

## Some Consequences of Invariance under Charge Conjugation

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The charge conjugation operator, which transforms charged fields to their charge-conjugate fields and reverses the sign of the electromagnetic field, is explicitly constructed. States of zero net charge are eigenstates of this operator; the eigenvalues are  $(-1)^n$  for states of  $n$  photons and  $(+1)$  and  $(-1)$  for the singlet and triplet  $S$  states of positronium, respectively. The effect of charge conjugation on neutral boson fields is also discussed. Physical consequences of the invariance of field theory under this transformation include:

1. For usual interactions a spin-zero particle (including the singlet  $S$  state of positronium) cannot decay into three photons, although this decay is shown to be allowed by angular momentum and parity considerations.
2. The  $m=1$  and  $m=-1$  levels of triplet positronium cannot be split by a magnetic field.

### I. INTRODUCTION

IN considering the decay of a neutral spin-zero particle into three photons, we were struck by the fact that, although this was not forbidden by standard selection rules, the decay probability calculated for the usual interactions was zero. For the case of a neutral scalar meson decaying via intermediate pairs of fermions, the process had been shown to be forbidden<sup>1</sup> as a consequence of Furry's theorem, and this was found to be still true if the fermions were replaced by charged bosons. It was also pointed out to us<sup>2</sup> that the probability for the decay of the singlet  $S$  states of positronium into three photons was zero to the lowest order in  $e^2/\hbar c$ , a result which cannot be derived from Furry's theorem. If these results represent absolute selection rules, we should expect them to be related to a general invariance principle. It is the purpose of this paper to show that these results do follow from the invariance of field theories under charge conjugation and to consider some of the consequences of this invariance principle.<sup>3</sup>

### II. CHARGE CONJUGATION IN SPINOR ELECTRODYNAMICS

Charge conjugation for a charged matter field is the operation of reversing the signs of the charges of all the particles, while not altering their other properties—momentum and spin, for example. It is well known that if  $U$  is this (unitary) charge conjugation operator the effect of charge conjugation on the field operators  $\psi(x)$ ,  $\bar{\psi}(x)$  of a charged spinor field is given by<sup>4</sup>

$$U\psi(x)U^{-1} = C\bar{\psi}(x), \quad (1a)$$

$$U\bar{\psi}(x)U^{-1} = C^{-1}\psi(x), \quad (1b)$$

<sup>1</sup> H. Fukuda and Y. Miyamoto, *Prog. Theoret. Phys.* 4, 392 (1949).

<sup>2</sup> R. Drisko, private communication.

<sup>3</sup> While writing this paper we learned of a paper by A. Pais and R. Jost [*Phys. Rev.* 87, 871 (1952)] on the same general subject. We have attempted to minimize the overlapping of the two papers, and in particular have omitted consequences of simultaneous use of charge conjugation and charge symmetry.

<sup>4</sup> J. Schwinger, *Phys. Rev.* 74, 1439 (1948). We use the same notation as is used in that paper; in particular,  $\bar{\psi}(x) = \psi^*(x)\gamma_4$ .

where  $C$  is a four-by-four matrix having the properties that

$$C^{-1}\gamma_\mu C = -\tilde{\gamma}_\mu, \quad \bar{C}C^{-1} = -1, \quad (2)$$

and is undefined to the extent of a constant of modulus unity. After this operation the Dirac equation for charged particles, and consequently the equations of motion for the field operators  $\psi(x)$  and  $\bar{\psi}(x)$ , become those for particles with charges of the opposite sign.

If it is required that the Hamiltonian of the interacting matter field and electromagnetic field  $A_\mu(x)$ ,

$$H = H_{\text{matter}} + H_{\text{photons}} - \frac{1}{c} \int d^3x j_\mu(x) A_\mu(x), \quad (3)$$

is invariant under charge conjugation, the transformation property of  $A_\mu(x)$  must be

$$UA_\mu(x)U^{-1} = -A_\mu(x). \quad (1c)$$

$U$  leaves invariant the commutation relations of the fields and commutes with the Hamiltonian (3); it is therefore a constant of the motion. Since two successive charge conjugations reproduce the initial state,  $U$  has two eigenvalues,  $+1$  and  $-1$ , just like the parity operator. Furthermore,  $U$  commutes with the total angular momentum and the parity, but it anticommutes with the total charge  $Q$ ,

$$Q = \frac{1}{c} \int d^3x j_0(x).$$

Hence  $U$  and  $Q$  can be made simultaneously diagonal only for states of zero charge.

