ground state of B^{10}, and he concluded from the shape of the distribution that the ground state is of even parity. Actually Schecter was observing the first four or five states of B^{10}, but his conclusion was not incorrect since apparently all these states are of even parity and therefore have the same characteristic distribution.

Throughout this paper, the Butler curves have been drawn neglecting the spin factor $(2J+1)/[(2s_p+1)\times(2j+1)]$ in Eq. 34 of Butler's paper.[2] It was decided to assume spins of 3, 1, 0, 1, and 2, respectively, for the first five states of B^{10} and to plot the ratio of the integrated Butler equations for the $l_p=1$ cases, with the nuclear factors considered constant, to the relative total experimental cross sections. Figure 14 indicates that if the assumed spin assignments were correct, then the nuclear factors affecting the stripping process are constant within a factor of about 3 for these states.

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The Energy Spectrum of Positrons from the Decay of the μ-Meson*

H. J. BRAMSON, A. M. SEIFERT, AND W. W. HAVENS, JR.
Columbia University, New York, New York

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The energy of the positrons from $301\pi\rightarrow\mu\rightarrow\beta$ decays have been determined in electron sensitive emulsions by measuring the multiple scattering of the positron tracks. The annihilation properties of the charged particle emitted in the decay of the μ^+ meson confirm its identity as a positron. This spectrum has a maximum about 36 Mev and a nonzero cutoff at the high energy end. A statistical analysis of the data, using the Michel theory, yields a value of $\rho=0.41\pm0.13$.

I. INTRODUCTION

EARLIER experiments[1] have demonstrated that the β-decay of the μ-meson is consistent with the scheme

$$\mu\rightarrow\beta+\nu+\nu. \quad (1)$$

However, the possibility that β-decay of the μ-meson might be merely one phase of a universal interaction between fermions has raised interest in the exact shape of the spectrum of the β-particle. It is here that previous experiments have given contradictory results, due either to poor energy resolution of individual β-particles, to the low number of such events obtained, or both. A brief report on the preliminary results of our measurements has been given previously.[2]

II. METHOD

A. Detector

Minimum ionization emulsions were used to detect the $\pi\rightarrow\mu\rightarrow\beta$ decay. Emulsions have several important advantages in a study of the β-decay of the μ-meson. The grain density for all β's of relativistic energies is substantially the same;[3] hence, all decay β's are recorded with virtually equal sensitivity. Whereas it is characteristic of other methods of detection that massive material is used to stop the μ-meson and the decay β observed only after emergence from this "dead mass," the β-track is unobscured at any stage when emulsions are employed. Emulsions are also able to be used inside the 381-Mev Nevis cyclotron, where the meson flux is such that even in restricting entrance to the emulsion to pions of an 11- to 14-Mev band and less than a $\pm10°$ spread, some 10^4 pions/sec/cm^3 of emulsion are obtained.

Problems peculiar to the photographic technique for this purpose include the substantial labor required for high energy resolution per track through multiple scattering measurements, the discrimination against low energy β's because of outscattering from an emulsion of finite depth, and the necessity to obtain an accurate calibration of energy with multiple scattering measurements. As with other techniques, the uncertainty in energy degradation must be evaluated. These points will be discussed in greater detail below.

B. Multiple Scattering Measurements

The technique of determining the energy of a fast particle through the average magnitude of Coulomb scatterings in emulsion has been analyzed by Gold-

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[1] Leighton, Anderson, and Seriff, Phys. Rev. **75**, 1432 (1949); Davies, Locke, and Muirhead, Phil. Mag. **40**, 1256 (1949); J. Steinberger, Phys. Rev. **75**, 1136 (1949); Sagane, Gardner, and Hubbard, Phys. Rev. **82**, 557 (1951).

[2] H. J. Bramson and W. W. Havens, Jr., Phys. Rev. **83**, 861 (1951).

