PHYSICAL REVIEW VOLUME 88, NUMBER 1 OCTOBER 1, 1952

Production Cross Section and Frequency of Neutral Mesons in Cosmic Rays*

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(Received June 9, 1952)

Nuclear interactions have been observed in a large cloud chamber operated at Echo Lake, Colorado, altitude 10,500 ft. The events were produced by ionizing particles (protons) in NaI crystals inside the cloud chamber, and were detected by photomultipliers.

In this paper the electromagnetic component associated with the nuclear events is studied as evidence for the properties of the neutral meson whose decay is assumed to be the source of this component. The results are consistent with the assumption that the π^0 mesons are the main source of the photons associated with the nuclear events. The probability of production of π^0 mesons is in agreement with the theoretical results of Fermi, within the limits of the large experimental errors. The ratio of the number of neutral π^0 mesons to the number of the charged $\pi^{\pm}$ mesons is estimated in events of different multiplicities of shower particles and it is found to be close to 0.5 for events with multiplicity varying from 1 to 5 or more (corresponding to energies of from 1 to about 20 Bev for the producing protons).

These results agree with the results of other experimenters, if all the possible systematic errors are taken into account. The estimated energy distribution of the π^0 mesons emitted is given.

INTRODUCTION

THIS paper reports the results of our experimental research performed during the summer of 1951, with a cloud chamber controlled by scintillation crystals inside. We studied the production of the neutral mesons in the nuclear events produced by the cosmic-ray protons.

In the following we will summarize the present state of knowledge of neutral mesons before giving and discussing our results.

The first indication of the existence of neutral mesons in cosmic rays came from the cloud-chamber work of Fretter, Chao, Bridge, Rossi, and Hazen.[1] From their results it appeared that the frequency of the electromagnetic[2] component associated with high energy nuclear events seemed too high to be accounted for by some kind of purely e.m. mechanism. Successive support for the neutral meson hypothesis came from the work of Fretter,[3] Tinlot and Gregory,[4] and Lovati *et al.*[5] The results of Tinlot and Gregory, for example, indicated that, in certain favorable cases, the e.m. component associated with nuclear events is in agreement with the neutral meson hypothesis, and the number of neutral mesons deduced from the frequency of the e.m. components is equal to about one-third of the penetrating particles in the penetrating showers.

Significant work on neutral mesons has been done by the Bristol group[6] by means of nuclear plates. They succeeded in determining (a) an indirect estimate of the mass of the neutral meson, (b) a ratio of the numbers of charged and neutral mesons, (c) an upper limit for the decay time of the neutral mesons, and (d) an indication of the energy distribution of the neutral mesons in the air, at high altitude. One may consider results (b) and (c) to be more direct and reliable than those of (a) and (d).

Strictly speaking, one cannot take these results as direct evidence for neutral mesons in cosmic radiation. The situation is the same for cloud-chamber observations: namely, one can only say that there are cases in which the general aspect of the e.m. component emitted really indicates a two-photon decay of a neutral particle of between 200–400 electron masses. However, since nearly conclusive evidence of the proposed neutral meson properties has been furnished by experiments involving machine made mesons,[7] one may take these cosmic-ray results as fairly reliable.

The nuclear plate technique has a low efficiency in detecting neutral mesons. In fact, with a decay into two photons: $\pi^0 \rightarrow 2\gamma$, most of the photons will go out of the region of detection before materializing. Even when photons are found to materialize within this region the inferences concerning the parent neutral mesons are rather difficult to make. In this respect, a multi-plate cloud chamber has an advantage over the nuclear plates since, here, one can in most cases follow the decay photons through their electromagnetic development and, moreover, can correlate them to a particular event in both time and space. But one must pay careful attention to the bias introduced in triggering the cloud chamber with a particular detection arrangement. Most of the previous cloud-chamber work employed G-M counter controlled triggering in one form or another and unavoidably introduced unknown biases in detection.

In using multiplate cloud chambers, one must look for all the events that could have contained charged or

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[1] W. B. Fretter, Phys. Rev. 73, 41 (1948); Bridge, Hazen, and Rossi, Phys. Rev. 73, 179 (1948); C. Y. Chao, Phys. Rev. 75, 581 (1948).
[2] Hereafter referred to as e.m.
[3] W. B. Fretter, Phys. Rev. 76, 511 (1949).
[4] B. P. Gregory and J. H. Tinlot, Phys. Rev. 81, 667 (1951). (References to previous work may be found in this article.)
[5] Lovati, Mura, Salvini, and Tagliaferri, Nuovo cimento 7, 943 (1950). Lovati, Mura, and Tagliaferri, Nuovo cimento 9, 205 (1952).
[6] Carlson, Hooper, and King, Phil. Mag. 41, 701 (1950).
[7] Steinberger, Panofsky, and Steller, Phys. Rev. 78, 802 (1950).

neutral mesons, irrespective of the number of penetrating secondary particles emitted. Therefore, if one triggers the cloud chamber by means of conventional G-M counters on the basis of the number of penetrating secondaries, the results obtained will be quite far from the real distribution, especially at the lower energies.

In order to get around this difficulty, an apparatus has been built to permit a relatively unbiased selection of the events to be studied. Such unbiased selection compares favorably with that obtained by nuclear plates, and allows us to correlate in a more definite manner the results of the cloud chamber with those obtained by nuclear plates.

I. EXPERIMENTAL DISPOSITION

The technique utilizes a proportional scintillation counter placed inside a cloud chamber. The cloud chamber is triggered by means of the signal from photomultipliers viewing the crystal, when that signal indicates that at least one heavily ionizing particle has resulted from an interaction in the crystal. In this way, selection is made on the basis of the energy of the heavily ionizing particles resulting from the interaction (the "evaporation" particles), regardless of the detailed behavior of the secondary particles of higher energies, so that the bias is no more severe in regard to protons of fairly high energy (100 Mev up), and to mesons, than the bias usually introduced in the nuclear plate technique.

The cloud chamber used in these experiments was a rear illuminated type, with an illuminated region $46\times46\times20$ cm^3. The general construction is not much different from other cloud chambers of this type.[8] NaI crystals, each $7\times10\times0.6$ cm^3, were used for the scintillation counter. They were enveloped with aluminum foil and glass and mounted in an aluminum plate. We placed this crystal plate under a glass window located in the cloud-chamber ceiling (Figs. 1 and 7). The light emitted by the crystal was collected by six 1P21 photomultiplier tubes. The high voltage supplied to the photomultiplier tubes was adjusted in order to discriminate against events giving up less than a certain amount of energy in the crystal. Most of the time the bias corresponded to about 12 Mev ionization energy loss in the crystal. This amount of energy loss is much lower than the average energy loss of a nuclear star caused by a proton in the crystal. Since the energy dissipation by a nuclear star in the crystal will be mainly that of the evaporation particles and gray particles, the triggering will be independent of the number or nature of the fast secondary particles emitted. Also the bias is enough to discriminate against the abundant fast cosmic-ray particles traversing the crystals such as μ-mesons, since at least three or four fast particles must traverse the crystal simultaneously in order that the energy dissipation exceeds that set by the bias.

