Elastic Scattering of 9.7-Mev Protons by Deuterium and by Hydrogen*

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The proton-deuteron and proton-proton differential scattering cross sections have been measured with a 9.7-Mev cyclotron external proton beam and with photographic plates as detectors of the scattered particles. In both interactions the angular distributions are similar to those observed at lower energies, and the total cross sections are in good agreement with reasonable extrapolations of the low energy data. The *p-d* data show some evidence for deuteron disintegration. Neglecting spin-orbit coupling, the *p-p* data are consistent with pure S wave scattering with a phase shift of $57.8°\pm1.2°$.

I. INTRODUCTION

THE investigation of *p-d* scattering was undertaken for the purpose of extending to 10 Mev the series of precise cross-section measurements made at Los Alamos and elsewhere in the energy region below 5.2-Mev incident proton energy.[1-4] It was hoped that accurate *p-d* scattering data up to 10 Mev would permit a check on existing theories[5-7] of the *p-d* interaction and aid in the development of new theories where existing theories appear inadequate.

The investigation of *p-p* scattering was originally performed to check the background determination in the *p-d* experiment. Because of the lack of data in this energy range, it seemed desirable to report our results in detail, although the accuracy attained was no better than about 4.5 percent.

II. EXPERIMENTAL PROCEDURE

Thin gas targets, spectroscopically pure, of deuterium for one experiment and hydrogen for the other, were bombarded with 9.7-Mev protons from the external beam of the Berkeley 60-inch cyclotron. The scattered and recoil nuclei were detected simultaneously over 2.5° intervals on photographic plates in a multiplate camera which has been previously described in detail.[8] In the *p-d* scattering investigation the range covered extended from laboratory angles of 10° to 170° with respect to the direction of the incident proton beam; in the *p-p* scattering experiment an upper limit of 60° in the laboratory system was imposed by the energetics of the interaction.

The direction and energy of the incident proton beam were determined for each run by a method previously described.[9] The energy averaged over all six *p-d* runs was 9.7 ± 0.15 Mev. The calculated energy for the last run of the *p-p* scattering experiment was consistently higher than that for the first two. Hence the cross-section values are listed separately instead of being averaged with the others. The average energy for runs *PP*-4 and *PP*-5 was 9.7 ± 0.15 Mev; that for *PP*-6 was $9.8_5\pm0.15$ Mev.

From examination of range distributions, the width at half-maximum of the incident proton beam was determined to be less than 0.15 Mev. The maximum divergence of the incident proton beam due to geometry of the system was 1.2° in the horizontal direction and 0.85° in the vertical direction. Protons scattered by more than 3.5° were prevented from entering the scattering chamber by a collimating system built into the camera. The angular resolution of the detection slit system was $\pm0.8°$.

Magnetic shielding was required to prevent the external proton beam from being deflected into one side of the magnetic yoke by the fringing field of the cyclotron. A 5-ft long channel of soft iron of rectangular cross section with the large face perpendicular to the magnetic field of the cyclotron provided shielding sufficient to reduce the fringing field of the magnet near the cyclotron tank to 10^3 oersted. Additional shielding of the camera itself by a soft iron case $\frac{1}{2}$-inch thick reduced the field inside the scattering chamber to 7 oersted.[10] The collimated proton beam entered the camera through a 0.5-mil Duraluminum window over the entrance port, passed through another 0.5-mil Dural window over the exit port, and was finally collected in an electrostatically shielded Faraday cup.

With the exception of the measurement of integrated current, the procedure for making a run was identical with that previously described.[4,8] A number of such runs were made with both H_2 and D_2 at different pressures and with various values of the integrated current in order to provide suitable track densities at all scattering angles and in order to check the purity and temperature constancy of the gas, the accuracy of the current inte-

