for meson production.

[8] Lebow, Feld, Frisch, and Osborne, Phys. Rev. 85, 681 (1952).

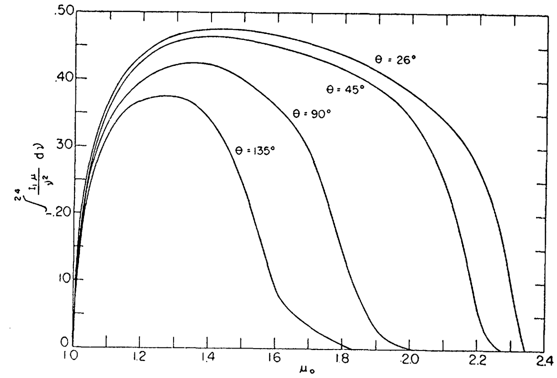

FIG. 4. The integral $\int_1^\nu(\mu I_1/\nu^2)d\nu$. To obtain the meson spectrum multiply the ordinate by $\mathcal{K}^2+\mathcal{L}^2$ and subtract $\frac{1}{3}\mathcal{K}^2+\mathcal{L}^2$ times the ordinate of the corresponding curve plotted in Fig. 5.

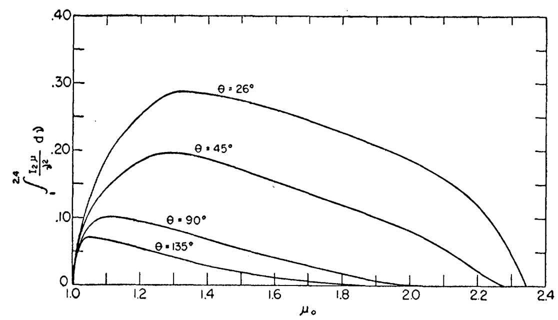

FIG. 5. The integral $\int_1^\nu(\mu I_2/\nu^2)d\nu$.

Cross Sections for High Photon Energies

A first approximation to the cross section at high energies may be based on the closure approximation (9). Comparison with the free proton cross section gives the ratio

$$\frac{\sigma_D}{\sigma_p}\rightarrow 1-\frac{\int(\frac{1}{3}|\mathbf{K}|^2+|L|^2)V\delta_p d\boldsymbol{\mu}}{\int(|\mathbf{K}|^2+|L|^2)\delta_p d\boldsymbol{\mu}}. \quad (24)$$

The second term gives the reduction of the cross section for large values of ν because of the Pauli exclusion principle and other interference effects.

There is another reduction of the cross section which enters because of the assumption on the k integration which was made in obtaining (24). It is only asymptotically true that the k integration may be extended over all of k space. Indeed, for energies sufficiently close to threshold the integration in k space is restricted by momentum and energy considerations. We may obtain an estimate of this effect by employing a technique developed in reference 2. Consider first the single

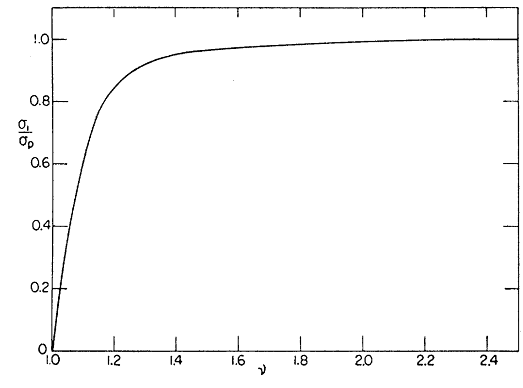

FIG. 6. The ratio of the "single" particle contribution to the total production cross section and the total production cross section in hydrogen.

particle contribution σ_1. Then from (13),

$$\sigma_1=(2\pi)^{-2}\int\int d\mathbf{\mu}d\mathbf{k}(|\mathbf{K}|^2+|L|^2)|C(\mathbf{k}-\mathbf{k}_0)|^2 \times\delta(\mu_0+\epsilon+(k^2+k_0^2)/M-\nu_0). \quad (25)$$