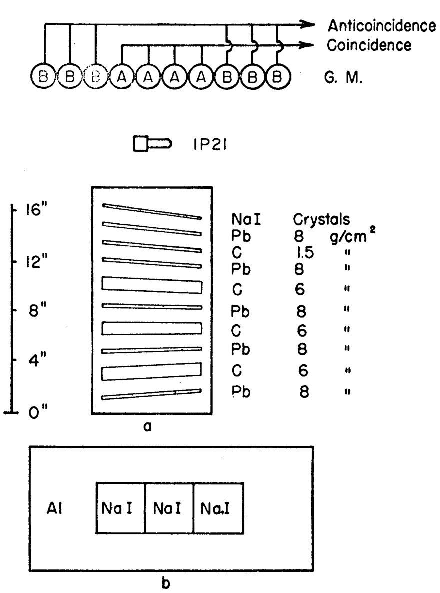

FIG. 1. (a) Crystals and plates assembly in the cloud chamber. (b) Top view of the Al plate containing the NaI crystals.

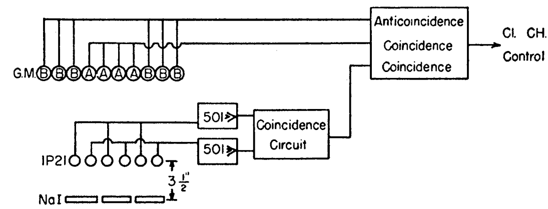

FIG. 2. Block diagram of the circuits and counters used for triggering the cloud chamber.

In order to reduce the background noise, the photomultipliers were arranged in two groups of three each in a twofold coincidence. The block diagram of the circuitry used for the scintillation crystals is given in Fig. 2, which also shows the Geiger counters involved.

The triggering occurred when there was a pulse in the crystal above the bias level in coincidence with one or more counters in A and in anticoincidence with B. We adopted this scheme in order to avoid the many small stars due to neutrons and the frequent air showers. Thus, we collected the events due mainly to proton interactions.

The experiment was carried out during the summer of 1951 at Echo Lake, Colorado (altitude 10,500 feet). The actual running time of the cloud chamber was about 1200 hours.

Part of the time we took oscilloscope pictures of crystal pulses in order to correlate the pulse heights with the different kinds of nuclear events. A short description of this pulse-height distribution for different

[8] R. R. Rau, Princeton Cosmic Ray Group, Tech. Report No. 4 (unpublished); M. B. Gottlieb, Phys. Rev. 82, 349 (1951).

kinds of events is given in the Appendix. The cloud chamber contained Pb plates and carbon plates mounted alternately, of the thickness indicated in Fig. 1. Carbon plates are intended to give information on the interactions of the secondary particles in light nuclei.

II. ANALYSIS

1. In the analysis of the photographs obtained with the apparatus described above, we were mainly interested in the observation of the fast protons and of all the π-mesons, charged or neutral, emitted from the crystal. For the following discussion it is important to distinguish clearly between these fast particles and the slow tracks which usually appear in nuclear interactions. This distinction, with somewhat different limits in energy will be similar to the distinction between "shower particles" and black and gray particles which was made by Camerini *et al.*[9] In the following, we will designate as penetrating particles (abbreviated p.p.) those charged particles emitted directly from the nuclear interactions, with an ionization loss close to the minimum. The estimate of the ionization alone is a poor index of the energy of the particle. In order to get a better definition of the p.p. we considered each track whose ionization was between one and two times the minimum on the assumption it was either a proton or a meson. This examination was made by estimating the Coulomb scattering from plate to plate and the eventual change in ionization of the track in going through the plates. With these assumptions concerning the nature of the particle, we could in turn estimate the ionization of the particle again, and assign an upper limit to it.

With the limits of our cloud chamber in the estimate of the ionization and of the Coulomb scattering, we could go so far as to consider as a p.p. a particle whose "estimated ionization" was less than 1.7 the minimum (>290 Mev for protons, and >45 Mev for the mesons). Thus we define for the purpose of this paper a p.p. as a particle ionizing ≤ 1.7 minimum. The possible error involved in this estimate is quite important in the estimate of the ratio of neutral to charged mesons and will be considered in detail in Sec. IV.

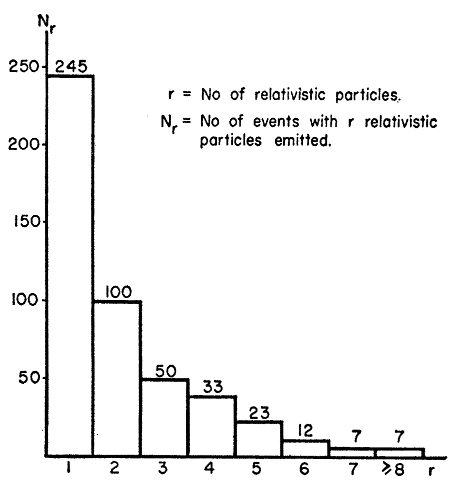

FIG. 3. Distribution of the nuclear events according to the total number of the relativistic particles (π^0 mesons+penetrating particles). Due to the character of our measurements this distribution can be considered the real one at 10,500 feet above sea level.

[9] Camerini, Davis, Fowler, Franzinetti, Muirhead, Lock, Perkins, and Yekutieli, Phil. Mag. 42, 1241 (1951).

The particles going out of the illuminated region after traversing only one or two plates below the crystals (Plates 2 and 3) were the most difficult to classify according to our method. But, due to the favorable location of the crystals (Fig. 1) these were only a small percentage of the total number of shower particles. We did not include in the statistics the events whose primary proton was inclined more than 50° with respect to the vertical.

Also an intense development of an electromagnetic component may be counted as an obstacle in judging the penetrating particles, since under this condition the picture will be more or less obscured. This effect, however, was not significant for our energy range.

The presence of electromagnetic components emitted from the nuclear events in the crystal was identified from the electromagnetic cascade generally starting from the first plate of lead (Plate 2). We classified an event as containing an electromagnetic component when at least one pair of electrons or at least one relativistic electron was observed, as judged by the strong scattering in going through the plates. Care was taken to rule out accidental electron tracks by comparing the different regions of the cloud chamber.

Gray tracks and heavy tracks were estimated from the density of ionization. They are, however, not too important for the present discussion. In each nuclear event in the crystal, we also looked for electrons emitted directly as well as other kinds of particles.

2. Classification of Events. All events in the crystal were considered in which we could identify at least one penetrating particle or at least one electromagnetic component. These events were classified according to the number of p.p.'s emitted (see Table I, row a). In each group we also listed the number of events which contained electromagnetic components (row b) and the total number of neutral mesons estimated (row c). For instance, we observed 239 of N_1 events (namely events containing one p.p.), and 35 events out of these 239 contained electromagnetic components. Our estimate of the corresponding total number of neutral mesons was 45 and the number corrected for missed cascades is 50. (The correction procedure is described below.)