[3] E. Pickup and L. Voyvodic, Phys. Rev. **80**, 89 (1950).

schmidt-Clermont.[4] A variation developed by Fowler[5] has been used, consisting of measuring the ordinates of a track at fixed intervals, and obtaining from the second differences a measure of the average scattering angle. The mathematically equivalent Molière[6] and Snyder-Scott[7] theories then permit a unique equivalence of average scattering angle with energy.

In principle, the only serious limitations in energy resolution of a particular trajectory are those given by the number of scattering angles n that can be obtained and by the abrupt radiation losses that may be masked by the normal fluctuations in Coulomb scattering. The first factor decreases with track length as $\sigma_{\text{stat}}=0.80/\sqrt{n}$, the latter increases with track length as $\sigma_{\text{rad}}=(\text{const})L$, where L is the track length. Thus ideally, maximum energy resolution would be obtained with an L sufficiently long so that $\sigma_{\text{stat}}=\sigma_{\text{rad}}$. In practice, track lengths employed are considerably shorter than this optimum length. First, the thickness of emulsions that can be used with the high resolution microscope lenses, which are necessary to minimize errors in ordinate readings, discriminate seriously against slow particles remaining in the emulsion for this optimum distance. Second, improved energy resolution must be balanced against obtaining a greater number of events.

For these reasons, a minimum track length of 1500 microns was adopted as the criterion of acceptance for analysis on the 600 micron thick emulsions, and 2500 microns long on the 1000 micron thick emulsions. The magnitude of this outscattering effect is shown in Fig. 1. For the minimum track lengths used the effect of outscattering on the determination of the Michel parameter ρ is negligible (see below).

Where a β-trajectory of a prescribed length is to be analyzed, the shortest interval between ordinate readings (cell length) is employed which for that trajectory will result in a nonobjectionable error in the final estimate of average scattering angle. Both by using a "straight line" obtained from the Bausch and Lomb ruling engine and a test devised by Corson,[8] the minimum average error in second differences of ordinates obtainable was found to be 0.03 micron. With this low value of reading noise, the effect on the square of the true scattering angle is less than 1 percent even for the fastest particles. The ability to use cell lengths of 25 to 60 microns (depending on track energy and length available) resulted in an average energy resolution due to the statistical factor n of 10 percent.

To reduce the tedium of realigning the β-trajectory relative to the base line during scattering measurements, and to provide a reliable, accurate movement of the trajectory along the base line, a special stage motion was devised for this experiment. The rotating turntable of a Spencer research microscope was removed from its stand and attached to a heavy carriage that rode on a lathe bed. Thus any arbitrary direction in the emulsion could be rotated about a mechanical axis that coincided closely with the optic axis, and aligned with the direction of motion of the carriage. This carriage rested on a vee and flat of the bed over a length of 18 inches to reduce rotational play, and was pushed by a precision screw.

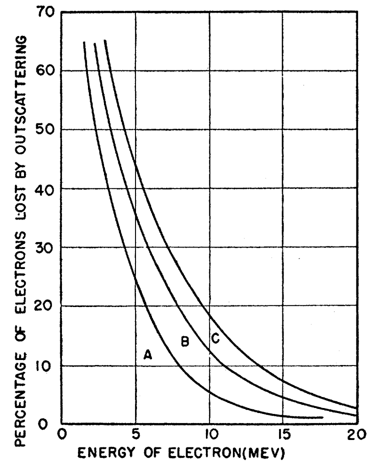

FIG. 1. Outscattering losses for various emulsion thicknesses for minimum track lengths adopted. Curve A gives the loss for the bulk of the data: 600μ thick emulsion and 1500μ minimum track length; curve B for the remainder of the data: 1000μ emulsion and 2500μ track length; curve C for a 200μ emulsion and 1000μ track length.

It is perhaps worth noting that the distance pushed by the screw is the cell length only if the dip of the track in the emulsion is ignored; otherwise, the cell length is greater by a factor of the secant of the angle of dip. This factor if ignored can cause an error in cell length of 1 percent for a track even as long as 4000 microns in a 600-micron emulsion, and as much as 8 percent in the extreme case of a 1500-micron track, which represents an error of 1.4 percent and 11 percent respectively in energy.

