tentatively be ascribed to the existence of a number of filaments so thin that their critical field has risen in a manner comparable with that observed in superconductors of small dimensions.[7]

Figure 3 shows the critical field-temperature curves for the bulk material and for the filaments as deduced from these measurements. It appears that the normal transition temperature of the filaments is the same as or slightly lower than that of the bulk material.

This work forms part of the program of the Low Temperature and Solid State Physics group, and the writer is grateful to Dr. D. K. C. MacDonald for very helpful discussions.

[7] R. B. Pontius, Phil. Mag. 24, 787 (1937).

PHYSICAL REVIEW VOLUME 88, NUMBER 2 OCTOBER 15, 1952

Neutron Capture Cross Sections*,†

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Cross section formulas for neutron capture are developed by means of statistical theory, for intermediate and heavy weight nuclei for energies of the order of 1 kev to a few Mev. The formulas do not exhibit resonances, but are rather averages over resonances in the energy region where these exist. The case is treated where the residual nucleus levels are well separated, although the method is applicable in principle where these levels form a virtual continuum. The processes competing with capture in the energy range considered are elastic and inelastic scattering. The energy dependence of radiation widths is estimated. The ratio of energy level spacing to radiation width at the dissociation energy of a neutron enters as a parameter in the cross-section formulas. This ratio is chosen to make experiment and theory agree as well as possible. Taking the spacing of levels of a given spin and parity at the dissociation energy of a neutron to be 25 ev, the values of the radiation widths are found to be 0.08 ev for Ag^{108} and In^{116}, 0.10 ev for Ag^{110}, and 0.20 ev for Au^{198}.

INTRODUCTION

THE cross sections of atomic nuclei for bombarding particles of low enough energy are characterized by resonances and are explained by the well-known Breit-Wigner type formulas. For increasing kinetic energy of the bombarding particles, the resonances become more and more closely spaced until finally they overlap, i.e., the cross sections become smoothly varying functions of energy. Formulas will be developed here for neutron capture cross sections representing an average over resonances in the resonance region and the actual cross sections where they become smooth functions of energy. The validity of the compound nucleus concept underlies the development so that one cannot use this treatment for excessively high energies. (For energies of the order of 50 Mev the compound nucleus ceases to have any meaning, since the mean free path of the bombarding particles becomes of the order of size of a nuclear diameter.[1]) The paper may be considered a sequel to those of Feshbach, Peaslee, and Weisskopf[2]; and Feshbach and Weisskopf,[3] which consider nuclear reaction theory in which the target and bombarding particles are taken to have spins of zero. The approach here follows a similar one for inelastic scattering of neutrons by Hauser and Feshbach.[4]

The treatment is subject to the validity of the usual assumptions of statistical theory. In particular, the expression (17) for the partial neutron widths is used, an expression which can be true only on the average. Fairly large fluctuations from the widths given by (17) must be expected for any individual level. Since the (n, γ) cross sections are the result of competitions among several emissions by the compound nucleus, it is expected that these fluctuations are cancelled out to a large extent.

DERIVATION OF CROSS SECTION FORMULAS

Let i be the spin quantum number of the target nucleus. This spin can combine with the incident neutron spin $s=\frac{1}{2}$ to give a "channel spin" quantum number $j=j^{(\pm)}=i\pm\frac{1}{2}$ if $i\neq 0$, with azimuthal quantum number $m=-j, -j+1, \cdots, j$.[5] Let the z axis be chosen in the direction of the incident neutron beam. The cross section for the formation of a compound nucleus of spin J, by incident neutrons of orbital angular momentum l and energy E in the channel of spin j with z component m then, following H.F., is written

$$\sigma(l, j, J, m, E)=(2l+1)\pi\lambdabar^2 T_l(E)\,|(lj0m|Jm)|^2, \quad (1)$$

where $2\pi\lambdabar$ is the wavelength of the incoming neutrons, $(lj0m|Jm)$ is the Clebsch-Gordan coefficient relating to

* This work was supported by the ONR and AEC.
† Part of a doctoral thesis submitted to the Physics Department at the Massachusetts Institute of Technology.
[1] V. F. Weisskopf, Helv. Phys. Acta **XXIII**, 187 (1950).
[2] Feshbach, Peaslee, and Weisskopf, Phys. Rev. **71**, 145 (1947).
[3] H. Feshbach and V. F. Weisskopf, Phys. Rev. **76**, 1550 (1949).
[4] W. Hauser and H. Feshbach, Phys. Rev. **87**, 366 (1952). This paper will be called H.F.
[5] If $i=0$ the channel spin is $\frac{1}{2}$.