In estimating the number of π^0 mesons from the number of observed electromagnetic components, we assumed the decay scheme for π^0 to be: $\pi^0 \rightarrow 2\gamma$. The energy and angle relation in this process has been discussed by many authors.[3,5]

In about 40 percent of the cases, we actually could see two e.m. cores originating from the nuclear events in the crystal, and in a few cases we were able to verify the relation:

$$(2 \sin\tfrac{1}{2}\psi)^2 = (\mu c^2)^2/E_1E_2, \qquad (1)$$

holding between the two cores. In Eq. (1), ψ is the angle subtended by the two decay photons and E_1, E_2 are their energies, and μc^2 is the π^0 rest energy. When three e.m. cores were clearly correlated to the nuclear event concerned, we assumed that there were two π^0 decays. In no case, was there evidence for the existence of more than four e.m. cores in a single nuclear event. This could be due to the difficulty in resolving the more complicated events. However, due to the low frequency of the complicated events this limitation will not alter our statistical results. This method of estimating the number of π^0 mesons is subject to two types of errors. One is the geometry factor; π^0 mesons emitted with angle wider than $60°\sim 70°$ from the vertical will escape detection. In Table I, we did not make any correction arising from this geometry factor, since we will also miss shower particles with about equal probability. However, we have to estimate a systematic error arising from the fluctuation of the electromagnetic cascade developments. The cascade process is essentially a statistical affair, therefore we may sometimes miss π^0 mesons because the decay photons did not convert to electrons at all. According to the recent work of Wilson[10] the average number of electrons to be observed from a photon initiated shower is smaller than the number one would expect from conventional shower theories, and this effect becomes important for low energy showers. Thus our error may become quite appreciable for low energy π^0 mesons. Using Wilson's data, we have estimated the probability P of missing a π^0 meson, emitted in an interaction in the crystal of our cloud chamber. For a π^0 meson of a given energy, we estimated this probability by considering various energy divisions among the two decay photons. We found that the probability P is relatively high ($P=0.4$) for a low energy π^0 and approaches zero for a π^0 of total energy above 400 Mev. Assuming that the π^0 energy distribution is similar to that of the charged mesons, we estimated the average missing probability P to be ~ 0.12 and have applied this correction to the number of observed neutral mesons. To understand the relatively low value of P, one must remember that we considered as evidence for the presence of one π^0 meson the appearance of one electron or one e.m. pair below the first Pb plate.

Another source of small error in the opposite direction may be the knock-on electrons from the p.p. traversing the Pb plates. An estimate of the knock-on error combined with P resulted in a final correction of 0.10 to the results of Table I, row c, leading to the final corrected values for the π^0 meson, given in row d of Table I.

[10] Robert R. Wilson, Phys. Rev. **86**, 261 (1952). We are very grateful to Professor Wilson for sending his results to us before publication.

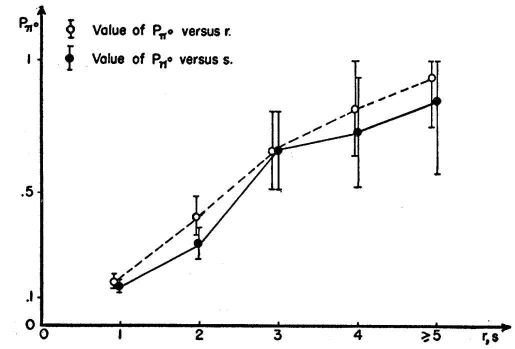

FIG. 4. Probability of emission of at least one neutral meson in nuclear events of different multiplicities. On the abscissa the numbers of the penetrating particles, indicated by s, and the numbers of relativistic particles, indicated by r, are reported.

In estimating the number of neutral mesons from the observed number of electromagnetic showers, we explicitly assumed that all the electromagnetic component comes from π^0 decays. If any appreciable amount of electromagnetic component arises from some other source besides π^0 decays, then the estimate of the number of π^0 mesons will be too high.

The number listed under the column $s=0$ is subject to a greater error since this kind of event is generally hard to identify. Therefore its normalization with the other numbers is doubtful.

From Table I, and again because of the unbiased condition of our triggering system, we can derive the frequency distribution of the number of nuclear events *versus* the number of the relativistic particles. Here, and in later discussions, relativistic particles include the neutral mesons and the p.p. The observed distribution is given in Fig. 3.

III. CROSS SECTION FOR PRODUCTION OF THE NEUTRAL MESONS

In comparing rows a and b in Table I, we notice the fact that the probability of emission of electromagnetic component becomes higher as the multiplicity increases. This indicates that the production of the neutral mesons becomes more probable for large size stars. To express this effect quantitatively we form the π^0 production probability $P_{\pi^0}(s)$ by dividing the number in row b by that of row a, i.e.,

$$P_{\pi^0}(s) = b/a,$$

for different multiplicities, s.

This will give us the probability of producing at least one π^0 meson in a nuclear event containing s p.p.'s. In Fig. 4 is shown a plot of P_{π^0} *vs* s. In the same Fig. 4 (broken line), we also plot $P_{\pi^0}(r)$, which is the probability of finding at least one π^0 meson in an event with r relativistic particles. The quantity $P_{\pi^0}(r)$ is not biased in favor of the charged particles as the quantity $P_{\pi^0}(s)$ is.

In order to obtain some information on the cross section σ_{pr} for the production of $\geq 1\pi^0$ meson, we now consider our results on the production of neutral mesons by protons, together with those obtained by Camerini *et al.*,[9] on the relation between the energy of the primary and the multiplicity of p.p.'s for proton-induced nuclear events. The results obtained will be affected by large statistical errors and will be only indicative. Nevertheless, we consider the attempt worthwhile, due to the present complete lack of direct information on this point.

According to the results of Camerini *et al.*,[9] for each multiplicity s, one can attribute a certain spread of the energy of the producing proton. Since it is not likely that our nuclear events differ statistically from those produced in the nuclear plates, we will use the energy-multiplicity relation derived in the emulsion work.

Thus, for each s, we estimate the region of energy inside which most of the events ($\sim$70 percent) with s p.p.'s fall. This region is the horizontal dimension of each rectangle indicated in Fig. 5; the corresponding multiplicity s is indicated by the number inside each rectangle. In the limits of our estimate we can assume that $P_{\pi^0}(s)$ is equal to the ratio between $\sigma_{\text{pr}}(E)$, cross section for π^0 production, and the total cross section for nuclear interaction of the impinging protons, which we can assume to be close to the geometrical, σ_{geo}:

$$\sigma_{\text{pr}}(E) = P_{\pi^0}(s)\sigma_{\text{geo}},$$

where E is an energy inside the limits of the width of the rectangle s. Each rectangle s is located in Fig. 5 at an ordinate equal to $\sigma_{\text{pr}}(E)$, and the height of the rectangle is equal to the statistical error of this quantity. So, the rectangles of Fig. 5 show the behavior of σ_{pr} *versus* the energy in the sense that the real cross section σ_{pr} for emission of at least one π^0 meson is a line which should be contained in the region of the diagram delimited from these rectangles.

The experimental point given at 345 Mev was obtained by Moyer.[11] The geometrical cross section in the crystal is the average of the geometrical cross sections in Na and I:

$$\sigma_{\text{geo}} = \tfrac{1}{2}\pi r_0^2(A_{\text{Na}}^{\frac{2}{3}} + A_{\text{I}}^{\frac{2}{3}}) \cong 10^{-24}\ \text{cm}^2.$$

In Fig. 5 we show also the energy dependence of the cross section for production of at least one charged meson, which we will indicate as σ_{pr}', obtained from the same results of Camerini *et al.* We can see from comparing the position of the dots with that of the rectangles that σ_{pr} is lower than σ_{pr}'. The cross section for production of $\geq 1\pi^0$ meson by protons of 1–2 Bev is about 0.1–0.2 the geometrical value of the cross section.

TABLE I. Distribution of the nuclear events, N_s.

No. of penetrating particles, s		0	1	2	3	4	5	≥ 6
N_s, no. of nuclear events containing s penetrating particles	a	?	239	103	50	26	11	9
No. of nuclear events N_s having electromagnetic component	b	(45)	35	32	33	19	9	8
No. of estimated π^0 mesons	c	(49)	45	41	43	27	15	14
No. of π^0 mesons corrected	d	54	50	45	47	30	16	15

In Fig. 5 we also plot the cross sections *versus* the primary proton energies predicted by the Fermi model.[12] In this calculation, we assumed that any of the three types of mesons will be emitted with equal probability in nuclear interactions. Curve I is the cross section for the production of at least one charged mesons, and Curve II is the cross section for the production of at least one neutral meson.