Finally, account was taken of the correction due to the averaging of grains about a given abscissa. This averaging, adopted to obtain a weighted mean ordinate and thus mitigate the grain fluctuations, causes the underemphasis of the peaks and valleys in a track, resulting in a falsely high energy. By restricting this region about the abscissa over which the grains were averaged to ± 10 percent, the error in the average scattering angle is less than 2 percent, and so has a negligible effect on the mean squared scattering angle.

C. Energy Degradation

In photographic emulsion where the energy of a particle is determined by a dispersion measure of a statistical distribution of the scattering along its track, a sudden loss of energy of considerable magnitude may be masked by the fluctuations normal to that distribution. Only when the number of scattering angles after the point of sudden loss is sufficient to establish a trend distinguishable from that of the earlier portion, can the loss be noted. Since the estimate of energy of the particle is based upon measurements made during energy degra-

[4] Y. Goldschmidt-Clermont, Nuovo cimento **7**, 1 (1950).
[5] P. H. Fowler, Phil. Mag. **41**, 169 (1950).
[6] G. Moliere, Z. Naturforsch. **2a**, 133 (1947); **3a**, 78 (1948).
[7] W. T. Scott, Phys. Rev. **85**, 245 (1952).
[8] D. R. Corson, Phys. Rev. **80**, 303 (1950).

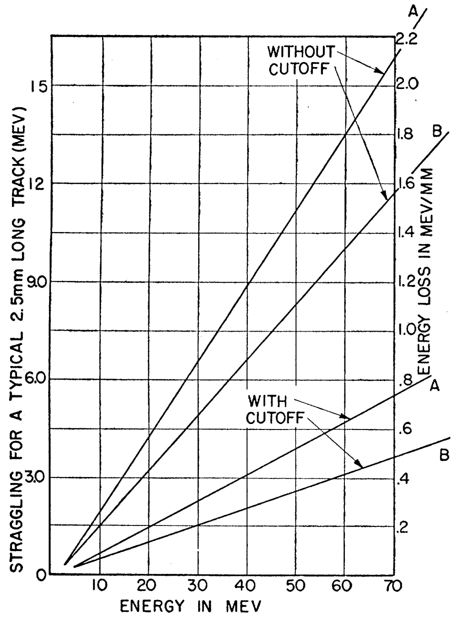

FIG. 2. A: Radiation loss and B: straggling of positrons in Ilford G-5 emulsion, with and without radiation cutoff.

dation, it becomes important to take into account the continual, but not continuous, loss of energy along the track.

The degradation of energy due to inelastic collisions resulting from excitation of atoms along the trajectory causes little difficulty, since the number of such collisions is sufficiently large ($>10^4$), for the shortest tracks observed, to reduce the straggling to a low value (about 0.3 Mev for the length tracks used). This is not the case for inelastic collisions resulting from radiation. While, as above, there is a high number of collisions in all tracks analyzed, the probability of losing a given amount of energy through many collisions of small loss each is approximately the same as of losing a large fraction in a single collision. This results in a large straggling about the mean radiation loss; e.g., for track lengths used in this experiment, about 8 percent of the time one radiative collision can be expected to occur with an energy loss of at least 30 percent.[9] Actually, in 301 positron tracks analyzed, some 21 were observed to suffer such large losses. If no allowances were made for these large losses, an average radiation loss of 2.7 Mev for a typical 2.5 mm long 40-Mev track would be accompanied by a straggling ($\sigma^2=\langle E^2\rangle_{\text{Av}}-\langle E\rangle_{\text{Av}}^2$) of 6.3 Mev about this average! Hence in a sense the small average radiation loss is deceptive; the large straggling about the average is more indicative of the accuracy to which energy losses are known.