* Work performed under the auspices of the AEC.
† Now at the Naval Research Laboratory, Washington, D. C.
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3 Karr, Bondelid, and Mather, Phys. Rev. **81**, 37 (1951).
4 Louis Rosen and J. C. Allred, Phys. Rev. **82**, 777 (1951).
5 R. A. Buckingham and H. S. W. Massey, Proc. Roy. Soc. (London) **A179**, 123 (1941).
6 Buckingham, Hubbard, and Massey, Proc. Roy. Soc. (London) **A211**, 183 (1952).
7 J. L. Gammel, Phys. Rev. **78**, 321 (1950).
8 Allred, Rosen, Tallmadge, and Williams, Rev. Sci. Instr. **22**, 191 (1951).
9 Rosen, Tallmadge, and Williams, Phys. Rev. **76**, 1283 (1949).
10 Details of this arrangement are given in a paper by T. M. Putnam to be published in a forthcoming issue of the *Review of Scientific Instruments*.

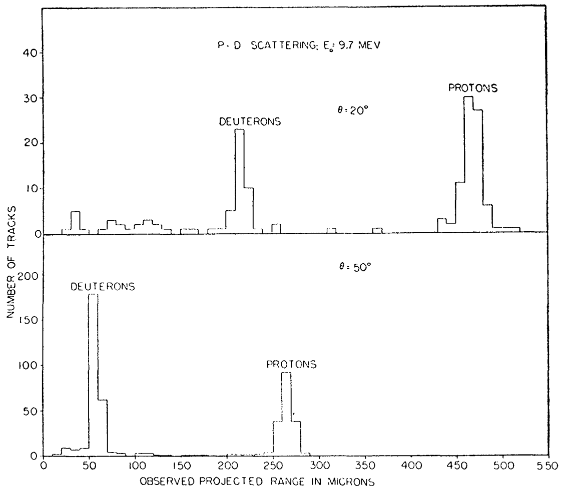

FIG. 1. Observed range distributions of scattered protons and recoil deuterons at laboratory angles of 20° (above) and 50° (below).

grator, and the multiple small angle scattering in the target gas. Six satisfactory runs were obtained with deuterium and three with hydrogen as the target. In those with deuterium the pressure ranged from a minimum of 5-cm Hg to a maximum of 21-cm Hg, and integrated current from a corresponding minimum of 2 microcoulombs to a maximum of 22 microcoulombs. For all three runs with the hydrogen target, the pressure was 5-cm Hg, while integrated current varied from 2 to 10 microcoulombs.

Several background runs without gas in the camera were made for the purpose of determining the background corrections for multiple scattering by the slits collimating the beam and defining the scattering angle, and for scattering by water vapor liberated from the emulsion.

The number of protons which passed through the scattering chamber during an exposure was determined by measuring the total charge accumulated on the faraday cup, the interior of which was at a pressure of 5×10^{-7}-mm Hg. In order to suppress secondary electrons from the aluminum foil which isolated the Faraday cup from the scattering chamber, a negative potential

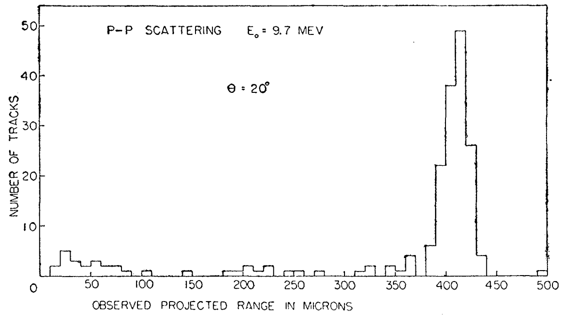

FIG. 2. Observed range distribution of protons scattered by protons.

TABLE I. Differential cross-section values in the center-of-mass system as a function of angle for *p-d* scattering at 9.7-Mev proton bombarding energy.