The estimate of σ_{pr} as a function of the energy given in Fig. 5 is only indicative, in view of the large horizontal spread of the energies that we mentioned, and also because of the many data that had to be supplied from experiments with the nuclear plates, which were performed at higher altitude.

IV. RATIO OF NEUTRAL TO CHARGED π-MESONS

1. We will indicate by R the ratio of the number of π^0 mesons to the number of charged π-mesons:

$$R = \pi^0/(\pi^+ + \pi^-).$$

A few remarks should be made before results are discussed. The measurement of R is of such a nature that the real error is likely to be bigger than the statistical error. This is because the identification of the neutral and the charged mesons is based on different kinds of evidence. The main difficulty in counting the charged mesons lies in the indistinguishability of the charged mesons from protons at high energy; whereas, in the case of the π^0 mesons, the main difficulty is that of missing low energy π^0's, because of the low energy of the decay photons and the large angular spread. For these reasons, the error attributed to each estimate of R will be increased in order to take into account the possible systematic errors.

The value of R has already been estimated by a few groups of experimenters, using several different methods. Carlson *et al.*,[6] gave a value $R = 0.45 \pm 0.1$ obtained by counting the pairs in the neighborhood of the nuclear explosions for events with three or more shower particles. (The error quoted is purely the statistical error.)

Gregory and Tinlot[4] found a value 0.3 for the ratio of neutral mesons to penetrating particles. If we assume that the charged π-mesons are about $\frac{3}{4}$ of the shower particles, their result gives $R = 0.4$. The numbers above refer to events with at least 3 shower particles, produced by protons or neutrons.

Recently Camerini *et al.*,[9] repeated the estimate of R from another point of view, which is more indirect, but still very interesting, for it at least makes it possible to fix an upper limit to the value of this difficult ratio R. Here an estimate was made of the number of π^0 mesons

[11] Burton J. Moyer (private communication). The result was obtained by bombarding carbon with 345-Mev protons.

[12] E. Fermi, Prog. Theor. Phys. 5, 570 (1950).

in events whose primaries had energies ≤ 10 Bev, and whose multiplicity s was ≤ 4. On the basis of the balance of energy between the primary proton and the charged secondaries, they concluded that the neutral mesons are about as numerous as the charged mesons. Due to the character of this measurement we would indicate their result in the following way: $R \leq 1 \pm 0.3$.

A preliminary estimate of R based on results using the present technique has already been given by us in a previous note.[13] This value was reported as $R_3 = 0.37 \pm 0.08$ (statistical error only) for nuclear events with three or more shower particles produced by protons. This measurement was affected by some slight systematic error which will be discussed in the following.

2. The data of Table I will now be considered, taking into account the possible sources of systematic errors which occur in the estimate of R. We consider first events with three or more penetrating particles.

These events show 368 shower particles, and for them we estimated 109 (corrected number[14]) neutral mesons, corresponding to 30 percent of the penetrating particles. If we want to estimate R we have to know how many of these penetrating particles are charged π-mesons.

This number can hardly be estimated directly in our cloud chamber, where we can distinguish between $\pi^{\pm}$ mesons and protons on the basis of the Coulomb scattering and the intensity of ionization only up to about 400 Mev in the most favorable cases. Therefore, we estimated the percentage of the $\pi^{\pm}$ mesons on the shower particles using the results of other experiments. As has already been stated (Sec. II), our shower particles are protons of energy higher than 290 Mev, and charged π-mesons of energy higher than 45 Mev. From the map of Camerini *et al.*,[15] giving the distribution in energy of the protons and charged π-mesons of the nuclear interactions, we deduce that 65 percent of our p.p. are charged π-mesons. On the one hand, an estimate of the percentage of charged π-mesons among the penetrating particles has been made by Annis and Bridge in a cloud chamber similar to, but larger than ours.[16] They found that 75 percent of the fast secondary particles of nuclear interactions are $\pi^{\pm}$ mesons. Their results corresponded to energies higher than ours, so we can consider that 75 percent is an upper limit for the percentage of the $\pi^{\pm}$ mesons among our p.p.'s.

On the other hand, a percentage of 50 (half the p.p. are $\pi^{\pm}$ mesons) would certainly be a lower limit. In fact, according to the map given by Camerini this would correspond to assuming that we considered as p.p. all the particles ionizing ≤ 2.2 times the minimum (protons ≥ 200 Mev). On the contrary, we can conclude, on the

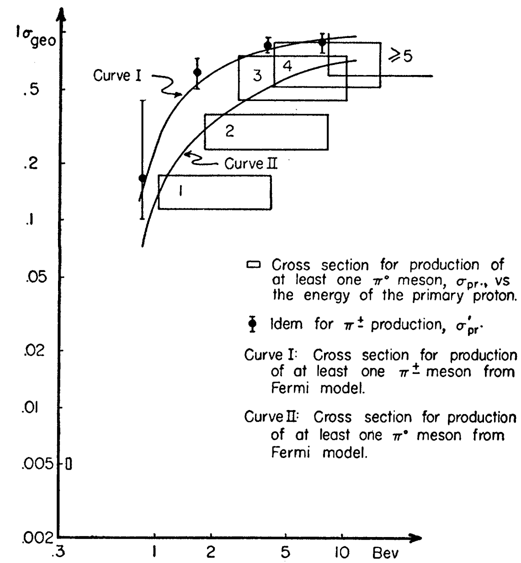

FIG. 5. π^0 production cross section *vs* primary proton energies. The points indicated by dots are values reported by Camerini *et al.* (see reference 9) in the nuclear plates work. Solid lines are the cross sections for charged (Curve I) and neutral mesons (Curve II) predicted by the Fermi model (see reference 12) of meson production.

basis of the comparisons made with electrons and recognized protons and π-mesons, that most of the tracks ionizing two times the minimum were rejected in our selection of the p.p., and therefore 50 percent for the relative number of $\pi^{\pm}$ mesons and p.p. is a lower limit.

On this basis, the values of R for nuclear events with three or more shower particles, which will be designated as R_3^{∞}, is as follows:

$$R_3^{\infty} = 109/(368)(0.5) = 0.59 \pm 0.08 \text{ upper limit of } R_3^{\infty},$$

supposing that 50 percent only of the p.p. are mesons:

$$R_{3P} = 109/(368)(0.65) = 0.45 \pm 0.07 \text{ most probable value according to our measurement}$$

$$R_3^{\infty} = 109/(368)(0.75) = 0.39 \pm 0.06 \text{ lower limit of } R_3^{\infty}.$$

3. We can now try to estimate the value of R for lower multiplicities, therefore lower energies. For this purpose, we consider the events of columns 0 to 4 in Table I (events with four or less shower particles). The ratio r_0^4 of the π^0 mesons to the number of the penetrating particles is:

$$r_0^4 = 226/699 = 0.32 \pm 0.03.$$

Again the most uncertain point is the percentage of the $\pi^{\pm}$ mesons among the pp. An indication of how much the percentage of the $\pi^{\pm}$ mesons among the shower particles changes with the size of the nuclear events comes from Camerini *et al.*[9] From these results we deduce that the $\pi^{\pm}$ mesons are about 60 percent of our p.p.'s in this range of multiplicity (the previous estimate for the events of multiplicity $3-\infty$ was 65

[13] G. Salvini and Y. Kim, Phys. Rev. **85**, 921 (1952).
[14] The number of the π^0 mesons is higher here than in the previous report (see reference 13). The main corrections come from the re-estimates of the probability of missing the neutral meson, based on the results of the Monte Carlo method (see reference 10).
[15] Camerini, Fowler, Lock, and Muirhead, Phil. Mag. **41**, 413 (1950).
[16] M. Annis and H. S. Bridge, Phys. Rev. **86**, 589(A) (1952).