This sizable straggling can be reduced by eliminating from consideration all positrons whose radiation losses exceed a given amount. The lower this cut-off radiation loss, the lower will be the average radiation loss and straggling. However, too low a cutoff will sacrifice many tracks already analyzed, again prejudicing against the total number of events in favor of better energy resolution per event. Since a cutoff at 30 percent loss maintains the radiation straggling small as compared to the statistical straggling, yet eliminates only 8 percent of the total number of analyzed events, this loss has been selected as cutoff. The effect of this cutoff upon the average radiation loss and straggling is shown in Fig. 2 for one mm of track and for a typical 2.5 mm long track, respectively. Since track lengths scanned are generally less than a tenth the radiation length, the total loss is very nearly proportional to the track length. The relative straggling, on the other hand, varies slowly with L, increasing by only 1 percent when L is doubled. For the example cited above, our cutoff reduces the average radiation loss to 1.0 Mev and the straggling to 2.0 Mev.

III. RESULTS

A. Annihilation of the Positrons

As has been shown previously,[2] the observed annihilation of the minimum ionization particle from μ-meson decay offers further substantiation of its positron nature. The study of these emitted charged particles has covered about 230 cm in emulsion in the course of which we have found five annihilations. According to Heitler,[10] the cross section for annihilation of a positron is given by

$$\phi=\frac{\pi r_e^2 m_e}{E}\left(\ln\frac{2E}{m_e}-1\right), \tag{2}$$

where m_e is the mass of the positron, and r_e the classical electron radius. Averaging over the constituents of G-5 emulsion and the energy spectrum of these particles, a value of 57 cm is obtained for the annihilation length. Thus the observed annihilation length is consistent with that of a positron.

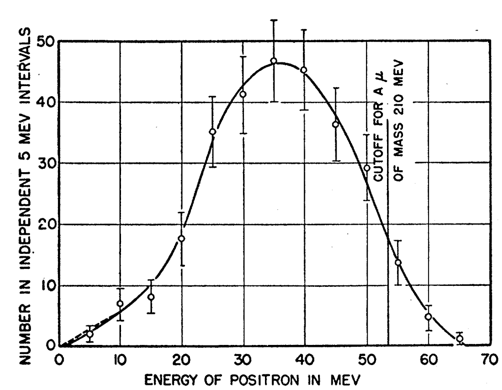

FIG. 3. Differential energy spectrum of 286 positrons from $\pi\rightarrow\mu\rightarrow\beta$ decay. Dashed portion at low energies gives correction for outscattering.

[9] H. Bethe and W. Heitler, Proc. Roy. Soc. (London) **146**, 83 (1934).

[10] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1947), second edition, p. 208.

B. Energy Spectrum of Positrons

Of the 301 tracks energy-analyzed by the technique of multiple-scattering measurements, 21 were observed to have lost more than the maximum energy (30 percent) acceptable under our criterion; of these, however, there were 6 for which the track length prior to the large energy loss was adequate to warrant using that section alone to salvage the event for the purpose of the energy spectrum. In Fig. 3, the unsmoothed spectrum for these 286 events is shown. The dashed portion of the curve at the low energy end indicates the extent of the calculated correction because of the outscattering referred to in Sec. IIB. This correction amounts to 0.4 percent of the mean of the untabulated, uncorrected energies, (35.7 Mev), and only 0.01 percent of $\langle E^4\rangle_{\mathrm{Av}}$ (see below).

In Michel's theory,[11] it is shown that for any linear combination of the five well-known β-decay interactions that may be responsible for the interaction energy of (1), the shape of the positron spectrum is given by

$$P(E)=(4E^2/W^4)[3(W-E)+\tfrac{2}{3}\rho(4E-3W)], \qquad (3)$$

where W represents the maximum energy available to the positron. The effect of a particular specification of the linear combination is contained in the single parameter ρ to a high degree of accuracy, which improves with increasing E. For the particular prescriptions similar to nucleon decay, e.g., $\mu\rightarrow\nu$, $e\rightarrow\nu$, ρ would be given by