We now change the variable of integration to $\mathbf{k}'=\mathbf{k}-\mathbf{k}_0$ and do the $\mathbf{\mu}$ integration first. The result may be expressed in terms of an integration of $|C(\mathbf{k}')|^2$ over an effective cross section:

$$\sigma_1=\int d\mathbf{k}'|C(\mathbf{k}')|^2\sigma(\mathbf{v},\mathbf{k}',\nu_0), \quad (26)$$

where

$$\sigma(\mathbf{v},\mathbf{k}',\nu_0)=(2\pi)^{-2}\int d\mathbf{\mu}(|\mathbf{K}|^2+|L|^2)\delta(\mu_0+\epsilon +((\mathbf{k}'+\mathbf{k}_0)^2+k_0^2)/M-\nu_0).$$

The range of integration on $\mathbf{k}'$ is limited by the energy delta-function to a sphere,

$$|\mathbf{k}'+(\mathbf{v}/2)|\lesssim[M(\nu_0-\nu_t)]^{\frac{1}{2}}=d, \quad (27)$$

where ν_t is the threshold energy. We note that for large ν, $C(\mathbf{k}')$ has a strong maximum at $k'=0$, so that $\sigma(\mathbf{v},\mathbf{k}',\nu_0)$ may be approximated by $\sigma(\mathbf{v},0,\nu_0)$, in other words, by the free proton cross section. Hence,

$$\sigma_1/\sigma_p \underset{\nu\gg 1}{\longrightarrow} \int|C(\mathbf{k}')|^2 d\mathbf{k}', \quad (28)$$

where the range of integration is limited according to (27). Roughly speaking, as (27) and (28) show, we are concerned with the overlap between two spheres in momentum space, one centered at the origin and of radius of the order of α, the other centered at $(-\nu/2)$ of radius d. The closure approximation will be valid when these spheres overlap completely, that is, when d is several times greater than $(\alpha+\nu/2)$ or for ν at about 200 Mev. Integration of (28) yields

$$\pi(1-\alpha\rho_1)\frac{\sigma_1}{\sigma_p}=\frac{\beta^2+3\alpha^2}{\beta^2-\alpha^2}\left[\tan^{-1}\frac{d-\frac{1}{2}\nu}{\alpha}+\tan^{-1}\frac{d+\frac{1}{2}\nu}{\alpha}\right] -\frac{\alpha(\alpha^2+3\beta^2)}{\beta(\beta^2-\alpha^2)}\left[\tan^{-1}\frac{d-\frac{1}{2}\nu}{\beta}+\tan^{-1}\frac{d+\frac{1}{2}\nu}{\beta}\right] -\frac{\alpha}{\nu}\left[1+\frac{2(d^2-\frac{1}{4}\nu^2+\alpha^2)}{\beta^2-\alpha^2}\right]\ln\frac{\alpha^2+(d+\frac{1}{2}\nu)^2}{\alpha^2+(d-\frac{1}{2}\nu)^2} -\frac{\alpha}{\nu}\left[1-\frac{2(d^2-\frac{1}{4}\nu^2+\beta^2)}{\beta^2-\alpha^2}\right]\ln\frac{\beta^2+(d+\frac{1}{2}\nu)^2}{\beta^2+(d-\frac{1}{2}\nu)^2}. \quad (29)$$

A curve giving σ_1/σ_p as a function of ν is plotted in Fig. 6, showing that 90 percent of the high energy limit is obtained at $\nu=1.3$ or at $\nu=180$ Mev.