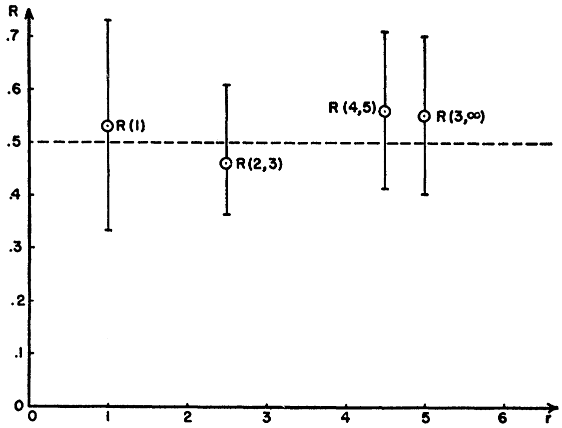

FIG. 6. The ratio of neutral to charged mesons *vs* multiplicities in relativistic particles.

percent). With the same criteria as in the estimate of R_3^∞ we give for R_0^4 the values (see Table I):

$R_0^4 = 226/699\times 0.5 \ = 0.65\pm 0.08$ (upper limit of R_0^4)
$R_0^4 = 226/699\times 0.6 \ = 0.54\pm 0.07$ (most probable value)
$R_0^4 = 226/699\times 0.75 = 0.43\pm 0.07$ (lower limit of R_0^4).

In our earlier estimate[13] we found $R_0^4 = 0.4\pm 0.1$. The earlier result was not corrected for the fluctuations in the cascade of the decay photons, for the final results of the Monte Carlo method[10] were not known to us at that time. The present value seems to be lower (but almost within the errors) than the one estimated by Camerini *et al.*[9] In the comparison of the two values, we have to keep in mind the following facts: The difference in the energy spectrum of the protons at high altitude (Camerini *et al.*) and at mountain altitude (our results) is likely to affect the difference between their value of R_0^4 and ours. Their value $R_0^4 = 1\pm 0.3$ has to be considered in our opinion as an upper limit, for there is no sure evidence yet that the "missing energy" goes only into production of neutral mesons.[17]

4. A less biased estimate of R can be obtained from the following considerations: The general question implicit in the value of R is the dependence of R on the size of the nuclear events, and therefore on the energy of the primary protons. Should we find that R is fairly constant for different multiplicities and close to $\frac{1}{2}$, this would indicate that the cross section for production of π^0 mesons goes with the energy as the cross section for production of the charged mesons of each sign. To resolve the question, we can now estimate the value of R in a way which differs from the previous estimates, and which can be used because of the very low bias of our disposition. It is easy to see that the selections of the type we used before (0 to 4, or 3 to ∞ p.p.'s) are going to favor the neutral mesons or the charged mesons in an appreciable way. This systematic error already was pointed out by Camerini *et al.*[9] We are free to a large extent from these errors if we obtain the ratio R from events of equal numbers of relativistic particles, without any choice in favor of the charged or neutral mesons. By relativistic particles we mean here the neutral mesons and the p.p. In computing, for instance, the number of the events with two relativistic particles, we will include the events with $2\pi^0$ only, $1\pi^0$ and 1 p.p., 2 p.p.'s only. Moreover, what we want now is the ratio of the π^0's to the total number of $\pi^\pm$ mesons. Therefore, we have to add to the number of $\pi^\pm$ mesons estimated among the p.p.'s the $\pi^\pm$ mesons of energy <45 Mev (see Sec. II). The number of these low energy $\pi^\pm$ mesons can be estimated from our results. It constitutes a small correction: the $\pi^\pm$ mesons below 45 Mev are about 10 percent of the total number of $\pi^\pm$ mesons. We will indicate by $R(m, n)$ the value of R referred to events with m, $m+1$, $\cdots n$ relativistic particles.

Value of $R(2, 3)$

We indicate by $R(2, 3)$ the ratio R obtained for the events with two or three relativistic particles, including the $\pi^\pm$ mesons with energy <45 Mev. From our events, reported in Table I and Fig. 3, we obtain

$$R(2,3) = 83/180 = 0.46\begin{matrix}+0.15\\-0.1\end{matrix}.$$

We assume in this computation that the $\pi^\pm$ mesons are 60 percent of the p.p.'s. The large errors attempt to include the possible systematic errors which have already been outlined in the preceding paragraph.

TABLE II. Ratio R of the number of the $\pi^\pm$ mesons, among nuclear events of different multiplicity. R_3^∞ = value of R for events with three or more penetrating particles. $R(2, 3)$ = value of R for events with two or three "relativistic particles." The other symbols are obvious.

	Authors	Value
R_3^∞	Gregory and Tinlot[a]	0.4 ± 0.1[d,e]
R_3^∞	Carlson *et al.*[b]	0.45 ± 0.1[f,e]
R_3^∞	Salvini and Kim present experiment	$0.45\begin{matrix}+0.15\\-0.1\end{matrix}$[g,e]
R_0^4	Camerini *et al.*[c]	1 ± 0.3[h]
R_0^4	Salvini and Kim present experiment	$0.54\begin{matrix}+0.15\\-0.1\end{matrix}$[g,i,h]
$R(3, \infty)$	Salvini and Kim present experiment	0.54 ± 0.15[g,i]
$R(4, 5)$	Salvini and Kim present experiment	0.55 ± 0.15[g,i]
$R(2, 3)$	Salvini and Kim present experiment	$0.46\begin{matrix}+0.15\\-0.1\end{matrix}$[g,i]
$R(1)$	Salvini and Kim present experiment	0.53 ± 0.2[g,i]

[a] See reference 4.
[b] See reference 6.
[c] See reference 9.
[d] Estimated by us on the basis of their results.
[e] This measurement is biased in favor of the charged π-mesons.
[f] Only the statistical error reported.
[g] The error given here is larger than the statistical, for allowing for the possible systematic errors (see the text).
[h] This measurement is biased in favor of the π^0 mesons.
[i] The $\pi^\pm$ mesons of energy ≤ 45 Mev were included.

[17] In a conversation with one of us Dr. Perkins kindly discussed our results and their previous results (see reference 9). He told us that in a recent, more direct estimate they found a value close to our value (2). We are indebted to him for a general discussion of the problem of the determination of R.

Value of $R(4, 5)$

In the same way as above we find, for events with four or five shower particles: (We assume now that the charged mesons are 65 percent of the p.p.).

$$R(4,5)=0.55^{+0.15}_{-0.1}.$$

For $R(3, \infty)$ we find:

$$R(3,\infty)=0.54^{+0.15}_{-0.1}.$$

It is interesting to note how this less biased value differs from R_3 given in (2).

5. A more direct estimate can be obtained in the following way: Consider first the value of $R(1)$. Among the events with only one p.p. (Table I), we identified $29\pi^{\pm}$ mesons of a kinetic energy lower than 100 Mev. On the basis of the energy spectrum of the $\pi^{\pm}$ mesons (Camerini *et al.*[15]) these mesons should be $\sim\frac{1}{3}$ of the total number of $\pi^{\pm}$ mesons. So we have about $87\pi^{\pm}$ mesons from the events with one p.p. only. We have to compare this number with the number of the events of Table I in column (0) with only one π^0 meson emitted. The ratio results:

$$R(1)=46/87=0.53\pm 0.2.$$

The large errors take into account the possible systematic errors in the estimate of the number of the π^0's and of the energy of the $\pi^{\pm}$ mesons.