TABLE I. Energies, spins, and parities of low-lying states of Ag^{107}, Ag^{109}, In^{115}, and Au^{197}.

Target	Ag^{107}			Ag^{109}		In^{115}			Au^{197}				
n	0	1	2	0	1	0	1	2	0	1	2	3	4
E_n, Mev	0	0.094	0.94	0	0.088	0	0.338	1.04	0	0.077	0.268	0.541	1.22
i_n	1/2	7/2	···	1/2	7/2	9/2	1/2	···	3/2	(3/2)	(5/2)	(11/2)	···
Parity	odd	even	···	odd	even	even	odd	···	even	(even)	(even)	(odd)	···

the probability that l and j with z components 0 and m combine vectorially to give a spin J with z component m. The $T_l(E)$ are wave mechanical penetration factors due to the sharp change in potential at the nuclear surface and to the centrifugal barrier. For electrically charged particles, there is in addition the effect of the Coulomb barrier. The $T_l(E)$ depend on the nuclear radius R. For neutrons Feshbach and Weisskopf[3] show that

$$T_l(E)=4xXv_l/[X^2+(2xX+x^2v_l')v_l], \qquad (2)$$

where $x=R/\lambda\!\!\!^{-}$, $X^2=X_0^2+x^2$ with $X_0\simeq 10^{13}\times(R \text{ in cm})$,

$$v_l=|x[j_l(x)+in_l(x)]|^{-2},$$
$$v_l'=|(d/dx)\{x[j_l(x)+in_l(x)]\}|,^2$$

$j_l(x)$ and $n_l(x)$ being spherical Bessel and Neumann functions. The notation in (2) is that of Blatt and Weisskopf,[6] not that of the original authors.

The part of (1) due to neutron capture is then

$$\sigma(l,j,J,m,E)\Gamma_r^{(J)}/\Gamma^{(J)}, \qquad (3)$$

where $\Gamma_r^{(J)}$ and $\Gamma^{(J)}$ are the radiation and total widths of the compound nucleus of spin J at the appropriate excitation energies. Summing over the possible J and l values and averaging over j and m, one gets the neutron capture cross section for unpolarized neutrons, if $i\neq 0$,

$$\sigma_{\text{cap}}=\frac{\pi\lambda\!\!\!^{-2}}{2(2i+1)}\sum_{l=0}^{\infty}\left[T_l(E)\sum_{J=0}^{\infty}\frac{\epsilon_{jl}{}^J(2J+1)\Gamma_r^{(J)}}{\Gamma^{(J)}}\right], \qquad (4)$$

$$\begin{aligned}\epsilon_{jl}{}^J&=2, \quad |J-l|\leqslant j\leqslant J+l \quad \text{for } j=j^{(\pm)},\\ &=1, \quad |J-l|\leqslant j\leqslant J+l \quad \text{for } j=\text{only one of } j^{(\pm)},\\ &=0, \quad \text{otherwise;}\end{aligned}$$

if $i=0$, one obtains

$$\sigma_{\text{cap}}=\frac{\pi\lambda\!\!\!^{-2}}{2}\sum_{l=0}^{\infty}\left[T_l(E)\sum_{J=|l-\frac{1}{2}|}^{l+\frac{1}{2}}\frac{(2J+1)\Gamma_r^{(J)}}{\Gamma^{(J)}}\right]. \qquad (5)$$