Center-of-mass angle (Degrees)	$\sigma(\Omega)$ (Barns)	Center-of-mass angle (Degrees)	$\sigma(\Omega)$ (Barns)
18.7	0.152 ±0.007	114.9	0.0231±0.0009
22.4	0.147 ±0.006	117.4	0.0208±0.0010
26.2	0.150 ±0.006	120.0	0.0206±0.0008
29.8	0.145 ±0.006	120.0	[a]0.0217±0.0010
33.6	0.145 ±0.006	122.4	0.0205±0.0009
37.2	0.127 ±0.005	124.9	0.0221±0.0009
44.5	0.122 ±0.005	127.2	0.0230±0.0010
50.0	[a]0.106 ±0.004	129.5	0.0246±0.0010
51.7	0.114 ±0.004	130.0	[a]0.0268±0.0011
55.2	0.104 ±0.004	131.7	0.0287±0.0012
58.8	0.0932±0.0035	133.9	0.0318±0.0013
60.0	[a]0.0967±0.0036	135.0	[a]0.0367±0.0015
65.0	[a]0.0920±0.0035	136.0	0.0350±0.0014
65.7	0.0835±0.0034	138.0	0.0392±0.0016
69.1	0.0826±0.0032	140.0	[a]0.0454±0.0017
70.0	[a]0.0817±0.0030	141.9	0.0510±0.0020
72.5	0.0699±0.0027	145.0	[a]0.0633±0.0027
79.2	0.0610±0.0024	145.6	0.0647±0.0027
80.0	[a]0.0645±0.0024	149.2	0.0780±0.0031
82.4	0.0592±0.0025	150.0	[a]0.0817±0.0029
85.0	[a]0.0557±0.0022	152.5	0.0882±0.0036
85.6	0.0521±0.0020	155.0	[a]0.103 ±0.005
90.0	[a]0.0484±0.0020	155.7	0.106 ±0.004
91.9	0.0451±0.0018	158.8	0.132 ±0.005
98.0	0.0372±0.0015	161.7	0.146 ±0.006
100.0	[a]0.0364±0.0014	164.5	0.162 ±0.006
103.9	0.0304±0.0012	167.2	0.180 ±0.007
105.0	[a]0.0301±0.0012	169.8	0.184 ±0.010
106.7	0.0278±0.0013	171.2	0.190 ±0.010
109.5	0.0247±0.0010	172.4	0.193 ±0.012
110.0	[a]0.0268±0.0010	173.7	0.180 ±0.012
112.2	0.0233±0.0010	175.0	0.190 ±0.015
		176.2	0.195 ±0.015

[a] Values obtained from recoil deuterons.

of 135 volts with respect to ground was placed on the current collector. An excessive leakage current between the outside of the current collector and ground was

TABLE II. Differential cross-section values in the center-of-mass system as a function of angle for *p-p* scattering at 9.7- and 9.85-Mev proton bombarding energies.

Center-of-mass angle (Degrees)	$\sigma(\Omega)$ at 9.7±0.15 Mev (Barns)	$\sigma(\Omega)$ at 9.85±0.15 Mev (Barns)
24		0.0592±0.0026
26		0.0555±0.0026
29	0.0517±0.0023	0.0536±0.0025
31		0.0509±0.0023
34	0.0528±0.0021	0.0496±0.0023
41	0.0543±0.0021	0.0548±0.0025
44	0.0528±0.0026	0.0568±0.0029
51	0.0543±0.0021	
54	0.0554±0.0023	
61	0.0554±0.0023	0.0541±0.0025
64	0.0567±0.0025	
71	0.0588±0.0024	
74	0.0599±0.0029	
81	0.0582±0.0023	
84	0.0578±0.0023	
91	0.0585±0.0024	
94	0.0584±0.0024	
101	0.0567±0.0022	
104	0.0599±0.0025	
111	0.0582±0.0024	
114	0.0578±0.0027	
121	0.0564±0.0024	