6. Conclusions on the value R. In Table II we give the values for the ratio R as estimated in different regions of multiplicity from different observers.

Figure 6 shows the dependence of R on the multiplicity. The circles represent the values $R(1)$, $R(2, 3)$, $R(3, \infty)$, $R(4, 5)$, which are the only ones of Table II really unbiased from selections in favor of the π^0 or of the $\pi^{\pm}$ mesons.

Our conclusion is that the value R is fairly constant for multiplicities from 1 to about 6, and is close to 0.5. In terms of the energy of the primary protons, this means that R is close to 0.5 for events produced by protons of energy between 2 and 20 Bev.

The differences in the previous results of many authors can be understood on the basis of the different selections and the systematic errors which can easily be made in a measurement as difficult as this one. These results for R are consistent with those found in Sec. III, for the probability of the production of the neutral and the charged π-mesons.

V. ENERGY OF THE π^0 MESONS. EXAMINATION OF THE ELECTROMAGNETIC CORES

We consider now the e.m. cores emitted from nuclear events in the crystals. This will lead to an estimate of the energies of the π^0's emitted in events of different multiplicities, and establish an upper limit for the percentage of the e.m. component which may be due to some source other than the π^0 decay.

TABLE III. Energy distribution of the π^0 mesons in events with different numbers of penetrating particles. E= total energy (rest mass+kinetic) of the π^0 meson. The energy was measured by the method described in the text. The numbers given are the numbers of the π^0 mesons for each energy interval. Generally speaking, row (2) refers to events produced mainly by protons of about 1–4 Bev, row (3), to events mainly above 4 Bev.

	(a)	(b)	(c)
(1) Energy interval (Mev)	$400\leq E<700$	$700\leq E<1000$	$E\geq 1000$
(2) $0\leq s\leq 2$	14	6	7
(3) $s\geq 3$	8	5	8

Energy of the π^0 Mesons Emitted from Events with Different Multiplicities

It is difficult to make an absolute estimate of the energy of each π^0 meson emitted in the nuclear events, but it is relatively easy to compare the e.m. cores observed in events with different multiplicities in order to conclude if the energy distributions of the π^0 mesons taken from events of different multiplicities are the same or not, and in what sense they differ.

For the following estimates, which mainly involve the estimate of the energy of the decay photons from their e.m. development, we established a scale of energy which probably is close to the correct one, but which, since it is the same for events with different multiplicities, may not coincide with the correct one. We summarized here the criteria used in establishing this scale.

We considered the events with at least two e.m. cores, each of them going through at least four plates (including two lead plates) corresponding to a total of at least three radiation lengths. We estimated the energy of each of these cores, on the basis of the results of the Monte Carlo method[10] allowing for the possible fluctuations, which can be very large, with standard rules prepared in advance for all the events. For energies larger than 500 Mev we extrapolated the results of Wilson.[9] This extrapolation was used up to about 1000 Mev and it was made by assuming that the number of the electrons at maximum was proportional to the energy of the primary photons from 500 to 1000 Mev. The correction for the presence of the carbon plates was made by adding to the estimated energy an amount corresponding to the ionization losses of the electrons in traversing the carbon plates.

In this way, the energy was estimated for the π^0 mesons in events with different multiplicity. The energy of the two e.m. cores was estimated using the criteria given above, and each case was considered on the basis of the relation (1). We estimate that the energy calculated by us is in each case within a factor 2 of the real value. The results for the events with multiplicities s from 0 to 2 and from 3 to ∞ are given in Table III. The energies which were measured here were from 400

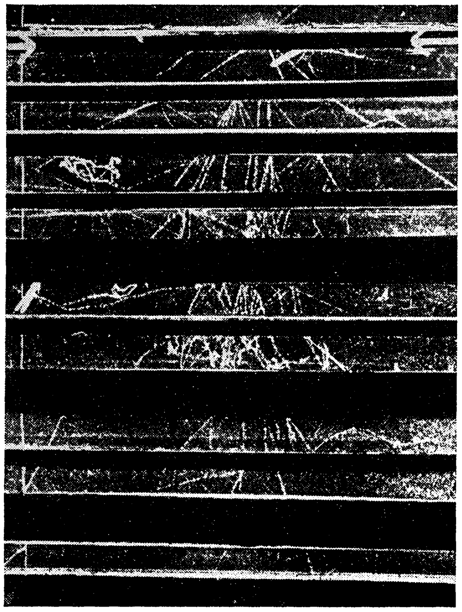

FIG. 7. A representative cloud-chamber photograph. The crystal plate is indicated by the arrows. This is a nuclear event in which no p.p. and one π^0 were created. The two electromagnetic cores are clearly due to photons: the π^0 had an energy higher than 1000 Mev.

Mev up. It is seen from the table that the energies of the π^0 mesons do not differ too much in the events of different multiplicities. This seems to agree with the corresponding result on charged π-mesons.[9]

An experiment designed to enable the estimate of the total energy distribution of the neutral mesons more directly is being prepared by one of us. A preliminary result of the present experiment is that the total energy distribution of the π^0 mesons is rather close to the energy distribution of the charged mesons.[15] In this sense the preliminary indications of Carlson *et al.*[6] concerning neutral mesons from 200 Mev to 600, and of our experiment concerning neutral mesons from 300 Mev and above[13] are confirmed.

Consideration of the Possibility of a Different Origin of the e.m. Component

Evidence that neutral unstable particles of mass close to 300 Mev are at least in good part responsible for the e.m. component in the cosmic radiation has already been given in an indirect but very suggestive way by Carlson *et al.*[6] The doubt that could still remain in their treatment is that the behavior of the e.m. cascade in the air and the Compton effect on the photons of low energy traveling in the air could simulate the existence of an unstable neutral mass decaying into two photons, and that part of the e.m. component from the nuclear events has an origin other than the π^0 mesons. It thus appears worthwhile to check this point again in a more direct way.

For this purpose we examined 39 e.m. cores, which traversed at least three lead plates (at least five plates in the cloud chamber). We searched around each of them for a second core, which should have been present if their only origin was from the decay photons of the π^0 mesons. We found the second core in 24 out of the 39 cases. Consideration of the fluctuations in the e.m. cascade and of the possible angles between the cores, as estimated on the basis of (1), and of the possibility for the second core escaping observation may explain at least 10 of the remaining 39–24 cases.

From this examination we conclude that the π^0 mesons are at least the main and may be the only source of the photons from the nuclear events. If single photons are emitted from the nuclear events of our energies (1–20 Bev), these photons should be ≤ 20 percent of the photons resulting from the decay of π^0's.

Nuclear Events with Only Electromagnetic Component Emitted (Events N_0)

In this category we include those events which do not show any ionizing penetrating particles. They were recognized on the basis either of the grey tracks usually coming out of the crystals, or of the presence of more than one core of e.m. component, pointing with a non-ionizing (photon) link to one point in the crystal. The first criterion was the more common. Examples of these pictures are shown in Figs. 7 and 8.