From σ_1/σ_p we must subtract σ_2 as obtained from

$$\sigma_2=(2\pi)^{-2}\int(\tfrac{1}{3}|\mathbf{K}|^2+|L|^2)I_2 d\mathbf{\mu}. \quad (30)$$

It was not possible to evaluate σ_2 analytically. Instead the following approximate evaluation was employed:

$$\sigma_2\simeq\sigma_2(\text{closure})R,$$

where

$$R=\lim(\sigma_2/\sigma_2(\text{closure})) \quad \text{as} \quad \alpha\to 0 \quad \text{and} \quad \beta\to\infty.$$

The quantity σ_2(closure) is given in Eq. (24) as $\int(\frac{1}{3}|\mathbf{K}|^2+L^2)V\delta_p d\mu$ where V is given in Eq. (9). Both σ_2(closure) and R may be easily evaluated if Eq. (22) is inserted for $|\mathbf{K}|^2$ and L^2.

The ratio $\sigma_2(\mathcal{K}^2+\mathcal{L}^2)/\sigma_p(\frac{1}{3}\mathcal{K}^2+\mathcal{L}^2)$ is plotted in Fig. 7. We see that it decreases like a (constant/ν) for ν large, rises to a maximum rather near threshold, and then drops to zero at threshold. We note that σ_2 is much smaller than σ_1, being at most a ten percent correction except near threshold for the case of no spin flip $K=0$, and being at most on third of this for $L=0$. Hence, in the deuteron the effect of the Pauli exclusion principle on the total cross section is negligible except near threshold in accordance with the assumptions made on reference 2.

Cross Section Near Threshold

We shall only quote the results of our calculations, since the approximation neglecting the neutron-neutron interaction is particularly poor here. We find that

$$\sigma_D/\sigma_p\simeq\frac{\pi^2}{3\sqrt{2}}\frac{(M+1)^2}{(2M+1)^{\frac{1}{2}}}\frac{\langle|\mathbf{K}|^2\rangle}{\langle|\mathbf{K}|^2\rangle+\langle|L|^2\rangle} \times|C(-\nu_t/2)|^2(\nu-\nu_t)^{\frac{3}{2}}, \quad (31)$$

where we have neglected the difference between free proton and deuteron thresholds. The quantities $\langle|\mathbf{K}|^2\rangle$

and $\langle |L|^2\rangle$ are the averages over angle. Inserting numerical values, (31) becomes

$$\sigma_D/\sigma_p \simeq 14.3(\nu-\nu_t)^{\frac{3}{2}}\langle|\mathbf{K}|^2\rangle/\langle|\mathbf{K}|^2+|L|^2\rangle. \tag{32}$$

We are indebted to G. Chew for many informative discussions. *Note added in proof:* Calculations similar to those of this paper are reported by Saito, Watanabe, and Yamaguchi, Prog. Theoret. Phys. **7**, 103 (1952).

APPENDIX

A comparison of the experimental ratio of photomeson production in deuterium to that in hydrogen cannot be made directly against the cross-sectional ratio σ_D/σ_p. The theoretical cross sections must first be averaged over a bremsstrahlung spectrum before taking the ratio. Using the assumption of Eq. (22), the experimental ratio of differential cross sections is to be compared with

$$\frac{\int (d\sigma_D/d\Omega_\mu d\mu_0)(d\nu/\nu)}{\int (d\sigma_p/d\Omega_\mu d\mu_0)(d\nu/\nu)} = \frac{\int (\mu I_1/\nu^2)d\nu - \gamma\int (\mu I_2/\nu^2)d\nu}{\int (\mu I_p/\nu^2)d\nu}, \tag{A1}$$

where

$$\gamma = (\tfrac{1}{3}\mathcal{K}^2+\mathcal{L}^2)/(\mathcal{K}^2+\mathcal{L}^2), \tag{A2}$$

$$I_p = \delta((M^2+(\boldsymbol{\nu}-\boldsymbol{\mu})^2)^{\frac{1}{2}}+\mu_0-\nu_0-M), \tag{A3}$$

and we have used Eq. (5) reference 2 for the proton cross section. (A factor $(2\pi)^{-2}(\mathcal{K}^2+\mathcal{L}^2)$ was cancelled in simplifying the right-hand side of (A1).