In the energy range considered below, the Coulomb barrier renders the emission probability of charged particles negligible so that the total width $\Gamma^{(J)}$ can be taken to be the sum of the partial neutron widths and the radiation width $\Gamma_r^{(J)}$. The radiation width is to be determined semi-empirically. In order to compare (4) or (5) with experiment, the form of the radiation width, in particular, its increase with energy is needed. Formula (16) in the appendix gives an estimate of this energy dependence. Among other things, it depends on the form used for the density of levels of the compound nucleus. Whatever the energy dependence of radiation widths, however, the general features of the work presented here are preserved. The width for emission of a neutron of orbital angular momentum l' and energy E' from a nucleus of spin J is denoted by $\Gamma_N^{(J,l')}(E')$. The following estimate for the ratio of neutron width to radiation width is derived in the appendix:

$$\frac{\Gamma_N^{(J,l')}(E')}{\Gamma_r^{(J)}(B+E)}\simeq\frac{D^{(J)}(B)}{\Gamma_r^{(J)}(B)}T_{l'}(E')\times\frac{\int_0^B \epsilon^{2\lambda+1}\rho(B-\epsilon)d\epsilon}{\int_0^{B+E}\epsilon^{2\lambda+1}\rho(B+E-\epsilon)d\epsilon}\equiv\frac{D^{(J)}(B)}{2\pi\Gamma_r^{(J)}(B)}T_{l'}(E')f_\lambda(E). \qquad (6)$$

In the above, B is the binding energy of the neutron emitted, $B+E$ is the excitation of the compound nucleus, $D^{(J)}(\epsilon)$ is the spacing of levels of the compound nucleus of spin J and given parity at excitation, ϵ, $\rho(\epsilon)$ is the density of levels of all kinds of the compound nucleus at excitation ϵ, and 2^λ is the multipole type of the radiation in the gamma-ray decay of the

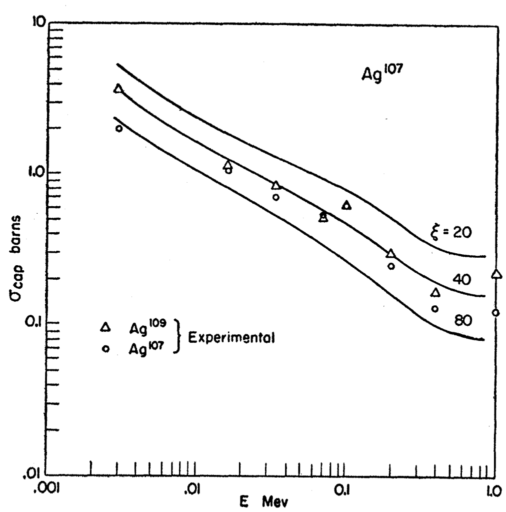

FIG. 1. Theoretical and experimental values for neutron capture cross sections for Ag^{107}, Ag^{109}.

[6] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952).

compound nucleus. It is assumed that a single multipole predominates.

Using (6) the form of (4) becomes

$$\sigma_{\text{cap}} \simeq \frac{\pi \lambda\!\!\!^{-2}}{2(2i+1)} \sum_{l=0}^{\infty} \left[T_l(E) \sum_{J=0}^{\infty} \right.$$

$$\left. \times \frac{\epsilon_{jl}{}^J (2J+1)}{1+\xi_J f_\lambda(E) \sum_{l'} \sum_n \epsilon_{j_n l'}{}^J T_{l'}(E-E_n)} \right] \equiv \sum_{l=0}^{\infty} \sigma^{(l)}, \quad (7)$$

where

$$\xi_J = D^{(J)}(B) / [2\pi \Gamma_r{}^{(J)}(B)]. \quad (8)$$

E_n is the energy of the nth excited state of the residual nucleus for $n>0$, $E_0=0$ being the ground-state energy; the corresponding spins are denoted by $i_n(i=i_0)$ and $j_n = i_n \pm \frac{1}{2}$. The sum over l', the angular momenta of the scattered neutrons must include only those terms that conserve the parity of the system, i.e., for each E_n there must be only odd or even l' values depending on whether the parity of the ground state and the nth excited state are opposite or the same. The sum over n is to include only excited states such that $E_n < E$, of course.