These events were examined quite carefully, since

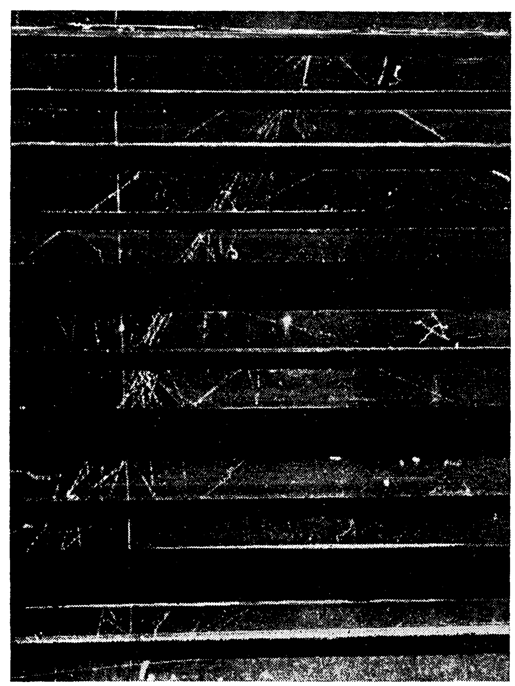

FIG. 8. A nuclear event with two protons and only e.m. component emitted. Events like this and like the one of Fig. 7 would escape the conventional Geiger-Müller control or the nuclear plates.

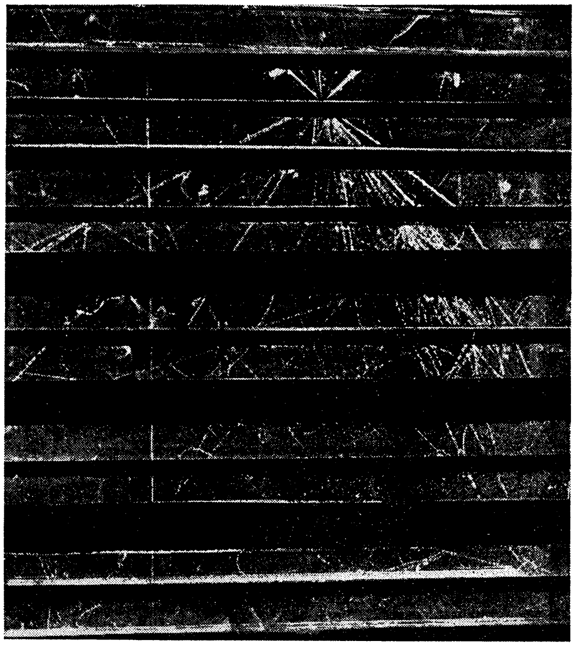

Fig. 9. An event produced in the first lead plate. We have a few of these events, which indicate how the crystal could be an efficient detector also for interactions produced outside.

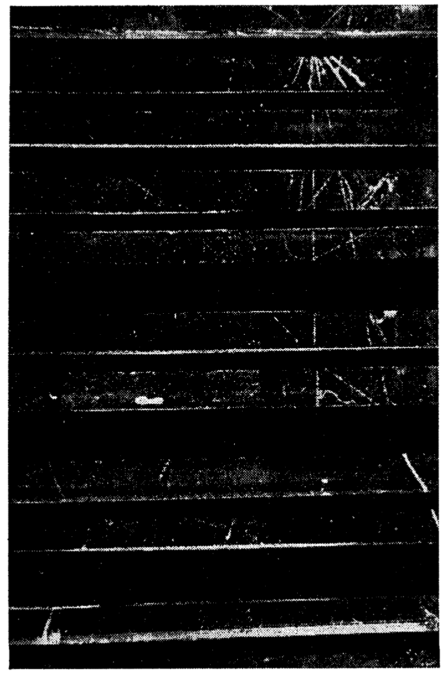

Fig. 10. A nuclear event in the crystal with two p.p. emitted. One of them interacts in the seventh plate (carbon) to give a heavy prong and one light particle, probably one $\pi^{\pm}$ meson directed upward.

they will usually escape observation in the nuclear plates and the Geiger-Müller triggered cloud chambers. The evidence for π^0 mesons in these events was about as strong as in the other nuclear collisions.

Two other events of interest are shown in Figs. 9 and 10.

CONCLUSIONS

On the basis of our experiment and of the evidence already existing we can now fix certain points regarding the existence and properties of the π^0 mesons.

The π^0 mesons are definitely the main source of the photons associated with the nuclear events. An upper limit for the number of photons associated with nuclear events having an origin other than π^0 decay can be set at 20 percent.

The cross section $\sigma_{\mathrm{pr}}(E)$ for production of the π^0 mesons increases with the energy of the producing protons, in the way indicated by Fig. 5. This estimate of the cross section was obtained in an indirect way, and involves rather large corrections. In these limits it appears that $\sigma_{\mathrm{pr}}(E)$ depends on energy in a way that would be expected from the assumption that the π^0 meson has, at energies above 1 Bev, the same probability of production as each of the positive and negative π-mesons. The predictions of Fermi[12] concerning the cross section for production of the neutral and charged π-mesons are confirmed. The value of the cross section for production of at least one π^0 meson is about 0.1–0.2 of the geometrical for incident protons of 1–2 Bev.

The ratio $R=\pi^0/(\pi^+ + \pi^-)$ of the total number of neutral mesons to the total number of charged mesons is close to 0.5 for multiplicities ranging from one to five or six, that is, for different energies of the producing protons between 2 and 20 Bev. This result agrees with almost all the results of previous experiments when these are corrected for the bias introduced in the selection of the events.

Table IV. Pulse-heights distribution for various kinds of events.

Pulse height: Events	12–17	18–23	24–29	≥ 30	Total
Nuclear interactions[a]	38	53	49	77	217
No visible track[b]	13	23	10	14	60
Charged slow particles stopping in the crystals[c]	26	32	38	9	105
Charged particles traversing the crystals[d]	74	94	53	27	248
Electromagnetic showers[e]	83	85	54	36	258
Doubtful cases[f]	35	34	18	13	100
					988

[a] Nuclear interactions occurred in the NaI crystals, with visible prongs coming out of the crystals. About 75 percent of them appear to be due to ionizing particles (protons).
[b] No visible track. Events included in this class may consist of the following types: (a) small stars produced by neutral particles whose prongs end in the crystal; (b) relativistic particles traveling in an almost horizontal direction.
[c] Charged slow particles stopping in the crystals. These events may consist of either simple stoppings or small evaporation stars.
[d] Charged particles traversing the crystals. The following kinds of events were included in this class: (a) unrelated penetrating particles going through the crystals; (b) particles from nuclear events produced above the crystals.
[e] Electromagnetic showers. Mostly produced above the crystal either by cloud-chamber walls or by materials above the cloud chamber.
[f] Doubtful cases. Cases in which it is hard to estimate the origin of the pulse. Some of them could be due to radioactive contamination.

The behavior of R *versus* the multiplicity of relativistic particles (π^0 mesons plus penetrating particles) calculated on the basis of our results, is given in Fig. 6.

The energy distribution of the π^0 mesons from events of low and high multiplicity is about the same, within the statistics of 48 cases selected only on the criterion of being near the center of the cloud chamber. This is similar to the results for $\pi^\pm$ mesons. Also, the total energy distribution of the π^0 mesons seems to be close to the total energy distribution of the charged mesons.[13]

ACKNOWLEDGMENTS

The authors are deeply indebted to Professor G. T. Reynolds for continuous discussion and encouragement. He worked with us at the beginning of this experiment, on the preparation of the scintillation crystals and the discussion of the project.

We also wish to express our gratitude to Professor R. Rau for his help in the solution of many technical problems connected with the technique of rear illuminated cloud chambers.

APPENDIX

We studied about one thousand oscilloscope pictures of chamber events. The results are tabulated in Table IV with the description of the terminologies used in the table. The pulse-height scale is comparative, and for pulse heights higher than 30, a saturation effect exists.

The useful events for us (nuclear events) are only about 14 percent of the cloud-chamber pictures taken. The actual events we took into our analysis are about 10 percent of the total pictures taken. It is true that if we used a higher bias (for instance, required pulses higher than 24 or so), then we could have increased the efficiency by eliminating a rather large percentage of undesired events at low pulse heights, but we feared that this might bring in some bias in our selection.