The numerator of (A1) is supplied by Eq. (17) and Figs. 4 and 5. The denominator can be integrated immediately:

$$\int (\mu I_p/\nu^2)d\nu = (M+\nu_c-\mu_0)\mu/[(M-\mu_0+\mu\cos\vartheta)\nu_c^2], \tag{A4}$$

where ν_c is the Compton photon energy (20),

$$\nu_c = \frac{\mu_0-(1/2M)}{1-(\mu_0-\mu\cos\vartheta)/M}. \tag{A5}$$

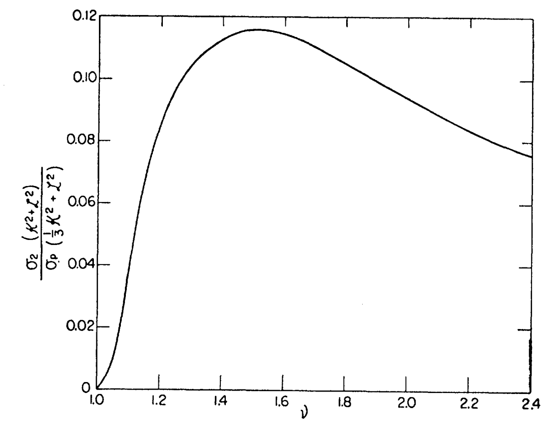

FIG. 7. $\sigma_2(\mathcal{K}^2+\mathcal{L}^2)/\sigma_p(\frac{1}{3}\mathcal{K}^2+\mathcal{L}^2)$.

Inserting (A5) into (A4) we obtain

$$\int (\mu I_p/\nu^2)d\nu = \mu\left(1-\frac{\mu_0}{M}\right)\left(1-\frac{\mu_0-\mu\cos\vartheta}{M}\right)\left(\mu_0-\frac{1}{2M}\right)^{-2} + M^{-1}\mu[\mu_0-(1/2M)]^{-1}, \tag{A6}$$

the lower limit on the spectrum (A6) is $\mu_0=1$. The upper limit is $\bar{\mu}_0=\mu_0^c(\bar{\nu},\vartheta)$ where $\mu_0^c(\nu,\vartheta)$ is the Compton meson energy obtained by solving (A5) for μ_0:

$$\mu_0^c = \frac{[\nu+(1/2M)][1+(\nu/M)] \pm (\nu/M)\cos\vartheta[(\nu+(1/2M))^2-\lambda]^{\frac{1}{2}}}{\lambda}, \tag{A7}$$

where $\lambda = 1+2(\nu/M)+(\nu/M)^2\sin^2\vartheta$ and $\bar{\nu}$ is the upper end of the bremsstrahlung spectrum.

A comparison of the angular distribution in the deuteron and proton case requires that we evaluate $\int(\mu I_p/\nu^2)d\nu d\mu_0$. Expanding (A6) in inverse powers of M, the result to terms of order M^{-2} is

$$\begin{aligned}\int (\mu I_p/\nu^2)d\nu d\mu_0 = {} & \ln(\mu+\mu_0)-(\mu/\mu_0) \\ & + M^{-1}[(\mu_0+\mu_0^{-1}-2)\cos\vartheta-\mu \\ & + (3/2)\cos^{-1}(1/\mu_0)-(\mu/2\mu_0^2)] \\ & + \tfrac{1}{2}M^{-2}[\mu_0\mu-\cos\vartheta(\mu_0^2-\mu_0^{-2} \\ & - 4\ln\mu_0)-\tfrac{1}{2}(\mu/\mu_0)-\tfrac{1}{2}(\mu/\mu_0^3)],\end{aligned} \tag{A8}$$

where μ_0 is to be replaced by its upper limit $\bar{\mu}_0=\mu_0^c(\bar{\nu},\vartheta)$ using (A7) with $\nu=\bar{\nu}$, and μ is to be replaced by $(\bar{\mu}_0^2-1)^{\frac{1}{2}}$.