EXAMPLES

Silver

The capture of neutrons by Ag^{107} will be considered in the energy region from $E=3$ kev up to the point where inelastic scattering due to the second excited state becomes important, near $E=1$ Mev. Table I lists the spins, energies, and parities of pertinent states of the residual nucleus. The spin and parity of the E_1 level has been determined by Goldhaber and Sunyar[7] from its lifetime (44 sec) and from its internal conversion coefficients. The other information on Ag has been obtained from *Nuclear Data.*[8] The following

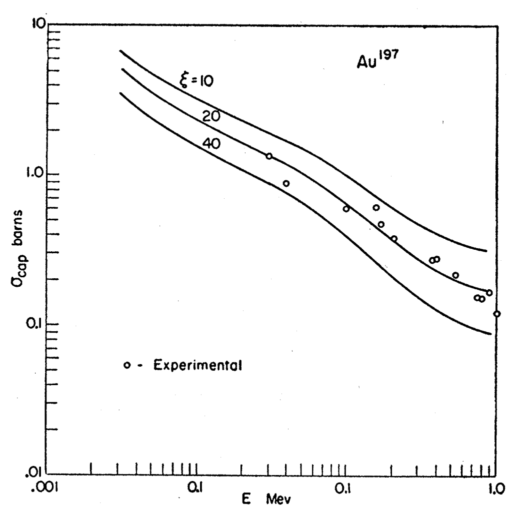

FIG. 2. Theoretical and experimental values for neutron capture cross sections for Au^{197}.

[7] M. Goldhaber and A. W. Sunyar, Phys. Rev. **83**, 906 (1951).
[8] *Nuclear Data*, Circular No. 499 (1950).

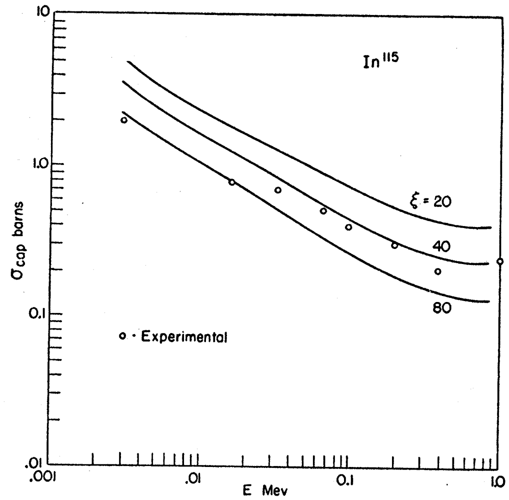

FIG. 3. Theoretical and experimental values for neutron capture cross sections for In^{115}.

are sample $\sigma^{(l)}$ for this case:

$$\sigma^{(0)} = \frac{\pi \lambda\!\!\!^{-2}}{4} T_0(E) \left[\frac{1}{1+\xi_0 a_0} + \frac{3}{1+\xi_1 a_1} \right],$$

$$\sigma^{(1)} = \frac{\pi \lambda\!\!\!^{-2}}{4} T_1(E) \left[\frac{1}{1+\xi_0 b_0} + \frac{6}{1+\xi_1 b_1} + \frac{5}{1+\xi_2 b_2} \right], \quad (9)$$

$$\sigma^{(2)} = \frac{\pi \lambda\!\!\!^{-2}}{4} T_2(E) \left[\frac{3}{1+\xi_1 a_1} + \frac{10}{1+\xi_2 a_2} + \frac{7}{1+\xi_3 a_3} \right],$$

where

$$\begin{aligned}
a_0 &= f_\lambda(E)[T_0(E)+T_3(E-E_1)],\\
a_1 &= f_\lambda(E)[T_0(E)+T_2(E)+2T_3(E-E_1)+T_5(E-E_1)],\\
a_2 &= f_\lambda(E)[2T_2(E)+T_1(E-E_1)+2T_3(E-E_1)\\
&\qquad +2T_5(E-E_1)],\\
a_3 &= f_\lambda(E)[T_2(E)+T_4(E)+2T_1(E-E_1)\\
&\qquad +2T_3(E-E_1)+2T_5(E-E_1)+T_7(E-E_1)],\\
b_0 &= f_\lambda(E)[T_1(E)+T_4(E-E_1)],\\
b_1 &= f_\lambda(E)[2T_1(E)+T_2(E-E_1)+2T_4(E-E_1)],\\
b_2 &= f_\lambda(E)[T_1(E)+T_3(E)+2T_2(E-E_1)\\
&\qquad +2T_4(E-E_1)+T_6(E-E_1)]. \quad (10)
\end{aligned}$$

