PHYSICAL REVIEW VOLUME 88, NUMBER 1 OCTOBER 1, 1952

A New Gyromagnetic Effect in Permalloy and Iron

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(Received September 11, 1950)

A long circular cylinder of compressed ferromagnetic powder is magnetized along its axis to an intensity of magnetization J_z, near saturation, so that the moments of all of the magnetic elements point essentially in the same direction Z. In a direction X, normal to Z, a small magnetic intensity H_x with frequency f, such that $1/f$ is much greater than the relaxation time, is applied. This causes the intensity of magnetization to oscillate synchronously through a small angle in the XZ-plane; and this leads to the development of an intensity of magnetization J_y in the transverse direction Y and proportional to H_x. From measurements of J_y, or the corresponding magnetic induction B_y, the longitudinal magnetization J_z, the frequency f, and the transverse permeability $\mu_x=\mu_y$, the gyromagnetic ratio ρ can be calculated for approximate saturation. For both Permalloy and iron rods, steadily magnetized longitudinally in very intense fields and cross magnetized at frequencies of 22 and 30 kc/sec in very weak fields, the values obtained for ρ do not differ by more than the experimental error from those obtained for the same substances (though in a different physical state) in experiments on the Barnett and Einstein-de Haas effects.

I. INTRODUCTION

It has been known since 1914[1] that a rod of iron becomes magnetized by mere rotation about an axis precisely as it would be magnetized by applying to it in the direction of the axis a magnetic field with the intensity

$$H=\rho d\theta/dt, \tag{1}$$

where $d\theta/dt$ is the angular velocity in radians per second and ρ is a constant for the given substance. The quantity ρ is the ratio of the angular momentum of the elementary magnet to its magnetic moment, and it is known as the *gyromagnetic ratio* of the substance.

Since 1914 many other ferromagnetic substances have been magnetized by rotation, and their gyromagnetic ratios have been found to be comprised approximately between the limits $1.00\times m/e$ (Heusler alloy) and $1.09\times m/e$ (cobalt), in electromagnetic units, m and e being the mass and the charge, respectively, of a Lorentz electron. The results have been confirmed by numerous experiments on the converse effect, first suggested in 1907 by O. W. Richardson, but first obtained experimentally by Einstein and de Haas in 1915 and 1916 (the rough magnitude of the ratio in 1915, the sign also in 1916).[2]

The magnetization produced by rotation as above cannot, however, be attributed to the rotation of the rod as a whole, but rather to the rotation impressed on its magnetic elements, which are entrained in the motion. Thus, the idea arose that if the rod were initially magnetized along a diameter and the vector intensity of magnetization rotated about the axis while the rod itself remained at rest, a gyromagnetic intensity of magnetization should be developed along the axis. *Conceivably* this could be done in at least two ways:

(1) The fixed rod might be placed in a steady magnetic field directed normally to the axis, and the field-producing agent could be rotated about this axis. This would rotate the vector intensity of magnetization and *might* be expected to produce a gyromagnetic intensity of magnetization along the axis.

(2) Or, if the rod were magnetized transversely by each of two similar coils with axes normal to the rod and perpendicular to one another and supplied with equal currents by a two-phase electric generator, rotation of the vector intensity of magnetization would result, and a gyromagnetic intensity of magnetization along the axis *might* be expected.

Closely related to (1) and (2) is a third process, (3). The rod might be steadily magnetized to approximate saturation along its axis so that the moments of all the elementary magnets would be aligned practically in this direction (Z) and placed in a weak alternating magnetic field with lines of magnetic induction normal to the rod (direction X). This would produce a small oscillation of the magnetic elements about axes in the direction Y normal to X and Z and thus produce an alternating gyromagnetic intensity of magnetization in the direction Y. This could be observed by means of a surrounding coil of wire with the planes of its turns normal to Y.

Experiments by method (1) offer no certain advantages over those in which the rotation is affected mechanically, and certain serious disadvantages are evident. Such experiments have apparently never been made.

Experiments by method (2) have an advantage over those involving the actual rotation of the rod in that very much higher frequencies can be used and the gyromagnetic intensity is proportional to the frequency.

Two groups of high frequency experiments by this method were made in the interval 1922–1925[3] by J. W. Fisher, who expected to find, even with rods only weakly magnetized at right angles to their axes, longitudinal magnetization equal to that which would have resulted

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[1] S. J. Barnett, Phys. Rev. **6**, 239 (1915).

[2] See S. J. Barnett, Proc. Am. Acad. Arts Sci. **73**, 401 (1940); **75**, 109 (1944); and Phys. Rev. **66**, 224 (1944).

[3] J. W. Fisher, Proc. Phys. Soc. (London) **34**, 177 (1922); Proc. Roy. Soc. (London) **A109**, 7 (1925).

from rotating the *rods*, initially unmagnetized, at the same high frequencies. However, only small deflections, a few mm and less, and in the wrong direction, were obtained at frequencies up to 5×10^4 per sec, at which the rotation of the rods would have given 13 to 35 cm.

It was shown by one of us,[4] however, that Fisher's expected deflection, on the most favorable hypothesis possible, should be multiplied by the small factor $3I_\perp/2I_\infty$, where $I_\perp$ is the cross magnetization and I_∞ the saturation magnetization of the rods. On this most favorable (and quite improbable) hypothesis, deflections of the order of one cm only could be expected.

More elaborate and precise experiments by one of us,[4] made on Permalloy and iron powders with the assistance of Dr. V. Hoover, also gave, as was expected, deflections far smaller than the values calculated on the most favorable hypothesis.

In discussing this type of experiment with one of us a number of years ago Professor Einstein suggested that the magnetic elements do not rotate with the intensity of magnetization vector at all, but have their moments periodically reversed by the rotating field.

Thus, for simplicity (in an XYZ coordinate system, X and Y horizontal, Z vertical) we may imagine the rod to be unmagnetized in the Z directions, while half the elements have moments with X components only, and half moments with Y components only. The alternating X intensity will then change the X cross magnetization by reversing the moments of the first group, and the alternating Y intensity will change the horizontal cross magnetization by reversing the moments of the Y group. Thus, in the two-phase system, the magnetization vector will rotate but without any rotation of the elements, and thus without the production of any longitudinal (Z) magnetization by a gyromagnetic process. Bloch and Becker have more recently suggested essentially the same explanation of the null effect.[5]

As is well known, there have long been other indications favoring the hypothesis that in the early part of the magnetizing process (weak fields) the magnetization proceeds by such quantum jumps, or reversals, of the elements, and not by changes in their orientation; and the work by method (2), which has to do with weak fields only, supports the other evidence.

In strong fields, however, the indications are that changes of magnetization are due to gradual changes in the orientation of the elements. If this is the case, a repetition of the investigations by this method (2), but with fields strong enough nearly to saturate the magnetic material, should yield positive results instead of a null effect. However, there are obviously considerable difficulties in the way of such an investigation.