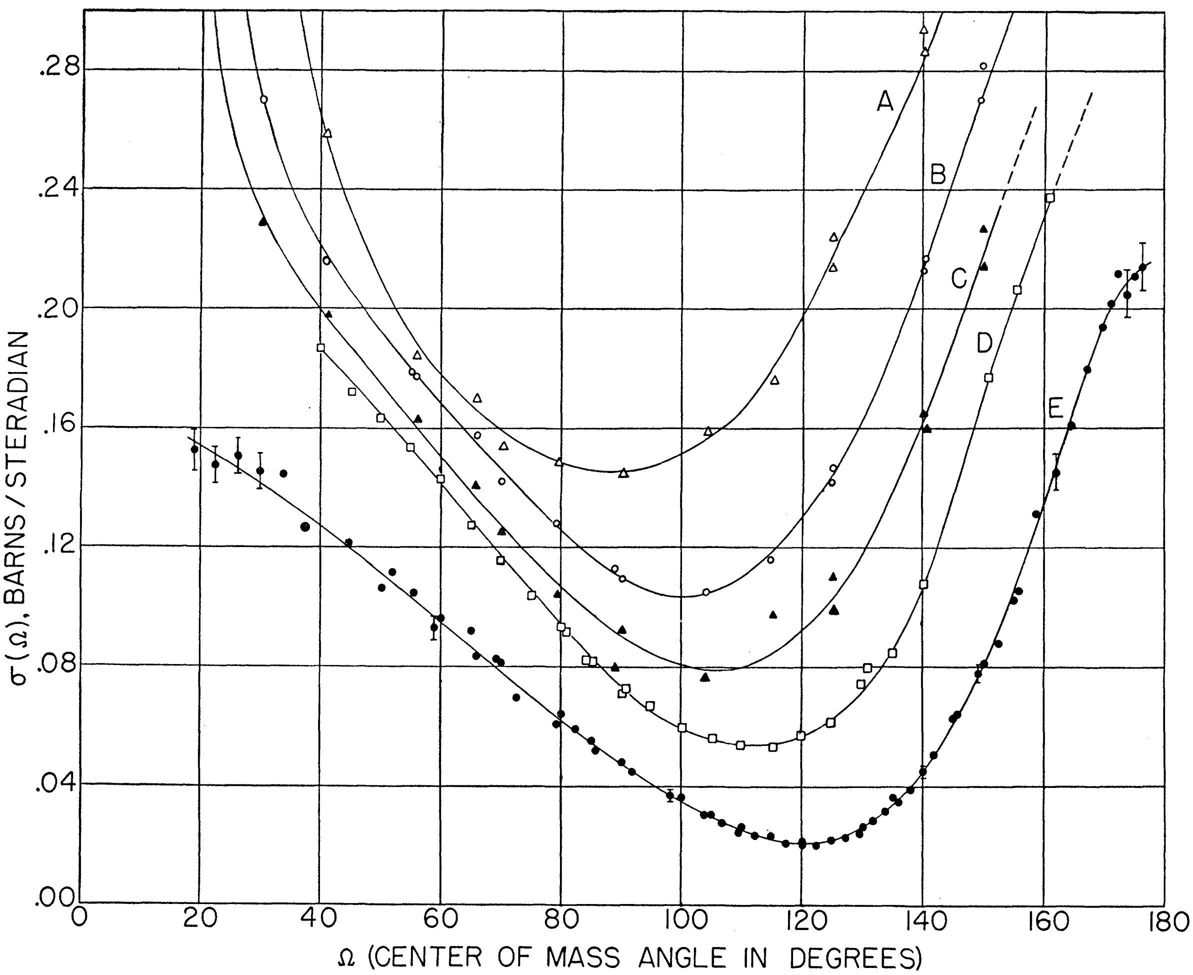

FIG. 3. Measured differential cross sections *versus* center-of-mass angle for p-d scattering, for protons of bombarding energy 1.5 to 9.7 Mev. A, B, C, Sherr *et al.* (reference 1), $E_p=1.51$, 2.53, 3.49 Mev, respectively; D, Rosen and Allred (reference 4), $E_p=5.18$ Mev; E, present data, $E_p=9.7$ Mev.

observed and found to be due to ionization of the air by the intense gamma-ray background of the cyclotron. This leakage current was eliminated by an electrostatic shield placed around the Faraday cup and maintained at its potential.

The current integrator used was made available to us by the Crocker Radiation Laboratory and has been described by Cork, Johnston, and Richman.[11] It is essentially a bridge device in which the Faraday cup is connected to one plate of a standard condenser, and the potential applied by a slide-rack voltmeter. The capacitance of the condenser (1.075 ± 0.005 μf) was determined by the Bureau of Standards and was checked against a secondary standard before and after the experiment. No corrections to the current integration data were made.