Indium and Gold

The information on the lowest three levels of In^{115} that appears in Table I has been taken from references 6 and 7. For Au^{197} Huber *et al.*[9] have found the first three excited levels given in Table I. In accordance with the lifetime of the 0.541-Mev level (7.3 sec) and the gamma-ray decay scheme, and using predictions of shell theory, spins and parities have been written down. These values are perhaps not as certain as those for In and Ag and are in parentheses. The $\sigma^{(l)}$ for In^{115} and Au^{197} will not be written down, but curves of σ_{cap} against

[9] O. Huber *et al.*, Helv. Phys. Acta **XXIV**, 127 (1950).

TABLE II. $f_\lambda(E)$, part of the energy dependence of radiation widths.

$x=R/\lambda\!\!\!^-$	E, Mev	$f_1(E)$	$f_2(E)$
0	0	1	1
0.1	0.00293	1.00	...
0.2	0.0117	0.99	...
0.3	0.0264	0.97	...
0.4	0.0469	0.95	...
0.7	0.144	0.84	...
...	0.384	0.64	0.60
1.2	0.422	0.61	...
...	0.769	0.41	0.36
1.7	0.847	0.38	...
	$\alpha B=210$, $\alpha=26$ Mev^{-1}		

energy are given. The energy range considered is the same as for Ag.

Before one can make calculations with (7), one must consider the set of parameters ξ_J. These will be taken to be independent of J, $\xi_J=\xi$. The validity of this will be discussed in the appendix.

COMPARISON WITH EXPERIMENT

Figures 1, 2, and 3 show the theoretical values of σ_{cap} plotted against the energy of the bombarding neutrons for Ag^{107}, Au^{197}, and In^{115} targets for several values of ξ. At the maximum energy considered, $E=0.847$ Mev, l values from 0 to 4 contribute appreciable $\sigma^{(l)}$. The value of the nuclear radius in all cases was taken to be $R=8\times10^{-13}$ cm. To calculate $f_\lambda(E)$ the form for the density of energy levels of the compound nucleus was taken to be

$$\rho(E)=C\exp[(\alpha E)^{\frac{1}{2}}], \tag{11}$$

with $\alpha=26$. B was taken to be 8 Mev. Values of $f_\lambda(E)$ are tabulated in Table II. The cross section plotted in Figs. 1, 2, and 3 are for $\lambda=1$.

Figure 4 has σ_{cap} curves for Ag^{107} with $\xi=40$ showing the effect of (a) neglecting inelastic scattering, (b) changing the parity of the E_1 level. There is also a curve (c) for Au^{197} with $\xi=40$ to show the sensitivity of σ_{cap} to the level scheme of the excited states of the target nucleus. From curve (a) it is seen that inelastic scattering is not a negligible effect in calculating σ_{cap}.

For Ag^{107} and In^{115} the best agreement of theory with the experiments of Segrè *et al.*[10,11] and Hughes and Sherman[12] is obtained with $\xi\simeq50$. For Ag^{109}, $\xi\simeq40$. (Experimental points for Ag^{109} as well as Ag^{107} are plotted in Fig. 1, since the lowest two levels of these isotopes have very similar properties.) For Au^{197} a good fit of theory to experiment is obtained with $\xi\simeq20$. The experimental results have been compiled by Adair.[11]

Using (8) and taking a level spacing $D^{(J)}(B)=25$ ev, the radiation widths are for In^{116} and Ag^{108}, $\Gamma_r(B)=0.08$ ev; for Ag^{110}, $\Gamma_r(B)=0.10$ ev; for Au^{198}, $\Gamma_r(B)=0.20$ ev. For dipole radiation using (16) the widths are 1.3 times as large at an excitation 0.85 Mev higher.