In connection with the discussion just mentioned, Professor Einstein suggested the type of experiment described under method (3), and this paper deals chiefly with an investigation made by this method.

In all the experiments mentioned above only relatively slow processes are contemplated, the times involved in producing the maximum changes in magnetization being very great in comparison with the relaxation times, so that in each change the magnetization is essentially and continuously in equilibrium with the impressed speed or field.

In quite recent years several related experiments using microresonance phenomena have been devised to measure gyromagnetic ratios of atomic nuclei and of electrons. Thus:

Rabi and others have developed atomic beam resonance methods to measure the gyromagnetic ratios of nuclei and other elementary particles.

Bloch and others have measured gyromagnetic ratios of nuclei by applying oscillating fields normal to steady fields and detecting the gyromagnetic intensity by induction, somewhat as in the experiments described in this paper. Closely related experiments have been made by Purcell and his collaborators.

Griffiths and others have used microresonance methods to measure gyromagnetic ratios for the elements in highly magnetized ferromagnetic substances. In some of these experiments, one plane wall of a resonant cavity is constructed of the substance to be studied and highly magnetized in a direction X parallel to the surface. The cavity is excited in a mode of oscillation such that the oscillating magnetic intensity close to the wall has the direction Y normal to X. In the combined fields the elements in the substance precess. When the frequency of the cavity is equal to the precessional frequency, the energy absorbed by the wall is increased to a maximum. By measuring the absorption of the cavity as a function of the intensity of the impressed field the gyromagnetic ratio can be calculated.

The fundamental principle involved in all these gyromagnetic experiments is the same. The nature of the magnetic element (whether simple spinning electron, spinning electron with orbital participation, nucleus, or other element), the strength of the supplementary or applied field if any is present, the degree of saturation of the material in such an applied field if any is present, the time involved in producing the gyromagnetic effect, and its relation to the relaxation time or to the resonance period, as well as the methods[6] of

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[4] See S. J. Barnett, Phys. Rev. **27**, 115 (1926); and Proc. Am. Acad. Arts Sci. **68**, 229 (1933).

[5] Or we may perhaps proceed as follows: With respect to the axis of the cylinder, some of the elements might revolve clockwise and some counterclockwise as they jumped from one orientation to the other. There is no *a priori* reason to expect either clockwise or counterclockwise rotations to predominate. The clockwise rotating elements would give rise to a gyromagnetic effect in one direction along the axis, while the counterclockwise rotating ones would give rise to an effect in the opposite direction. The macroscopic gyromagnetic intensity of magnetization would be close to zero.

[6] These methods can be varied greatly. For example, in experiments on magnetization by rotation, the rod may be oscillated about its axis instead of being set into continuous unidirectional rotation, and the alternating electromotive force thus developed

detection, differ from one type of experiment to another. Different values of the gyromagnetic ratio may thus be obtained for a substance from different types of experiment. But the fundamental idea in all is identical with that of the experiments on magnetization by rotation.

Not very long after Einstein's suggestion was made, the approximate theory was developed and experiments were instituted to test its validity and to measure the gyromagnetic ratios of continuously and highly magnetized compressed iron and Permalloy powders. These materials were chosen on account of the necessity of freedom from eddy currents in the rather high frequency fields employed. Circumstances, however, made it necessary to postpone the work in its early stages, and it was only recently that it became practicable to resume it. Although in the recent work many improvements have been made, we had hoped to make still further and important changes, but this has been prevented by the pressure of other matters. For the sake of brevity, the discussion here will be restricted, so far as experiments are concerned, almost entirely to the newer work.

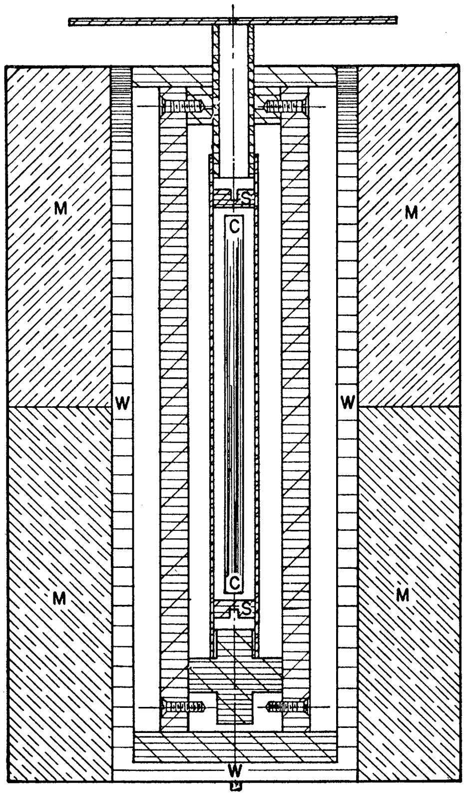

FIG. 1. Central cross section showing rod, rod holder, magnetizing coil, water jacket, etc.

II. GENERAL DESCRIPTION AND THEORY

In the experiments a long circular cylinder c of compressed iron powder, or Permalloy powder, is placed with its axis vertical (Z direction of a right-hand rectangular coordinate system XYZ) and is magnetized in this direction to intensity of magnetization J_z, approximately that of saturation, by a large coaxial solenoidal electric coil M[7] surrounding it symmetrically (see Fig. 1).

The magnetic cylinder lies symmetrically at the center of a second coil PP', Fig. 2, Helmholtz type, but long, narrow and rectangular, with its turns in vertical planes parallel to YZ. When the rod is absent there is produced through the agency of this coil, traversed by an alternating electric current I, with frequency $f=\omega/2\pi$, throughout the region normally occupied by the rod, a small and approximately uniform magnetic intensity H_x in the X direction. When the rod is in place there results in the rod an approximately uniform intensity of magnetization J_x in the X direction, synchronous with I and H_x. An equilibrium theory is assumed (see above) because the period $1/f$ is large in comparison with the relaxation time.

It is easy to show that when the length of the rod, with transverse permeability μ_x $(=\mu_y)$, is great in comparison with its diameter, and the demagnetizing factor is thus essentially equal to 2π, the intensity of magnetization J_x is given by the relation

$$J_x=\frac{1}{2\pi}\left(\frac{\mu_x-1}{\mu_x+1}\right)H_x. \tag{2}$$

When the ratio of the length of the rod to its diameter is not very great, as is the case in this investigation, the mean demagnetizing factor N_x will be somewhat less than 2π. We shall write

$$N_x=2\pi(1-\delta), \tag{3}$$

where δ is small in comparison with unity. Thus, in place of (2) we may write with all sufficient precision

$$J_x=\frac{1}{2\pi}\left(\frac{\mu_x-1}{\mu_x+1}\right)(1+\delta)H_x. \tag{4}$$

The alternating horizontal intensity H_x causes the axes of the elementary magnets—always lined up almost completely with their axes in the direction Z—to execute small oscillations in planes normal to X and Z about horizontal axes in the direction Y. Thus, there exists a gyromagnetic intensity G in the direction Y, producing a gyromagnetic intensity of magnetization J_y, proportional to G.