$$\frac{3[(g_s-g_t)^2+(g_t+g_{ps})^2+2(g_v+g_{pv})^2]}{4[g_s^2+4g_v^2+6g_t^2+4g_{pv}^2+g_{ps}^2]}, \qquad (4)$$

and

$$\frac{3[g_s+g_{ps}-2(g_v+g_{pv})]^2}{2[(g_s+6g_t-g_{ps})^2+16(g_v-g_{pv})^2+2(g_s+g_{ps}-2g_v+g_{pv})^2]}, \qquad (5)$$

according to whether the neutrinos are distinguishable or indistinguishable[12] and where the g's are the usual coupling constants. For all combinations, $\rho<\frac{3}{4}$ for identical neutrinos, and <1 otherwise. For the particular case of the Critchfield-Wigner interaction,[13] unique in that it is independent of the order of the particles in the Hamiltonian, ρ always equals $\frac{1}{2}$.

The most that can be concluded from any positron spectrum is the value of ρ. For this purpose, two experimental limitations must inevitably be considered: the effect of the limited number of events comprising the spectrum, and the uncertainty involved in the energy measurement of each event. To compensate for the latter effect, the distribution of uncertainties in the measuring process is folded into the theoretical Michel curves to yield, for various values of ρ, the curves shown in Fig. 4. The few events observed at low energies, the large number at very high energies, and the position of the mode are all indicative of a ρ near $\frac{1}{2}$.

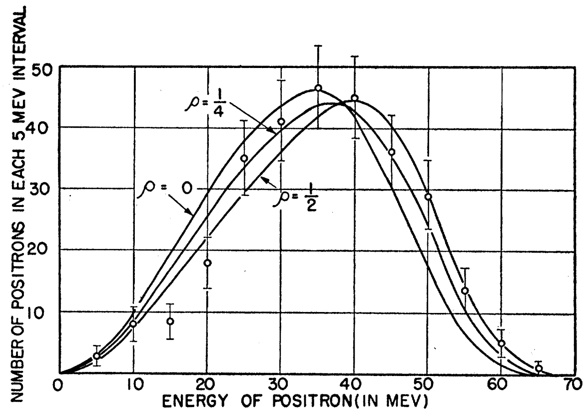

FIG. 4. Comparison of data with Michel curves for $\rho=0$, $\frac{1}{4}$, $\frac{1}{2}$, with the experimental error folded in.

C. A Statistical Estimation of ρ

The photographic and cloud chamber techniques are similar in that both yield a low number of satisfactory events (<1000). It therefore is desirable to exercise care in the choice of an optimum statistical criterion to estimate ρ. Two procedures have been offered which are based on the fact that $P(3W/4)$ is independent of ρ: (a) One[14] then compares the theoretical ratio of events before and after this E, (which is a linear function of ρ), with the experimental ratio, thus determining ρ. While this has the advantage of simplicity, it does not extract the maximum available information from the spectrum, effectively ignoring the spectral shape away from the $E=3W/4$ point. (b) The second[15] takes advantage of the fact that $P(E)/E^2$ is linear in E, and finds ρ as the slope of a least squares line. Unfortunately, the fewest number of events occur at the extreme energies, and although these have the least statistical accuracy, they exercise a high "lever-arm" on a least squares curve. A different approach is to use various moments of E of the experimental distribution. On taking these as estimates of the true moments, ρ follows from the relation

$$\mathrm{expectation}(E^i)=\frac{4W^i}{(i+3)(i+4)}\left\{3+\frac{2i\rho}{3}\right\}, \qquad (6)$$

which follows from (3). If we denote the value of ρ estimated in this fashion by ρ', then the straggling is a function of ρ itself:

$$\sigma_{\rho'}^2=\sigma_{\rho'}^2(\rho)$$
$$=\frac{(i+3)^2(i+4)^2}{16i^2n}\,\frac{(9+4i\rho)3}{(2i+3)(2i+4)}-\frac{4(9+2i\rho)^2}{(i+3)^2(i+4)^2}, \qquad (7)$$

[11] L. Michel, Proc. Phys. Soc. (London) **63**, 514 (1950).
[12] See, however, E. R. Caianiello, Phys. Rev. **86**, 564 (1952).
[13] L. Critchfield and E. Wigner, Phys. Rev. **60**, 412 (1941); L. Critchfield, Phys. Rev. **63**, 417 (1943).
[14] A. Lagarrique and C. Peyrou, J. phys. et radium **12**, 848 (1951).
[15] H. W. Hubbard, thesis, unpublished University of California report UCRL 1623 (1952).