III. ANALYSIS OF DATA AND EVALUATION OF ERRORS

The photographic plates were analyzed in the same manner as in the d-p scattering experiment of Rosen and Allred.[4] After making certain corrections which are described below, the differential cross sections are obtained directly from the number of tracks of proper range per unit swath width observed on the plates. Figure 1 shows typical range distributions of the scattered protons and recoil deuterons for the p-d interaction; and Fig. 2 shows a characteristic distribution of proton ranges resulting from the p-p interaction.

In both experiments a considerable amount of low energy background is apparent at the smaller angles. This is due partly to interaction particles whose energy has been degraded by penetration of the slit edges, and partly to protons from the incident beam which have penetrated or been scattered by the beam-collimating slits. Comparison of the p-d with the p-p data shows that a part of the "background" in the p-d interaction can be attributed to the disintegration of the deuteron. The correction for background tracks originating from all these sources was based on the shape of the range distribution in the neighborhood of the peak. In the p-p interaction it never exceeded 3 percent. No such

[11] Cork, Johnston, and Richman, Phys. Rev. 79, 71 (1950).

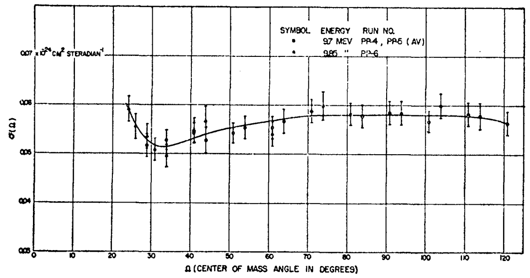

FIG. 4. Measured differential cross section *versus* center-of-mass angle for *p-p* scattering, for protons of bombarding energy 9.7 and 9.8_5 Mev.

correction was necessary for the proton data of the *p-d* interaction. The data obtained from the short range recoil deuterons, however, were in general subject to background corrections of the order of 7 percent; the uncertainty in this background correction accounts for almost the entire error in the cross-section values obtained from the recoil deuteron data. The range distributions showed that above 20° no correction was necessary for scattering by heavy gas impurities such as oxygen.

The errors which were common to all the runs of these two experiments are estimated as follows: geometry of slit system of multiplate camera, 1.0 percent; measurements of temperature and pressure, 0.5 percent; current integration, 2.5 percent; microscope calibration and personal factors, 1.0 percent. The errors which varied with angle are the statistical error and the uncertainty in assignment of background correction. The average uncertainty in the background corrections is 2 percent. Since in general at least 2500 tracks were counted at each angle, the statistical error is 2 percent or less. Thus, the absolute error varies from a minimum of 3.5 percent, at angles where background correction is unnecessary, to an average of 5 percent, at the angles where the correction is needed. In both experiments the greatest errors occur in the values for the smallest and the largest angles where large background corrections introduce the greatest uncertainties.

The final results for the differential elastic scattering cross sections as a function of angle are given in Tables I and II for the *p-d* and *p-p* interactions, respectively.

In Fig. 3 are shown our measured differential cross-section values for *p-d* scattering at 9.7 and 5.2 Mev, together with the measurements of Sherr *et al.* (reference 1) from 1.5 to 3.5 Mev. The errors in the values of our 9.7-Mev data are indicated at the extreme ends of the curve, where the uncertainties are largest, as well as for representative points at other parts of the curve. The values calculated from the recoil deuterons are in general slightly higher than those obtained from the scattered protons. The difference is probably due to errors in applying the background corrections.

The angular distributions of cross section for *p-p* scattering at 9.7 and 9.85 Mev are shown in Fig. 4. Although the values obtained at the higher energy were not averaged with the others, a single smooth curve has been drawn through the points.

IV. INTERPRETATIONS OF THE DATA

A. *p-d* Interaction

Buckingham, Hubbard, and Massey[6] have recently extended the prevous work of Buckingham and Massey[5] on *n-d* scattering to *p-d* scattering. In their later paper these authors have calculated the differential cross section for *p-d* scattering at various energies, taking into account relative angular momenta up to $l=2$. We have interpolated their phase shifts for our energy ($k=4.38\times10^{12}$ cm^{-1}) and calculated the differential cross section which Buckingham, Hubbard, and Massey would predict on their MHWB model (symmetrical exchange force).