A third coil S, which may or may not be similar to PP', is similarly situated but with the planes of its turns normal to those of PP' and linked with the flux ϕ due to J_y. It has, therefore, induced in it an electro-

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in the surrounding coil of wire may be compared with the electromotive force produced by an alternating field of known intensity and the same frequency, an amplifier being used to increase the sensitivity if needed. See S. J. Barnett, Bull Nat. Research Council 3, Part 2, 245 (1922).

[7] S. J. Barnett, Phys. Rev. 27, 425 (1908).

motive force E which may be measured, after amplification if necessary, and will make possible the determination of J_y or the corresponding magnetic induction B_y. For the intensity of magnetization J_y, we have

$$J_y=\frac{1}{2\pi}\left(\frac{\mu_x-1}{\mu_x+1}\right)(1+\delta)G. \tag{5}$$

Let θ designate the mean angle made by the axes of the elementary magnets with Z at any time t, and θ_0 its amplitude. Then θ is given by the relation

$$\theta=\frac{J_x}{J_z}=\frac{1}{2\pi J_z}\left(\frac{\mu_x-1}{\mu_x+1}\right)(1+\delta)H_x. \tag{6}$$

The angular velocity of the axes of the elements at the time t is

$$\frac{d\theta}{dt}=\frac{1}{J_z}\frac{dJ_x}{dt}=\frac{1}{2\pi J_z}\left(\frac{\mu_x-1}{\mu_x+1}\right)(1+\delta)\frac{dH_x}{dt}. \tag{7}$$

In accordance with Eq. (1), the gyromagnetic intensity G is given by the relation

$$G=\rho\frac{d\theta}{dt}=\frac{\rho}{2\pi J_z}\left(\frac{\mu_x-1}{\mu_x+1}\right)(1+\delta)\frac{dH_x}{dt}. \tag{8}$$

From Eqs. (5) and (8) we get

$$J_y=\frac{\rho}{4\pi^2 J_z}\left(\frac{\mu_x-1}{\mu_x+1}\right)^2(1+2\delta)\frac{dH_x}{dt}. \tag{9}$$

The magnetic induction B_y produced in the rod by G is

$$B_y=4\pi J_y-NJ_y=2\pi(1+\delta)J_y. \tag{10}$$

Suppose the intensity H_x produced (primarily by the coil PP') in the space to be occupied by the rod to be given by the equation

$$H_x=H_{x0}\sin\omega t, \tag{11}$$

so that

$$dH_x/dt=\omega H_{x0}\cos\omega t; \tag{12}$$

then

$$J_x=\frac{1}{2\pi}\left(\frac{\mu_x-1}{\mu_x+1}\right)(1+\delta)H_{x0}\sin\omega t\equiv J_{x0}\sin\omega t, \tag{13}$$

$$J_y=\frac{\rho\omega}{4\pi^2 J_z}\left(\frac{\mu_x-1}{\mu_x+1}\right)^2(1+2\delta)H_{x0}\cos\omega t\equiv J_{y0}\cos\omega t, \tag{14}$$

and

$$B_y=\frac{\rho\omega}{2\pi J_z}\left(\frac{\mu_x-1}{\mu_x+1}\right)^2(1+3\delta)H_{x0}\cos\omega t\equiv B_{y0}\cos\omega t. \tag{15}$$

When Eqs. (14) and (15) are solved for ρ we obtain

$$\rho_J=\frac{4\pi^2 J_z J_{y0}}{\omega H_{x0}}\left(\frac{\mu_x+1}{\mu_x-1}\right)^2(1-2\delta), \tag{16}$$

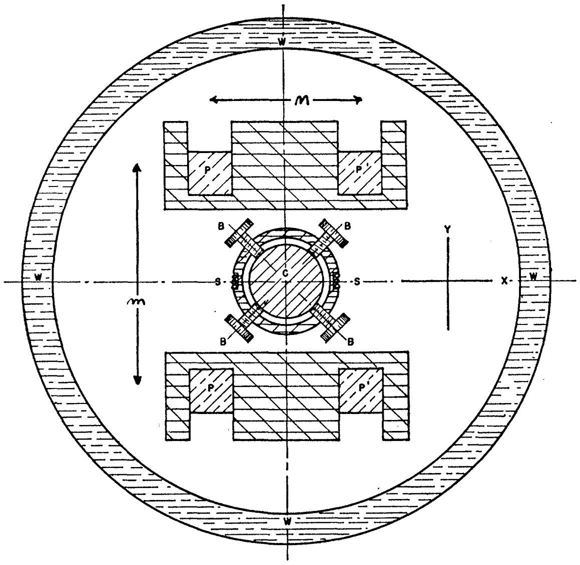

Fig 2. Cross section of primary coils PP, P'P', rod C, secondary coil S, adjusting screws B, and cooling jacket W.

and

$$\rho_B=\frac{2\pi J_z B_{y0}}{\omega H_{x0}}\left(\frac{\mu_x+1}{\mu_x-1}\right)^2(1-3\delta). \tag{17}$$

All the unknown quantities on the right-hand side of either Eq. (16) or Eq. (17) can be found experimentally by methods whose principles are well known, and thus ρ can be determined. The phase relations between X, J_y, etc., can be determined with an oscilloscope. The methods used are described below. The quantity J_y or the quantity B_y is determined, as indicated above, by measuring the emf E induced in the coil S when H_x varies.

If the coil S is so constructed that its constant γ is uniform throughout the space normally occupied by the material of the rod, Eq. (14) will be used. The flux ϕ through the coil will be given by the relation

$$\phi=\gamma VJ_y=\gamma VJ_{y0}\cos\omega t, \tag{18}$$

where V is the volume of the rod and γVJ_y is its magnetic moment, with the amplitude γVJ_{y0}.

If the coil S is a narrow flat coil wound symmetrically in (essentially) a plane through the axis of the rod and infinitely close to its surface, the flux will be given by the relation

$$\phi=B_y dl\cdot N=B_yS=B_{y0}S\cos\omega t, \tag{19}$$

where d is the diameter of the rod, l its length, N the number of turns in the coil, and S the total area of the coil.