PHYSICAL REVIEW VOLUME 88, NUMBER 1 OCTOBER 1, 1952

The Variational Method for Problems of Neutron Diffusion and of Radiative Transfer

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(Received June 17, 1952)

The functional as proposed by R. E. Marshak for solving certain inhomogeneous integral equations by the variational method is modified in such a way that the simultaneous equations which determine the parameters become linear. Thus we can solve this type of integral equations with an accuracy as high as required without much labor. As an illustration we obtain by the present method an approximate solution of Milne's integral equation for the neutron density (or for the source function in the problem of radiative transfer) in a simple form. Our approximate solution agrees (for the first four figures) everywhere with the one computed by C. Mark from the exact solution.

MARSHAK[1] has formulated a variational method for certain inhomogeneous integral equations concerned with neutron diffusion problems. Let the inhomogeneous integral equation be

$$q_0(x)=\int K(x,x')q_0(x')dx'+f(x), \tag{1}$$

where $K(x,x')$ is a positive symmetric kernel, and $f(x)$ is bounded so that $\int|f(x)|dx$ exists. Marshak finds that the functional,

$$\frac{\int q(x)\left\{q(x)-\int K(x,x')q(x')dx'\right\}dx}{\left[\int q(x)f(x)dx\right]^2}, \tag{2}$$

is an extremum[2] for the actual solution, $q_0(x)$, and can be equated to $k_1/[q_0(\infty)-k_2]$ provided that we can write

$$q_0(\infty)=k_1\int q_0(x)f(x)dx+k_2, \tag{3}$$

where k_1 and k_2 are two constants. LeCaine[3] applies this method to Milne's problem and obtains an approximate solution in simple analytical form with high accuracy.

As can be seen from (2), variation of Marshak's functional leads to a set of simultaneous equations of second degree for the varying parameters. Hence it would be very difficult to solve these simultaneous equations when we have more than two parameters in the trial function. We shall here slightly modify the functional in order to linearize the equations which determine the parameters.

Assume that $q_0(x)$ approaches zero rapidly so that $\int x^n q_0(x)dx$ exists (n is a positive integer). We define

* J. S. Guggenheim Memorial Foundation Fellow.
[1] R. E. Marshak, Phys. Rev. 71, 688 (1947).
[2] B. Davison, Phys. Rev. 71, 694 (1947), has discussed the conditions under which (2) is a minimum.
[3] J. LeCaine, Phys. Rev. 72, 564 (1947).

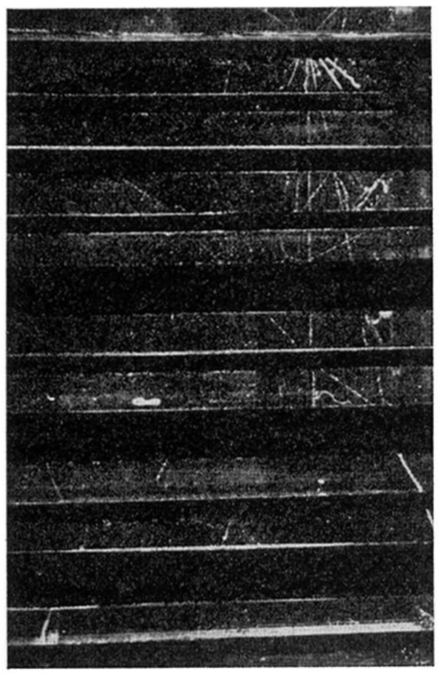


FIG. 10. A nuclear event in the crystal with two p.p. emitted. One of them interacts in the seventh plate (carbon) to give a heavy prong and one light particle, probably one $\pi^{\pm}$ meson directed upward.

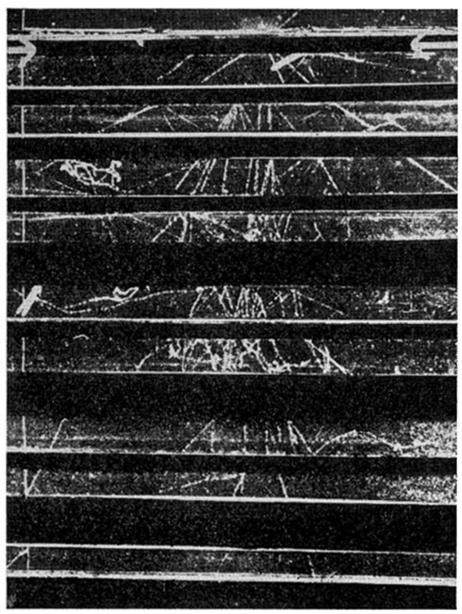
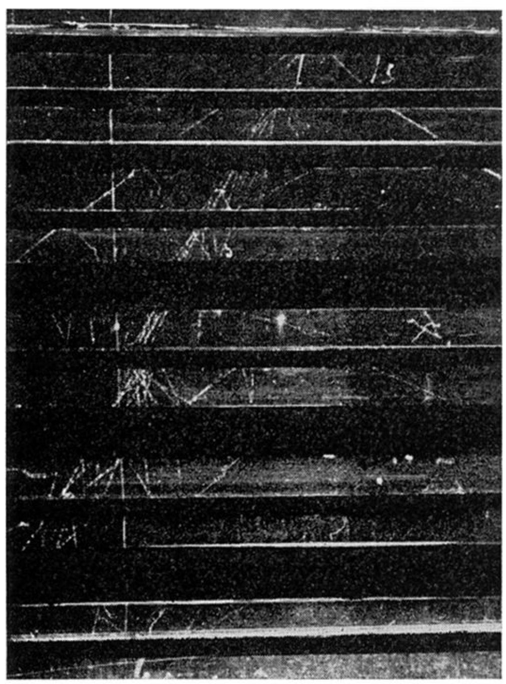


FIG. 7. A representative cloud-chamber photograph. The crystal plate is indicated by the arrows. This is a nuclear event in which no p.p. and one π^0 were created. The two electromagnetic cores are clearly due to photons: the π^0 had an energy higher than 1000 Mev.



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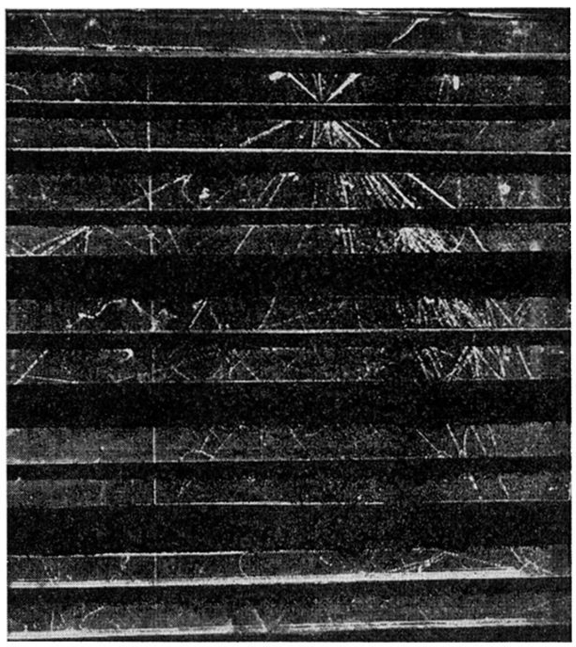

FIG. 9. An event produced in the first lead plate. We have a few of these events, which indicate how the crystal could be an efficient detector also for interactions produced outside.