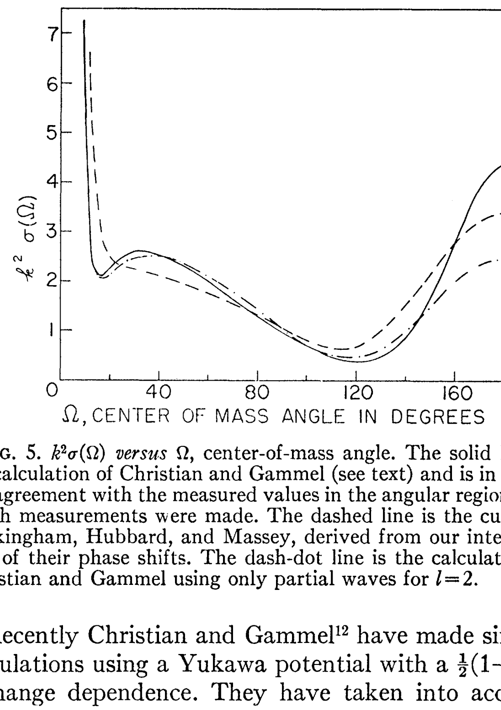

FIG. 5. $k^2\sigma(\Omega)$ *versus* Ω, center-of-mass angle. The solid line is the calculation of Christian and Gammel (see text) and is in essential agreement with the measured values in the angular region over which measurements were made. The dashed line is the curve of Buckingham, Hubbard, and Massey, derived from our interpolation of their phase shifts. The dash-dot line is the calculation of Christian and Gammel using only partial waves for $l=2$.

Recently Christian and Gammel[12] have made similar calculations using a Yukawa potential with a $\frac{1}{2}(1+P_x)$ exchange dependence. They have taken into account partial waves of order higher than $l=2$. The results of the two calculations are shown in Fig. 5. The solid line is the calculated cross section of Christian and Gammel which includes waves of higher order. This curve fits the experimental data, in the region in which measurements were made, with the rms deviation of 2 percent and a maximum deviation of 4 percent. The dashed line is the cross section of Buckingham, Hubbard, and Massey calculated from our interpolation of their phase shifts. It is seen that while there is agreement with the data in a general way, the fit is not nearly so good as that of Christian and Gammel. In the figure, the dash-dot line shows the result of a calculation by Christian and Gammel using only partial waves for $l=2$. Evi-

[12] R. S. Christian and J. L. Gammel, private communication.

dently there is an important contribution to the scattering by waves of higher order.

The phase shifts for $l=2$, for the quartet and doublet scattering are, in the order $\delta_0{}^q$, $\delta_0{}^d$, $\delta_1{}^q$, $\delta_1{}^d$, $\delta_2{}^q$, $\delta_2{}^d$, for Christian and Gammel: 68.6°; 90.0°; 33.4°; −2.9°; −8.0°; 6.9°. The interpolated phase shifts of Buckingham, Hubbard, and Massey are, similarly; 86.7°; −79.7°; 24.6°; −14.7°;−10.3°; 5.2°.

B. The *p-p* Interaction

The *p-p* scattering data have been analyzed by the procedures developed by Jackson and Blatt[13] for the determination of the S wave phase shift, possible contributions from P and D waves, and the shape of the potential well.

From the angular distribution shown in Fig. 4 and assuming no scattering other than S wave, an apparent S wave phase shift is calculated at each angle. The apparent S wave phase shift (δ_a) is given by

$$2\delta_a=\sin^{-1}\{\sin\omega-q[(\sigma(\theta)/\sigma_M(\theta))-1]\}-\omega,$$

where $\sigma(\theta)/\sigma_M(\theta)$ represents the ratio of the observed scattering cross section to the calculated cross section for Mott scattering in the c.m. system at angle θ, and ω and q are functions of the incident proton energy.

TABLE III. Calculated values of δ_a for *p-p* scattering.