The emf E induced in the coil in the first case is

$$E=-d\phi/dt=-(d/dt)(\gamma VJ_{y0}\cos\omega t)$$
$$=\omega\gamma VJ_{y0}\sin\omega t=E_0\sin\omega t, \tag{20}$$

so that

$$J_{y0}=E_0/\omega\gamma V. \tag{21}$$

The emf E induced in the coil in the second case is

$$E=-\frac{d\phi}{dt}=-\frac{d}{dt}(SB_y)=-\frac{d}{dt}(SB_{y0}\cos\omega t)$$

$$=\omega SB_{y0}\sin\omega t=E_0\sin\omega t, \tag{22}$$

so that

$$B_{y0}=E_0/\omega S. \tag{23}$$

In practice, of course, corrections have to be made because the coils do not satisfy exactly the conditions assumed and the demagnetizing factor differs somewhat from 2π.

III. THE APPARATUS AND ITS ARRANGEMENT

The apparatus may be divided into several groups as follows:

(A) The first group includes the magnetic materials that were studied.

(B) The second, the large coil M of copper wire which, when traversed by a steady electric current, produced the magnetic intensity H_z and the intensity of magnetization J_z.

(C) The third, the equipment for producing the high frequency power necessary to energize the coils which produced the oscillating magnetic intensity H_x.

(D) The fourth, the (primary) coils for producing the magnetic intensity H_x, the (secondary) coils used for the measurement of the gyromagnetic induction B_y or gyromagnetic magnetization J_y and for other measurements; also thermal devices to control the temperatures of these coils and the magnetic materials.

(E) The fifth, the amplifier used to increase the strength of the gyromagnetic signal appearing in the secondary coil, and devices to measure the magnitude, phase, and frequency of this signal.

(A) The Magnetic Materials

Three rods of compressed powdered ferromagnetic materials were used in the experiments. These same rods, from material supplied by the Bell Telephone Laboratories, were used by one of us in the experiments with rotating magnetic fields referred to above. They were cylindrical in shape, about 25 cm in length, and 1.41 cm in diameter. The rods were formed from small disks of compressed iron and compressed Permalloy dust. Disks were cut from the compressed material and packed closely, with cement between them, in Bakelite tubes ⅝-inch outside diameter and 1/32 inch thick. The disks were squeezed together in a vise while the cement hardened. The lengths of the rods are listed below:

(1)[8]	Permalloy	25.1 cm
(2)	Permalloy	24.0 cm
(3)	Iron	24.0 cm.

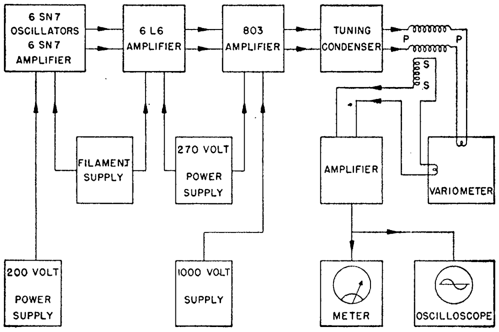

FIG. 3. Block diagram of primary and secondary circuits.

(B) The Coil for Producing the Steady Magnetic Field

This coil M (Fig. 1), a circular cylinder in form, was wound on a brass bobbin from about 300 pounds of *DCC* copper wire in 3900 turns, and had a resistance of 10.4 ohms at room temperature. It produced a magnetic field of approixmately 90.2 gausses per ampere averaged over the region in which the sample was placed. The coil was mounted with the axis approximately vertical.

On top of the bobbin, screw adjustments were provided which made it possible to rotate the sample and secondary coil through small angles, as described below.

The intensity external to the large coil fell off so rapidly with distance that its effect upon the meters and the rest of the apparatus could be neglected.

The large coil M, as well as the primary coil, at first became very hot and made it necessary to install a cooling jacket (see Fig. 1). For this jacket, coils were wound in the form of a helix from ⅜-inch copper tubing, on the curved surfaces and bottom of a brass tube 4½ inches in diameter and 20 inches long, with large quantities of soft solder filling the space between turns. The entire assembly fitted tightly in the circular hole of the large coil. Couplings were provided so that tap water could be forced through the coils from bottom to top. The primary coil and assembly were placed inside this tube and held there by machined bakelite disks bolted to the primary coil and fittng closely to the brass tube. The remainder of the space inside the brass tube was filled with transformer oil. In this way, temperature changes in the region inside the cylinder could be kept to less than a tenth of a degree during all experiments.

(C) Equipment for Producing the High Frequency Power (Fig. 3)

It was necessary to employ frequencies low enough to prevent disturbances from eddy currents and to insure that the time of an oscillation was much greater than the relaxation time of the material. On the other hand, since the gyromagnetic voltage is proportional to the square of the frequency, it was desirable to use as high frequencies as practicable. From these considerations frequencies ranging from 20 to 32 kc/sec were chosen. To prevent purely electrical disturbances from getting through it was necessary to use screen grid tubes and to make the apparatus as nearly symmetrical as possible. Push-pull tube arrangements were used throughout. The arrangements adopted produced steady voltages at constant frequencies and provided ready means for changing the frequency at will.

The oscillator consisted essentially of a 6SN7 vacuum tube in a multivibrator circuit, with an attached tank, as first used by Reich. We have had considerable experience with this kind of oscillator and have always found it to perform very satisfactorily. The oscillator could be so adjusted that its frequency changed by only 0.1 percent in the course of an experiment.

The oscillator was followed by a loosely coupled 6SN7 push-pull buffer amplifier, and this was followed by the other equipment necessary to produce the high frequency power. All this was of standard design, built from high grade materials, and properly screened in iron and copper boxes. The output of the final power amplifier was fed through a transformer into a tuned series resonant circuit. The inductance was that of the Helmholtz coil pair (described in the next section) which furnished the oscillating magnetic field and a small variometer in series therewith; the capacity was that of two variable condensers, with a small trimmer attached to one of them.

The oscillator could be so adjusted that its frequency changed by only about 0.1 percent in the course of an experiment.

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[8] In the course of the experimental work, rod No. (1) was accidentally broken so that it became necessary to make part of the Permalloy experiments on rod No. (1) and part on rod No. (2). The existence of the gyromagnetic effect was established with all the rods.

(D) The Alternating Current Coils

The fourth group of apparatus consisted of several coils and thermal devices. Some of the components showed microphonic behavior and were placed on special tables, separate from the rest of the apparatus.

Primary Coils

These coils *PP′*, the two halves of the Helmholtz pair referred to in the last section, were constructed with precision from highly insulating materials and had the approximate sizes and shapes indicated in Fig. 2. In the framework accurately machined holes and screws were provided by means of which the specimen under study could be inserted, removed, rotated, aligned, and centered.

Secondary Coils

The main secondary coil *C* was constructed much like one-half of the primary, but with shape, size, and windings suitable for its different purpose (see Fig. 2).

The coil, with the sample inside, was placed in the center of the primary Helmholtz coils *PP′* in such a position that the mutual inductance between primary and secondary was close to zero. A Bakelite bearing was provided to fasten the bottom of the secondary coil to the bottom of the frame of the primary coil. At the top of the secondary coil another Bakelite tube was fastened to the secondary coil with Bakelite screws. This tube served to hold and turn the secondary coil into any selected azimuth.