[10] Segrè, Greisen, Linenberger, and Miskel (unpublished).
[11] R. K. Adair, Revs. Modern Phys. **22**, 249 (1950).
[12] D. J. Hughes and D. Sherman, Phys. Rev. **75**, 632 (1950).

Professor H. Feshbach suggested this problem. I wish to thank him and Professor V. F. Weisskopf for valuable suggestions and criticisms. I thank too Miss Hannah Paul who made many of the calculations.

APPENDIX

Weisskopf[13] shows that the probability per unit time of a radiative transition of multipole 2^λ (electric or magnetic) from a nuclear quantum state a to a state b of energy ϵ lower is given by

$$T_\lambda(a,b)\propto\epsilon^{2\lambda+1}|M_\lambda|^2, \tag{12}$$

the proportionality constant depending only on λ, the nuclear radius R, and fundamental physical constants. M_λ is the matrix element for the transition, and Blatt

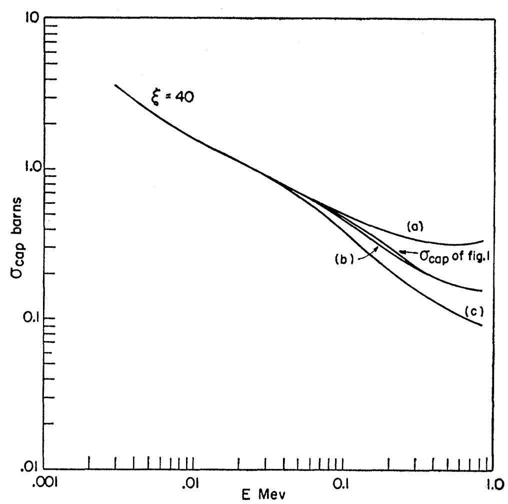

FIG. 4. Theoretical curves showing the effect on the neutron capture cross section for Ag^{107} of (a) neglecting inelastic scattering and (b) changing the parity of the first excited state. The curve (c) is the cross section for Au^{197}, to show the sensitivity of σ_{cap} to the level scheme of the target nucleus. All curves are for $\xi=40$.

and Weisskopf[6] show that its magnitude can be written in the form

$$|M_\lambda|^2=n_\lambda(a,b)D_a, \tag{13}$$

where D_a is the spacing of energy levels of the same spin and parity as a at the energy of a, a strongly varying function of energy, and $n_\lambda(a,b)$ is a slowly varying function. One can write then an approximation

$$T_\lambda(a,b)\simeq K_\lambda\epsilon^{2\lambda+1}D_a, \tag{14}$$

where K_λ is a constant.

Consider a nucleus with initial excitation B and having many levels between this initial excitation and the ground state. Then on a statistical basis the probability per unit time of a transition of multipole 2^λ from the state of energy B to one of energy ϵ to $\epsilon+d\epsilon$ lower is given by

$$P(\epsilon)d\epsilon\simeq K_\lambda\epsilon^{2\lambda+1}D(B)\rho_\lambda(B-\epsilon)d\epsilon, \tag{15}$$

[13] V. F. Weisskopf, Phys. Rev. **83**, 1073 (1951).

where $D(B)$ is the spacing of levels of the same spin and parity as the initial state at energy B, and $\rho_\lambda(B-\epsilon)$ is the density of levels at energy $B-\epsilon$ that can be reached through the radiative transitions of multipole 2^λ. The radiation widths are obtained by integrating $\hbar P(\epsilon)$ from $\epsilon=0$ to B.

The ratio of radiation widths of states of spin J and given parity with excitations B and $B+E$ then takes the following form:

$$\frac{\Gamma_r^{(J)}(B+E)}{\Gamma_r^{(J)}(B)} \simeq \frac{D^{(J)}(B+E)}{D^{(J)}(B)} \cdot \frac{\int_0^{B+E} \epsilon^{2\lambda+1}\rho(B+E-\epsilon)d\epsilon}{\int_0^{B} \epsilon^{2\lambda+1}\rho(B-\epsilon)d\epsilon} \equiv \frac{1}{f_\lambda(E)} \cdot \frac{D^{(J)}(B+E)}{D^{(J)}(B)}, \quad (16)$$

where $D^{(J)}(B)$ is the spacing of levels of spin J and given parity at excitation B. It is assumed that for all spins J the energy dependence of the level densities is as in (11), the constant preceding the exponential alone having J dependence. This allows the use of ρ, the density of levels of all types, instead of ρ_λ on the right side of (16).