θ	$\sigma(\Omega)\times 10^{24}$ (cm²)	δ_a (Apparent S wave phase shift)	Absolute error in δ_a (energy uncertainty included)	Relative error in δ_a
25°	0.0590	57° 54′	±1.5°	0.96°
30°	0.0522	57° 30′	±1.5°	0.96°
35°	0.0516	56° 33′	±1.6°	1.0°
40°	0.0524	56° 50′	±1.7°	1.0°
50°	0.0546	57° 27′	±1.7°	1.0°
60°	0.0564	58° 00′	±1.7°	1.0°
70°	0.0579	58° 37′	±1.7°	1.1°
80°	0.0580	58° 28′	±1.8°	1.1°
85°	0.0580	58° 23′	±1.8°	1.1°
90°	0.0580	58° 23′	±1.8°	1.1°

The values calculated from the smooth curve of Fig. 5 at 10 angles are listed in Table III, together with the absolute and relative errors.

The difference between the apparent and true S wave phase shifts depends, of course, on the phase shift of the higher partial waves. According to Jackson and Blatt, it may be approximated as

$$\delta_a-\delta_0=p_1\delta_1+p_2\delta_2,$$

where p_n are functions of the angle, energy, and S wave phase shift.

Neglecting D and higher shifts and assuming no tensor forces, the intercept of a plot of δ_a *vs* p_1 should give the S wave phase shift and the slope the value of the P wave phase shift. This plot is shown in Fig. 6, from which the S wave phase shift is deduced to be 58.1° ±1.2°. Figure 6 also indicates a small attraction in the ${}^3P$ state. It appears, however, that, within the accuracy of our measurements, the nuclear scattering can be said to occur only in the ${}^1S$ state, and, under this condition, the S wave shift is simply the average value of δ_a, which is 57.8°±1.2°. The large error arises from the fact that since the data at all angles are taken simultaneously, systematic errors such as those due to uncertainty in beam energy and charge measurement enter into the cross-section value at each angle in the same way. The error in δ_0 due to such systematic errors is thus not lessened by averaging over all angles.

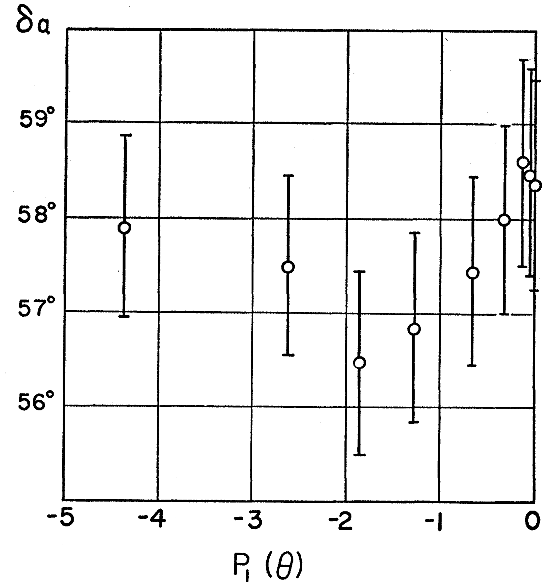

FIG. 6. δ_a *versus* $p_1(E,\theta)$ for the *p-p* scattering measurements (see text).

It should be noted that, for a central force, the repulsive P wave would tend to increase the cross section over pure S wave for small scattering angles, while an attractive P wave would be evidenced by a decrease in the cross section over pure S wave for small scattering angles. Either contribution would go to zero at 90° c.m. Our calculated apparent S wave phase shift from all of our cross sections is only 0.5° less than the S wave phase shift calculated from the data in the region around 90°. (Since the S wave phase shift is a slowly varying function of energy, the phase shifts obtained from the 9.85-Mev data were averaged with those obtained from the 9.7-Mev data.)