In order to detect the small gyromagnetic signal it was important to reduce to small values all extraneous voltages in the secondary coil. To do this, first the secondary coil was mechanically rotated into a position of approximate minimum mutual inductance with the primary and clamped in this position. To reduce the coupling between the two coils further, a variable mutual inductance was used by which any induced voltage could be reduced to a small quantity.

(E) Apparatus and Arrangements for Measuring the Gyromagnetic Electromotive Force ψ

The apparatus in this group included an amplifier to increase the strength of the gyromagnetic signal from a few tenths of a millivolt to a magnitude more convenient to measure by standard instruments. The amplifier finally adopted employed a 6SJ7 vacuum tube with a tuned-grid tuned-plate circuit. For this single stage amplifier the gain was 10.6 at 22 kc/sec and 17.5 at 30 kc/sec.

High *Q* parallel resonant circuits were used to peak the amplifier and allow only a narrow band of frequencies to be amplified. In this way background noise, always troublesome, was greatly reduced.

An oscilloscope and an electron voltmeter were attached to the output of the amplifier to measure the phase and magnitude of the signal. The oscilloscope was operated on external sweep, with a synchronizing signal tapped from the output of the 803 power amplifier.

IV. MEASUREMENT OF THE QUANTITIES NEEDED FOR THE DETERMINATION OF THE GYROMAGNETIC RATIO

If the secondary coil *S* were a narrow flat coil wound symmetrically in (essentially) a plane through the axis of the rod, and (infinitely) close to its surface, the quantity ρ could be determined from an equation formed by combining Eqs. (17) and (23) as follows:

$$\rho=\frac{2\pi E_0 J_z}{S\omega^2 H_{x0}}\left(\frac{\mu_x+1}{\mu_x-1}\right)^2(1-3\delta), \tag{24}$$

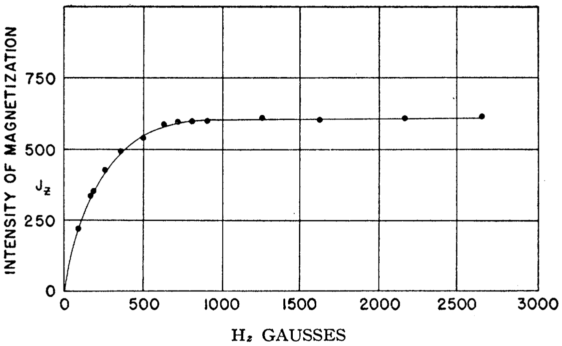


FIG. 4. Magnetization curve for powered Permalloy.

where *S* is the total area of the ideal coil. Corrections resulting from the departure of the shape and location of the coil from ideal conditions will be applied later.

(*1*) The frequency $f=\omega/2\pi$ of the 6N7 Reich oscillator was determined by comparison with the highly constant 60/sec frequency of the mains by standard procedure with the assistance of two commercial oscillators, an oscilloscope, and a frequency meter. The two frequencies used in most of the work were found to be 21.75 kc and 30.50 kc with an error not greater than about 0.1 percent.

(*2*) The amplitude H_{x0} of the oscillating horizontal intensity produced by the current I, with amplitude I_0, in the primary coil was determined as follows:

A specially constructed test coil was placed in the region occupied in the main experiment by the rod being tested, and a high impedance voltmeter was placed across its terminals. The voltage could then be read as a function of the current in the primary coil. In a preliminary experiment the test coil was rotated until the voltage was a maximum, when it was locked in place.

Numerous readings of the voltage E' were then taken with various values of I', the primary current, at each of the frequencies of the experiment. Then for each frequency an average value of the amplitude of H_x was computed for each value of E' and the corresponding value of I'. With this information it was easy to determine the effective constant K of the coil for each frequency used. Then we have

$$H_{x0}=KI_0, \tag{25}$$

where I_0 is the primary current producing the gyromagnetic emf in the main experiment.

It was, of course, impossible to proceed in the usual way to determine the constant of the coil on account of the presence of the surrounding conducting tube. Also, it is to be noted that in the method adopted an absolute calibration of the ammeter is not needed.

(*3*) *The longitudinal magnetization* J_z. In the case of each rod, placed symmetrically in the large coil M, J_z was determined ballistically for various magnitudes of the current in the coil. For Permalloy a curve for J_z against H_z is plotted in Fig. 4.

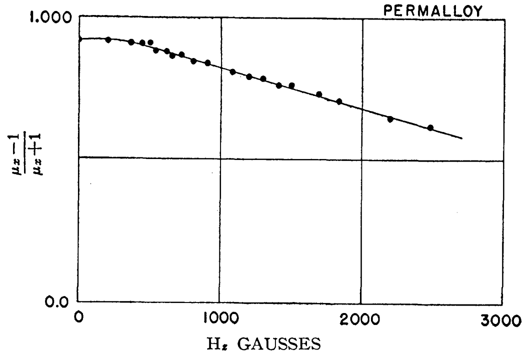


FIG. 5. Transverse permeability function for Permalloy.

(*4*) *The longitudinal permeability* μ_z *in weak fields.* While this quantity is not needed directly for the calculation of ρ, we have used it indirectly for the calculation of μ_x which, in weak fields where our rods are essentially isotropic, is equal to μ_z. The quantity μ_z is determined as follows.

Let H_{z_i} designate the mean magnetic intensity in the rod in the field of strength H_z. Then,

$$H_{z_i}=H_z-N_zJ_z,$$

and

$$\mu_z=4\pi J_z/(H_z-N_zJ_z)+1.$$

The demagnetizing factor N_z, which is easily measured but which can also be obtained with sufficient precision from the tables, is 0.07.

(*5*) *The transverse demagnetizing factor* $2\pi(1-\delta)$. The demagnetizing factor is a purely geometric quantity, but, of course, it requires magnetic measurements for its experimental determination. To determine δ we use the approximate equation

$$\delta=\frac{\mu_x+1}{2\mu_x}\frac{B_x{}^i}{H_{x0}}-1, \tag{26}$$

and proceed with weak fields, in which the transverse permeability is identical with the longitudinal permeability, already determined as indicated in (4). We measure B_{H_i}/H_{x0} for both small and large magnitudes of H_z, either ballistically or with the high frequencies of the principal experiments.

The results gave for δ the value 0.01.

(*6*) *The transverse permeability and the permeability function* $(\mu_x-1)/(\mu_x+1)$. When δ is small it is readily shown that

$$(\mu_x-1)/(\mu_x+1)=(B_x{}^i/H_{x0}-1)(1-2\delta), \tag{27}$$

an equation from which we can obtain both the transverse permeability μ_x and the permeability function $(\mu_x-1)/(\mu_x+1)$. The vertical curve of $(\mu-1)/(\mu+1)$ against H_z is plotted in Fig. 5.