It is of interest to construct the operator explicitly; to do this we factor  $U$ :

$$U = U_D U_{ph}, \quad (4)$$

where  $U_D$  and  $U_{ph}$  operate on the matter field and the electromagnetic field, respectively. The potentials  $A_\mu(x)$  are expanded in plane waves,

$$A_\mu(x) = \sum_{\mathbf{k}, \lambda} \left( \frac{2\pi\hbar c}{V|\mathbf{k}|} \right)^{\frac{1}{2}} (a_{\mathbf{k}, \lambda}(\mathbf{e}_{\mathbf{k}, \lambda})_\mu e^{i\mathbf{k}\cdot\mathbf{x}} + a_{-\mathbf{k}, \lambda}^*(\mathbf{e}_{\mathbf{k}, \lambda})_\mu e^{-i\mathbf{k}\cdot\mathbf{x}}),$$

where  $a_{\mathbf{k},\lambda}$  and  $a_{\mathbf{k},\lambda}^*$  are absorption and emission operators for photons with wave-vector  $\mathbf{k}$  and polarization vector  $\boldsymbol{\epsilon}_{\mathbf{k},\lambda}$ . Then an operator which anticommutes with  $A_\mu(x)$ , and which is therefore a suitable representation for  $U_{ph}$ , is

$$U_{ph} = (-1)^N; N = \sum_{\mathbf{k},\lambda} a_{\mathbf{k},\lambda}^* a_{\mathbf{k},\lambda}. \quad (5)$$

The Dirac matter field operators may be expanded in plane waves using positron theory:<sup>5</sup>

$$\psi(x) = V^{-\frac{1}{2}} \sum_r [a_r u_r(x) + b_r^* v_r(x)],$$

where we choose  $u_r(x)$  and  $v_r(x)$  to be related by

$$v_r(x) = C \bar{u}_r(x), \quad u_r(x) = C \bar{v}_r(x).$$

Here  $u_r(x)$  and  $v_r(x)$  are electron spinors of positive and negative energies, respectively,  $r$  standing for a particular value of the momentum  $\mathbf{p}$  and one of two values for the spin  $s$ .<sup>6</sup> The relations (1) can now be expressed in terms of the creation and annihilation operators  $a_r^*$ ,  $b_r^*$ ,  $a_r$ ,  $b_r$ ; from (1a),

$$\begin{aligned} V^{-\frac{1}{2}} \sum_r [U_D a_r U_D^{-1} u_r(x) + U_D b_r^* U_D^{-1} v_r(x)] \\ = C V^{-\frac{1}{2}} \sum_r [a_r^* \bar{u}_r(x) + b \bar{v}_r(x)] \\ = V^{-\frac{1}{2}} \sum_r [b_r u_r(x) + a_r^* v_r(x)], \end{aligned}$$

so that

$$U_D a_r U_D^{-1} = b_r, \quad U_D b_r^* U_D^{-1} = a_r^*. \quad (6a)$$

Similarly from (1b)

$$U_D a_r^* U_D^{-1} = b_r^*, \quad U_D b_r U_D^{-1} = a_r. \quad (6b)$$

In order to construct  $U_D^7$  we define operators  $\alpha_r^*$ ,  $\beta_r^*$ ,  $\alpha_r$ ,  $\beta_r$ , by

$$\begin{aligned} \alpha_r &= (a_r + b_r)/\sqrt{2}, \\ \beta_r &= (a_r - b_r)/\sqrt{2}, \end{aligned}$$

and the Hermitian conjugate equations. These operators satisfy the relations

$$\begin{aligned} U_D \alpha_r U_D^{-1} &= \alpha_r, \\ U_D \beta_r U_D^{-1} &= -\beta_r, \end{aligned} \quad (7)$$

and the usual anticommutation rules among themselves; by applying these rules it may be shown that (7) is satisfied if  $U_D$  is replaced by

$$1 - 2\beta_r^* \beta_r.$$

Thus  $U_D$  may be written

$$\begin{aligned} U_D &= \prod_r (1 - 2\beta_r^* \beta_r) \\ &= \prod_r (1 - a_r^* a_r - b_r^* b_r + a_r^* b_r + b_r^* a_r). \end{aligned} \quad (8)$$

The effect of  $U_D$  on a state vector may be found either by using the explicit construction (8) or by expressing the state vector in terms of creation operators acting

<sup>5</sup> See, for example W. Pauli, *Revs. Modern Phys.* **13**, 203 (1941). Our notation is not the same as his.