Following Jackson and Blatt, we observe that information as to the shape of the interaction potential can be obtained from a plot of energy *versus* the quantity K of the effective range theory. This quantity is determined from the average value of δ_a to be 7.68±0.20 and from δ_0, as determined from Fig. 6, to be 7.60±0.20. Jackson and Blatt have plotted this quantity as a function of energy for the square and Yukawa wells in such a

[13] J. D. Jackson and J. M. Blatt, Revs. Modern Phys. **22**, 77 (1950).

way as to give equivalent good fits to the very precise low energy Van de Graaff data. On the basis of this plot, and again assuming only central forces, our results appear to favor the Yukawa type potential. However, it should be pointed out that Jackson and Blatt have investigated the allowable uncertainties in cross-section and bombarding energy required to discriminate between the two shapes, and that our uncertainties are somewhat greater than those given by these authors.

An analysis of these data in terms of the f function of Breit, Condon, and Present[14] yields a value for this function, as determined from the 57.8° S wave phase shift, of 15.7 ± 0.5, which is in fair agreement with the value 16.5 ± 0.17 predicted by Yovits, Smith, Hull, Bengston, and Breit[15] for the Yukawa well.

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Note added in proof:—The points at angles from 169.8° to 176.2°, inclusive, on Fig. 3E are plotted incorrectly. They should be lowered 5 to 10 percent to conform with the values given in Table I.

[14] Breit, Condon, and Present, Phys. Rev. **50**, 825 (1936).

[15] Yovits, Smith, Hull, Bengston, and Breit, Phys. Rev. **85**, 540 (1952).

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Number Theory and the Magnetic Properties of an Electron Gas

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Theorems involving the correction terms of lattice point problems in the theory of numbers are interpreted to derive the orders of magnitude of the oscillatory (de Haas-van Alphen effect) and non-oscillatory (Landau and surface diamagnetism) terms in the magnetic moment of a Fermi gas in a finite cylindrical container. The results are valid for systems from atomic dimensions up, and all values of the magnetic field. The different types of moment are different from strong and weak fields, and may depend, for small particles, on the nature of the surface potential at the walls of the container. The applicability of the method to physical problems, and the difficulties associated with statistical mechanical problems involving magnetic fields are discussed.

I. INTRODUCTION

THE problem of the diamagnetism of an electron gas was first examined from a fundamental standpoint by Bohr[1] and Van Leeuwen,[2] who showed for a rather general class of conditions that no magnetic properties whatever were to be found on the basis of classical statistics. Bohr, in particular, showed that this conclusion was a consequence of exact cancellation between the large diamagnetic properties of electrons whose orbits did not collide with the wall and the paramagnetic properties of those orbits which did collide with the walls. Landau[3] re-examined the problem on the basis of quantum statistics and showed that a small diamagnetism was to be expected when the levels were quantized on the basis of either Boltzmann or Fermi-Dirac statistics. However, it was not at all clear how the conclusions of Bohr were related to those of Landau via the correspondence principle. The reason for this was that Landau assumed a very strong magnetic field (orbit radius very much less than dimensions of the container) and did not attempt to satisfy the boundary conditions at the walls of the container, but merely counted those quantum states which had the center of gravity of their probability distribution inside the container. If one attempts to follow the details of the Landau derivation, it appears that the results obtained are quite sensitive to such apparently trivial details as the order of integration over the different quantum numbers and of differentiation with respect to H to obtain the moment, the choice of origin for the energy level, and the choice of several possible forms for the Euler-McLaurin formula for replacing a sum by an integral.

Moreover, there is the added embarrassment that, if higher terms in the Euler-McLaurin formula are included, one may find infinite contributions to the moment because certain derivatives are infinite at the ends of the range of integration. Thus one can obtain the Landau result but one can also obtain quite different results which one has no *a priori* reason for rejecting.

The discrepancies can be roughly divided into two classes. First, a large difference in the moment per unit volume is computed by

$$M/V=kT(\partial/\partial H)\Sigma_i \log[1+\exp(E_0-E_i)/kT)]$$

as opposed to

$$M/V=-kT\ \Sigma_i(\partial E_i/\partial H)1/\{\exp[(E_i-E_0)/kT]+1\};$$

[1] N. Bohr, unpublished dissertation, Copenhagen, 1911. For an account of this work, see reference 5, p. 97.

[2] J. H. Van Leeuwen, J. phys. et radium **2**, 361 (1921).

[3] L. Landau, Z. Physik **64**, 629 (1930).