(*7*) *The gyromagnetic* emf ψ *developed in the actual secondary coil when the* emf E *would be developed in the ideal coil of area* S. In all of the final work the secondary coil was flat and rectangular like the ideal coil of area S, but it was longer than the rod and considerably wider. This coil will be designated in this section as coil (1).

From the actual emf ψ developed in this coil in the main experiment we have to find the emf E which would have been developed in the ideal coil of Eq. (24).

For this purpose experiments were made with three secondary coils, *viz.*, (1) the actual secondary coil, (2) the coil wound closely on the rod and approximating closely the ideal coil of Eq. (24), and (3) a coil exactly like coil (2) but wound on Bakelite. Each coil had the same number of turns N as the ideal coil and in the experiments with it was symmetrically mounted with the planes of its turns parallel to those of the primary coil, which was traversed by an alternating current of known amplitude. The fluxes through the three coils will be designated by ϕ_1, ϕ_2, ϕ_3.

For ϕ_1 we have

$$\phi_1=\phi_{1x}+\phi_{1Jx}, \tag{28a}$$

and thus

$$\phi_{1Jx}=\phi_1-\phi_{1x}, \tag{28b}$$

where ϕ_{1x} is the flux when the rod is absent, and ϕ_{1J} is the part of the flux due to the presence of the rod.

For ϕ_2 we have similarly

$$\phi_2=\phi_{2x}+\phi_{2J}=\phi_3+\phi_{2J}; \tag{28c}$$

since ϕ_3 is identical with the flux ϕ_{2x}, the flux would thread coil (2) if the rod were removed. Thus, we obtain

$$\phi_{2J}=\phi_2-\phi_3 \tag{28d}$$

for the flux through coil b due to the rod only.

The flux ϕ_{2J} (due to the intensity of magnetization only) is made up of two parts, *viz.*, the flux ϕ_b which crosses the central section of the rod and would be the total flux through the coil (2) if it were wound from infinitely thin wire, and ϕ_Δ, the reverse flux through the small area $2\Delta l$ between the coil and the rod.

The flux through the first is

$$\phi_b=+BS, \tag{28e}$$

and that through the second is

$$\phi_\Delta=-BS\Delta/[b(1+\delta)]. \tag{28f}$$

Thus,

$$\phi_{2J}=BS\{1-\Delta/[b(1+\delta)]\}=BS(1-\Delta/b). \tag{28g}$$

The fluxes ϕ_1, ϕ_{1x}, ϕ_2, and ϕ_3 were calculated from the voltages induced in the appropriate coils, and then the quantity C was calculated from the equation

$$C=(\phi_2-\phi_3)/(\phi_1-\phi_{1x}).$$

Then

$$\phi_{2J}=\phi_{1J}C. \tag{28h}$$

From Eqs. (28g) and (28h) we obtain

$$\phi_{2J}=BS[1-(\Delta/b)]=\phi_{1J}C, \tag{28i}$$

and thus

$$BS=\phi_{1J}C(1+\Delta/b). \tag{28j}$$

From Eq. (28j) we see that

$$E/\psi=BS/\psi_{1J}=C[1+(\Delta/b)], \tag{28k}$$

or

$$E=C\psi[1+(\Delta/b)]. \tag{28l}$$

Substituting in Eq. (24) the vlaue of E from (28l), we obtain

$$\rho=\frac{C\psi_0 J_z}{\omega^2 SH_{x0}}\left(\frac{\mu+1}{\mu-1}\right)^2\left(1-3\delta+\frac{\Delta}{b}\right). \tag{29}$$

Many experiments also were made with direct currents in the primary coil, a galvanometer being used to measure the fluxes.

The values for C obtained for the Permalloy and iron rods are contained in Table I. The voltage across the terminals of the secondary coil, which would equal the gyromagnetic voltage if there were no extraneous voltages in the secondary, was increased by the amplifier, whose gain was carefully determined for both frequencies, and then measured by an electron voltmeter.

V. EXTRANEOUS ELECTROMOTIVE FORCES IN THE SECONDARY CIRCUIT AND PRELIMINARY ADJUSTMENTS

In what follows, for convenience, emfs in phase with or in opposition to the gyromagnetic emf will be referred to as in-phase emfs and those in quadrature therewith as quadrature emfs.

The extraneous effects originate fundamentally through residual mutual induction and capacity coupling between primary and secondary circuits. These were minimized by making the arrangements as nearly symmetrical as practicable; partly by the use of variometers and variable condensers. But it would have been difficult, if not impossible, to eliminate their effects sufficiently except that the induction effect is in quadrature with the gyromagnetic effect and that all the disturbing effects are independent of the direction of J_z, while the gyromagnetic effect reverses its sign with J_z.

In preliminary experiments a secondary coil A, wound on a Bakelite frame, was placed inside the primary coil. This coil assembly was, in turn, placed inside the hole in the bobbin of the coil M. The secondary coil was rotated in azimuth, varying the mutual inductance between the primary and secondary coils, until an induced (quadrature) voltage of convenient amplitude was obtained.

The amplifier was connected to the terminals of this coil A. The electron voltmeter and the vertical deflection terminals (Y deflection) of an oscilloscope were connected to the output of the amplifier. In order to synchronize the sweep (X deflection) of the oscilloscope with the current in the primary circuit, the oscilloscope was operated on "external sweep." The synchronizing signal, obtained from the terminals of the transformer in the primary circuit, was attenuated by a resistance network, and applied to the oscilloscope.

The positions (X coordinate) of the maximum and minimum of the induced voltages were noted on the oscilloscope screen. These extremes furnished a standard with which to compare phase relations of any other sine wave appearing across the output of the amplifier.

The secondary coil A was then removed from the apparatus, and the ferromagnetic sample, with a surrounding secondary coil B, was placed inside the hole in the bobbin of the coil M. The amplifier was connected to the terminals of this secondary coil. The electron voltmeter and synchronized oscilloscope were again connected to the amplifier output.

The secondary coil B and magnetic sample were rotated by means of the controls provided until an approximate minimum mutual inductance was obtained between the primary and secondary coils. With the screw adjustments previously described, this coil could be adjusted to within ten minutes of arc of any desired azimuth.

The residual voltage in the secondary circuit contained chiefly a quadrature component arising from the remaining mutual inductance and an in-phase component arising from the gyromagnetic effect, and other causes referred to below. To reduce the voltage still further, the variometer was adjusted until the output voltage was brought to a minimum by reduction of its quadrature component.

When this minimum was reached, it was found that the remaining voltage was an in-phase voltage and could not be removed by readjusting the variometer. It contained both the gyromagnetic effect and an in-phase disturbing effect.