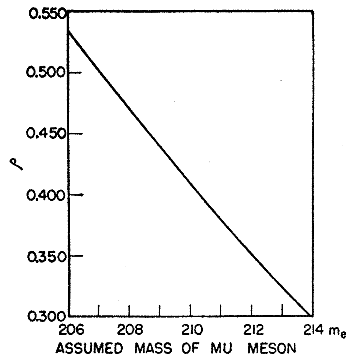

FIG. 5. Dependence of estimate of ρ with μ-meson mass assumed.

where n is the number of events ($=286$). For $\rho=\frac{1}{2}$, this has a minimum of 2.09 for the fourth moment ($i=4$), for an $n=1$.

One might wonder whether perhaps another means of estimating ρ might lead to a still smaller value of $\sigma_{\rho'}^2$. It is possible to compute the lower bound of this variance. Following Cramer,[16] for $\rho=\frac{1}{2}$ and $i=4$, this is found to be 2.08 for $n=1$. Hence, there is no other means of estimation which is theoretically capable of much further refinement. This in turn may be compared with the variance resulting from test (a), e.g., which for $\rho=\frac{1}{2}$, $i=4$, $n=1$ is 3.04.

A preliminary check of our data shows that $\rho\approx 0.5$. Hence from the above, it is most advisable to choose the fourth moment to estimate ρ. Further, this moment will tend to eliminate the slight effect of outscattering and the effect of small discrepancies in (2) at lowest energies. Our fourth moment of energy is 2.59×10^6 (Mev)^4, leading to an estimate of $\rho=0.41$ from the folded curves with a standard deviation of about 0.13. As shown in Fig. 5, this estimate depends on the mass assumed for the μ meson. Since the latter is known with considerable accuracy,[17] the resultant variation in ρ is small.

IV. DISCUSSION

Our earlier result of a distinct finite cutoff has persisted with little change, being quite compatible with a ρ of $\frac{1}{2}$, or a Critchfield-Wigner interaction. Since this value is higher than generally reported elsewhere, it is of interest to see where in this experiment bias could have been introduced to lead to higher energies. The usual pitfalls of the multiple-scattering technique—"noise" distortion and dip effects—would tend to lower the energy. The radiation correction does tend to increase energy slightly, but as indicated in II C, if any error were present one would tend to suspect that the loss was underestimated. Therefore, it is difficult to see an intrinsic error which will increase the energies. On the other hand, in techniques where the positron is observed only after emerging from a "dead mass," there is the possibility discussed above that occasional tracks are accepted which have lost more than average energy, before emerging from the absorber. This effect would be in the direction of lower ρ, and it is perhaps worthy of note that more recent Wilson cloud-chamber experiments with less dead mass present have resulted in higher values of ρ.

The authors are indebted to Dr. Helen Friedman and Dr. L. James Rainwater for the use of their exposure chamber and some of their plates, to Dr. H. Yukawa and Dr. R. Serber for helpful discussions, to Mr. R. Beckhofer and colleagues for a study on the statistical analysis of ρ, to Mrs. Carol Major and Marjorie Boehlert for painstaking scanning of our plates, and to members of the cyclotron crew, the nuclear emulsion group, and others too numerous for individual mention but whose assistance is deeply appreciated.

[16] H. Cramer, *Mathematical Methods of Statistics* (Princeton University Press, Princeton, 1946), pp. 477–485.

[17] Lederman, Booth, Byfield, and Kessler, Phys. Rev. **83**, 685 (1951).