<sup>6</sup> For a given value of  $r$ ,  $v_r(x)$  is the spinor of the negative-energy electron whose absence corresponds to a positron with spin and momentum the same as those of the electron whose spinor is  $u_r(x)$ .

<sup>7</sup> We are indebted to Professor G. C. Wick for this construction of  $U_D$ .

on the vacuum state and employing (6). By either method it can be seen by inspection that  $U_D$  multiplies each empty state  $r$  by unity, and for each occupied state  $r$  changes an electron to a positron and vice versa; for the exceptional case when both an electron and a positron are present in the state  $r$ ,  $U_D$  multiplies the state by  $-1$ , as is to be expected from the antisymmetry properties of Dirac particles.

### III. STATES OF TWO AND THREE PHOTONS

It will first be shown that the decay of a scalar or pseudoscalar particle into three photons is not forbidden by angular momentum and parity considerations. This has previously been stated,<sup>8</sup> but no explicit construction of a scalar or pseudoscalar state of three photons was demonstrated.

In order to illustrate the method to be used, the states of two photons previously discussed by Landau<sup>9</sup> and Yang<sup>10</sup> will be considered. The state of two photons must be described in terms of three vectors: the polarization vectors of the two photons,  $\boldsymbol{\epsilon}_1$  and  $\boldsymbol{\epsilon}_2$ , and the relative momentum vector  $\mathbf{p}$ . The polarization vectors are directly associated with the photon creation operators which act on the vacuum state. Since each creation operator acts only once our expression must be bilinear in  $\boldsymbol{\epsilon}_1$  and  $\boldsymbol{\epsilon}_2$ . Because of the transversality condition ( $\boldsymbol{\epsilon}_1 \cdot \mathbf{p} = 0 = \boldsymbol{\epsilon}_2 \cdot \mathbf{p}$ ) the only spherically symmetrical states are

$$\boldsymbol{\epsilon}_1 \cdot \boldsymbol{\epsilon}_2 f(|\mathbf{p}|), \quad (\text{scalar}) \quad (9a)$$

$$(\boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_2) \cdot \mathbf{p} f(|\mathbf{p}|), \quad (\text{pseudoscalar}) \quad (9b)$$

where  $f(|\mathbf{p}|)$  stands for an arbitrary function of the magnitude of  $\mathbf{p}$ . It is seen from these representations of the states that the planes of polarization of the photons are parallel and perpendicular in the scalar and pseudoscalar states, respectively. Since the only vector state satisfying the above conditions,

$$\boldsymbol{\epsilon}_1 \cdot \boldsymbol{\epsilon}_2 \mathbf{p},$$

is antisymmetrical on interchange of the two photons (which changes  $\mathbf{p}$  to  $-\mathbf{p}$ ), it is evident that there exists no such state of two photons.<sup>10</sup> Similarly there is no pseudovector state.

The states (9) can be expressed in a manifestly Lorentz-invariant and gauge-invariant manner by using the field strengths  $F_{\mu\nu}(x)$  instead of the potentials  $A_\mu(x)$ .

For three photons there are the three polarization vectors  $\boldsymbol{\epsilon}_1$ ,  $\boldsymbol{\epsilon}_2$ ,  $\boldsymbol{\epsilon}_3$ , and two independent vectors to be chosen from  $\mathbf{p}_1$ ,  $\mathbf{p}_2$ , and  $\mathbf{p}_3$ , the three momentum vectors. From these it is possible to construct spherically symmetrical states satisfying all the above conditions; two of the simpler ones are

$$\boldsymbol{\epsilon}_1 \cdot (\mathbf{p}_2 - \mathbf{p}_3) \boldsymbol{\epsilon}_2 \cdot (\mathbf{p}_3 - \mathbf{p}_1) \boldsymbol{\epsilon}_3 \cdot (\mathbf{p}_1 - \mathbf{p}_2) g(p_1, p_2, p_3), \quad (\text{scalar}) \quad (10a)$$

$$\boldsymbol{\epsilon}_1 \cdot (\boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_3) g(p_1, p_2, p_3), \quad (\text{pseudoscalar}) \quad (10b)$$

<sup>8</sup> D. Peaslee, *Helv. Phys. Acta* **23**, 845 (1950).