This in-phase disturbance was attributed mostly to electric asymmetry in the amplifier and to the eddy currents flowing in the conducting walls of the container surrounding and close to the primary and secondary coils. The coils of the variometer, on the other hand, were placed in a large iron box, in such a manner that the coils were comparatively remote from the walls and their field affected but little by them. In seeking to remove the residual in-phase voltage, it was found that altering the capacity of one of the terminal condensers in the primary circuit could be made to change symmetry conditions in such a way as to produce (if desired) a large voltage, consisting of both a quadrature component and a small in-phase component (less than one-twentieth as large). The quadrature component could be eliminated by readjusting the variometer, leaving a small in-phase component added to or subtracted from the residual voltage.

By a series of adjustments, first by turning the condenser one way or the other, and then by removing the quadrature voltage by adjusting the variometer, the magnitude of the in-phase voltage by adjusting the variometer, the magnitude of the in-phase voltage in the output of the amplifier could be set to any desired small value E_1, and the quadrature component could be eliminated.

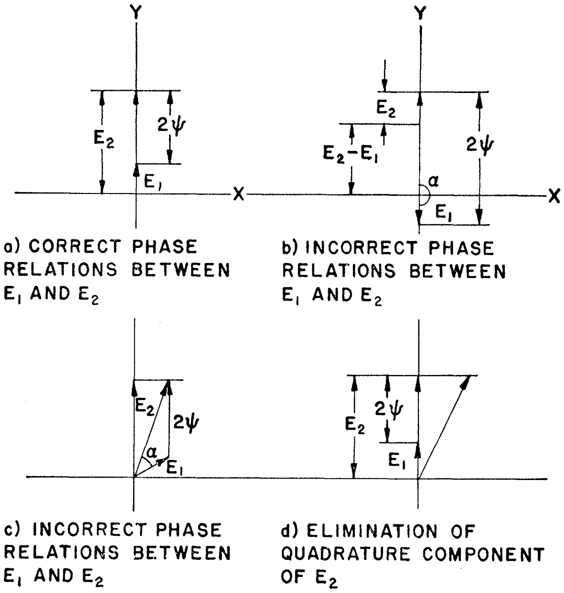
VI. THE MEASUREMENT OF THE GYROMAGNETIC EMF

The method finally adopted enabled the measurements to be made in spite of a troublesome quadrature disturbance due to the action on the magnetic material of the intense magnetic field in which it was placed. The method will be described first as if the disturbance were not present. Then the modifications necessitated by it will be discussed.

The sample and the secondary coil were placed in the apparatus, as described above. The current in the large coil M was set to some predetermined value, producing a steady magnetic intensity H_z and an intensity of magnetization J_z along the axis of the rod. The variometer and condenser were adjusted, as described above, until the output meter indicated a small in-phase voltage E_1. The current in the coil M was then reversed, changing H_z to $-H_z$, and also changing J_z to $-J_z$. With this change in J_z, the gyromagnetic voltage changed from ψ to $-\psi$. The output meter, which was originally reading E_1, would then change its reading to E_2, where $E_2-E_1=2\psi$ if the above-mentioned dis-

TABLE I.

	Permalloy	Iron
Direct current	2.27	2.21
23 kc/sec	2.17	2.12
32 kc/sec	1.98	1.93



a) CORRECT PHASE RELATIONS BETWEEN E_1 AND E_2

b) INCORRECT PHASE RELATIONS BETWEEN E_1 AND E_2

c) INCORRECT PHASE RELATIONS BETWEEN E_1 AND E_2

d) ELIMINATION OF QUADRATURE COMPONENT OF E_2

FIG. 6. Correct and incorrect phase relations.

turbances were absent. Certain precautions were necessary to eliminate possible errors due to disturbances. They can be explained with the help of the diagrams in Fig. 6. In these diagrams in-phase voltages are plotted along the Y axis, and quadrature voltages along the X axis.

Figure 6a shows the relations desired between E_1 and E_2, with the phase angle between E_1 and E_2 equal to zero. With this phase relation $E_2 - E_1 = 2\psi$.

Figure 6b displays an incorrect phase relation between E_1 and E_2, where the phase angle between them is an arbitrary value α. Here $E_1 - E_2 \neq 2\psi$.

In order to insure that the phase angle between E_1 and E_2 was zero, it was necessary to check continually this phase relation with the oscilloscope. The residual voltage E_1 was not reduced to zero, but was made large enough to permit its phase to be determined on the oscilloscope. To eliminate any possible quadrature component of E_1, the variometer was adjusted until a minimum voltage appeared on the output meter. It was found that this minimum occurred when all of the quadrature component was eliminated, only an in-phase component of E_1 being left.

In general, the quadrature disturbance modified the procedure. If E_1 was in phase with the gyromagnetic effect, the voltage E_2 appearing upon reversing the magnetic field H_z would, in general, not be exactly in phase but would have a quadrature component (Fig. 6d).

The appearance of this quadrature component can be understood by considering the effect of H_z and of its reversal upon the apparatus.

We made a very elaborate study of this quadrature effect and concluded that it was due at least chiefly to the change of mutual inductance caused by motions of the rod as a whole and the relative motions of its various parts produced by the entire magnetic field of the coil. Neither rod nor field was entirely homogeneous, symmetry was not perfect, and neither the rod nor its mountings possessed perfect rigidity. Minute motions of the rod could be observed with a microscope when the magnetic field was produced, and the emfs resulting were similar to those produced by giving to the rod mechanically displacements of the same order. These motions due to the magnetic forces occurred in spite of great care to make the construction as rigid as possible. The application of the magnetic field produced similar electromotive forces—even when the whole tube containing the coils was filled with solid paraffin, which may indicate that internal distortion of the rod was largely responsible.[9]

Fortunately, while the gyromagnetic emf reverses with H_z (or J_z), the distortion or motion of the materials due to the action of the intense magnetic field, like all the other disturbances, is identical for both directions of the field, and thus can be eliminated. It is not to be expected, however, that after the reversal of the field the material will return precisely to its previous configuration, and thus the quadrature effect produced by the reversal must be removed before reading E_z. The actual procedure for obtaining 2ψ was as follows: The current in the coil M was set to some predetermined value, and the emf E_1 was measured as indicated above. Next, the reversing switch in the circuit of the coil M was thrown, and the meter reading E_z was obtained, but not until the variometer had been adjusted to remove any quadrature emf imparted by the abrupt reversal of H_z. Then the reversing switch was thrown back to its original position, and another reading for E_1 was obtained. This process was repeated for four or more throws of the reversing switch, accompanied by four or more readings of E_1 and E_2, within a minute's time. The switch was thrown at approximately equally spaced times so as to minimize the effects of systematic drifts. It is exceedingly important to remark that the value of 2ψ obtained from this process is independent of the magnitude of E_1 and also of the magnitude of the quadrature emf which had to be eliminated before reading E_2.