Many authors, for example, Feshbach, Peaslee, and Weisskopf,[2] have shown that

$$\Gamma_N^{(J,l')}(E') \simeq T_{l'}(E')D^{(J)}/2\pi, \quad (17)$$

where the value of $D^{(J)}$ is to be taken at the excitation energy of the compound nucleus. Equation (6) follows directly from (16) and (17).

ρ_λ has no obvious J dependence. Hence, from (15) it can be seen that the main dependence of $\Gamma_r^{(J)}(B)$ is in its proportionality to $D^{(J)}(B)$. This is the basis for taking ξ_J to be independent of J.

PHYSICAL REVIEW VOLUME 88, NUMBER 2 OCTOBER 15, 1952

The Spread of the Soft Component of the Cosmic Radiation

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(Received June 3, 1952)

The fundamental diffusion equations describing the three-dimensional development of the electron-photon component of the cosmic radiation, are formulated. These equations take into account exactly ionization loss and variation of density in the medium, and enable one to determine all angular and radial moments of the distribution functions, as explicit functions of depth and energy for arbitrary initial conditions.

The exact general solution of these equations obtained in this paper are readily adapted to any physical situation of interest. The method is similar to that devised by the authors in considering the three-dimensional development of the nucleon component. The only approximations involved are those inherent in the Bethe-Heitler cross sections in the full screening approximation, and the neglect of angular deflections in processes other than elastic Coulomb scattering.

It is shown that all previous work is subject to very large errors on the following counts: (1) the neglect of fourth and higher angular moments for the Coulomb scattering in Landau's equation and equivalent integral equations; (2) the neglect of variation of density in the atmosphere—which alone can lead to errors as high as 5000 percent; (3) elimination of the depth dependence either by integration over all depths, or evaluation in the neighborhood of the cascade maximum; (4) miscellaneous errors introduced in the evaluation of already approximate integrals; (5) use of results due to Moliere, hitherto unpublished in detail, which involve errors of several orders of magnitude in the higher moments; and also in the distribution functions concerned.

No calculation of the actual distribution functions in the atmosphere or elsewhere has yet been made on the basis of a realistic physical model. The results obtained in this paper will allow the authors to do this in the future.

1. INTRODUCTION

It is difficult to find any topic in theoretical physics which has received so much attention, with results so lacking in precision, as the spread of the soft component of the cosmic radiation in the atmosphere. Both the physical assumptions and the mathematical techniques employed have been of such a crude nature that no confidence whatever can be placed in either the qualitative or quantitative aspects of the theories advanced.

All theories hitherto put forward which have not consisted of purely qualitative considerations, have been based either on equations due to Landau[1] or on completely equivalent integral equations due to Roberg and Nordheim.[2] These equations are actually inapplicable to the atmosphere, since they relate only to media of constant density. Janossy[3] has argued that "no great error arises from the fact that the variation of the cascade unit with air density has been neglected," but our exact calculations will show that the error actually attains a maximum of 5000 percent. Another defect of the equations of Landau, and Roberg and Nordheim, is the neglect of the higher angular moments of the Coulomb scattering of the electrons. In consequence of this neglect, the mean square angular deviation of particles from the shower axis can be calculated accurately for a medium of constant density, but the mean fourth power obtained is in error by 18 percent, the mean sixth power by 45 percent, and higher moments are completely inaccurate. Similar corrections apply to

[1] L. Landau, J. Phys. (U.S.S.R.) 2, 234 (1940).
[2] J. Roberg and L. W. Nordheim, Phys. Rev. 75, 444 (1949).
[3] L. Janossy, *Cosmic Rays* (Oxford University Press, London, 1948).