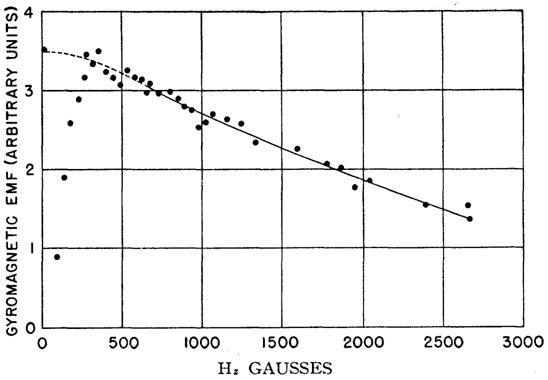


FIG. 7. Gyromagnetic emf for Permalloy at 30 kc/sec.

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[9] The very great effect on the mutual inductance produced by even a minute change in the orientation of the rod with regard to the direction of H_z can be seen as follows:

Suppose the axis of the rod, in the plane of the paper, to have the direction of the magnetic intensity H_z; suppose the long axis of the primary coil, in the direction CD, and the planes of its turns to make the small angle θ_1 with AB; and suppose the axis EF of the secondary coil and the planes of its turns to make the small angle θ_2, in any plane with AB. Then the primary coil, traversed by an electric current, will produce in the rod an (extra) longitudinal moment

$$M_1 = A\theta_1.$$

where A is a constant.

The moment M_1 will send through the secondary coil a flux

$$\phi_2 = BM_1\theta_2,$$

where B is a constant.

Therefore, on account of the asymmetry, that is, on account of the existence of the angles θ_1 and θ_2, the primary will send through the secondary a flux

$$\phi_2 = AB\theta_1\theta_2.$$

The flux ϕ_2 vanishes when either θ_1 or $\theta_2 = 0$, but when neither vanishes it may have a consideralbe value on account of the small longitudinal demagnetizing factor of the rod, and the consequent large value of A.

Thus, even very small angular displacements of the rod relative to the coils may produce large changes in the mutual inductance of the two.

For the single value of the primary current, an average value for E_1 and an average value for E_2 were calculated. The differences between these averages was equal to 2ψ.

For each set of four or more readings, the value of the primary current I_p was recorded so that H_z could be computed as described in Sec. IV. Also, the value of the current in the coil M was recorded, so that the proper value of $(\mu_x+1)/(\mu_x-1)$ could be selected.

Many repeated readings of 2ψ were taken for many values of H_z between zero and 2700 gausses. Figures 7 and 8 display graphs of $2\psi/H_{0x}$ plotted against H_z for Permalloy and iron, respectively. Also, in these figures are shown the curves expected for $2\psi/H_{0x}$ in a heavy black line, as predicted by Eq. (27). To calculate this curve, the value $\rho=0.96$ for Permalloy and $\rho=0.98$ for iron were used in Eq. (27), with other quantities appearing on the right-hand side of the equation determined as described earlier.

VII. THE GYROMAGNETIC RATIO AND ITS EXPERIMENTAL ERROR

The experimental values of ρ for Permalloy and iron were obtained by substituting in the right side of Eq. (24) the experimental values of the various quantities involved. Multiplication by the standard value of e/m gave $\rho e/m$, which is the gyromagnetic ratio in terms of that of a Lorentz electron spinning on a diameter. In the case of Permalloy the voltage rises to a maximum before approximate saturation at $H_z=ca$ 800 gausses is reached (see Figs. 4 and 7)—while values computed from the data obtained between 400 and 800 gausses are in close agreement with the results obtained in stronger fields. Only these latter data were included in the calculations of the final value of ρ since the equations are not strictly valid except for essential saturation.

The experimental error was computed in the usual way from the square root of the sum of the squares of errors arising from various sources, and was found to be $\delta\rho/\rho=\pm 0.08$.

As a result of the work on Permalloy we find for its gyromagnetic ratio

$$\rho=(0.96\pm 0.08)m/e.$$

It was not possible to saturate the iron sample with the magnetic fields obtainable, so that the formula could not be applied with certainty. However, it was possible to obtain a rough determination of the gyromagnetic ratio for iron in the region available. (See Fig. 8.) As H_z increased, the emf started from zero, reached a maximum, and then decreased in magnitude in much the same way as did the emf for Permalloy.

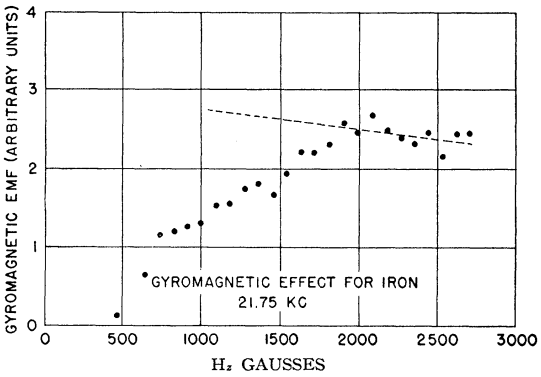

FIG. 8. Gyromagnetic emf for iron at 21.75 kc/sec.

TABLE II.

	ρ (Permalloy)	ρ (Iron)
Present study	$(0.96\ \pm 0.08\ )m/e$	$(0.98\ \pm 0.11\ )m/e$
Barnett effect	$(1.048\pm 0.006)m/e$	$(1.038\pm 0.010)m/e$
Einstein-de Haas effect	$(1.046\pm 0.003)m/e$	$(1.032\pm 0.004)m/e$

Since saturation J_z could not be obtained, it was not possible to apply, with certainty, Eq. (27) to iron. By assuming that the gyromagnetic ratio could be calculated from the values of E obtained after the maximum value of E had been reached (as was observed for Permalloy), and by assuming also that the correct saturation value of J_z could be obtained by extrapolating the data obtainable, the value

$$\rho=(0.98\pm 0.11)m/e$$

was obtained for iron.

The error was computed in the same way as for Permalloy.

In view of the large experimental error, there is no significant disagreement between these results, obtained with very intense steady fields, and the results obtained from the Barnett effect, with rotational frequencies equivalent to exceedingly minute magnetic intensities, and from the Einstein-de Haas effect, with both weak and moderately strong fields.

Table II lists the results obtained from the three effects.

On account of the magnitude of the experimental errors, no differences between the values of ρ which were observed at the two frequencies can be considered significant. The value of ρ presented is a weighted value obtained from measurements at both frequencies. Also, in view of the large experimental errors, no significance can be attached to the differences between the numerical results presented here and those given in a preliminary report[10] published before the completion of new, more extensive, and more reliable calibrations on which the above calculations are based.

The work described here has been carried out in the Norman Bridge Laboratory of the California Institute of Technology under the auspices of the University of California and the Institute, and with important help from the Carnegie Institution of Washington, the National Research Council, and the ONR.

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[10] S. J. Barnett and L. A. Giamboni, Phys. Rev. **76**, 1542 (1949).