<sup>9</sup> L. D. Landau, *Doklady Akad. Nauk, S.S.S.R.* **60**, 207 (1948).

<sup>10</sup> C. Yang, *Phys. Rev.* **77**, 242 (1950).

where  $g$  is a function antisymmetric on interchange of any two of the three (independent) scalars  $p_1$ ,  $p_2$ , and  $p_3$  [for example,  $(p_1 - p_2)(p_2 - p_3)(p_3 - p_1)$ ]. Thus it is possible for three photons to exist in scalar and pseudo-scalar states.

By applying the charge conjugation operator (5) to state vectors for a given number of photons we obtain the result that for an even number of photons  $U_{ph}$  has the eigenvalue  $+1$ , and for an odd number of photons the eigenvalue  $-1$ .

#### IV. THE EFFECT OF CHARGE CONJUGATION ON STATES OF POSITRONIUM

It has been pointed out in Sec. II that  $U$  can be diagonalized for states of zero charge, and can be diagonalized simultaneously with angular momentum  $\mathbf{J}$  and parity  $P$ . Therefore, states of positronium with specified values of  $\mathbf{J}$  and  $P$  may be chosen so that  $U$  is diagonal. In fact,  $U$  also commutes with the orbital angular momentum  $\mathbf{l}$  and the spin  $s$ . Hence the  $^1S$  and  $^3S$  states, which are specified uniquely by their  $\mathbf{J}$  and  $\mathbf{l}$  values, must be eigenstates of  $U$ . The state function of any state of positronium can be expanded in terms of free-particle state functions,

$$\Psi_{\text{positronium}} = \left( \sum_{\mathbf{p}, s_1, s_2} c(\mathbf{p}, s_1, s_2) a_{s_1}^*(\mathbf{p}) b_{s_2}^*(-\mathbf{p}) + \sum_{\mathbf{r}} \cdots a_{r_1}^* a_{r_2}^* b_{r_3}^* b_{r_4}^* + \cdots \right) \Psi_{\text{vac}}, \quad (11)$$

where the second term represents the effect of virtual pair production. Since  $U$  commutes with the number of free particles, it will be sufficient to determine the behavior under  $U$  of the first term in (11). Performing the operation, we have

$$\begin{aligned} U_D \Psi_{\text{positronium}} &= \sum_{\mathbf{p}, s_1, s_2} c(\mathbf{p}, s_1, s_2) U_D a_{s_1}^*(\mathbf{p}) U_D^{-1} \\ &\quad \times U_D b_{s_2}^*(-\mathbf{p}) U_D^{-1} U_D \Psi_{\text{vac}} \\ &= \sum_{\mathbf{p}, s_1, s_2} c(\mathbf{p}, s_1, s_2) b_{s_1}^*(\mathbf{p}) a_{s_2}^*(-\mathbf{p}) \Psi_{\text{vac}} \\ &= - \sum_{\mathbf{p}, s_1, s_2} c(\mathbf{p}, s_1, s_2) a_{s_2}^*(-\mathbf{p}) b_{s_1}^*(\mathbf{p}) \Psi_{\text{vac}} \\ &= - \sum_{\mathbf{p}, s_1, s_2} c(-\mathbf{p}, s_2, s_1) a_{s_1}^*(\mathbf{p}) b_{s_2}^*(-\mathbf{p}) \Psi_{\text{vac}}. \end{aligned}$$

For  $S$  states  $c(-\mathbf{p}, s_1, s_2) = +c(\mathbf{p}, s_1, s_2)$ ; thus

$$U_D \Psi_s = - \sum_{\mathbf{p}, s_1, s_2} c(\mathbf{p}, s_2, s_1) a_{s_1}^*(\mathbf{p}) b_{s_2}^*(-\mathbf{p}) \Psi_{\text{vac}}. \quad (12)$$

The three triplet  $S$  states must behave in the same way on operation by  $U_D$ , and for the particular states with  $s_1 = s_2$ ,  $c(\mathbf{p}, s_2, s_1) = +c(\mathbf{p}, s_1, s_2)$ . Consequently from Eq. (12) the triplet states must belong to the  $-1$  eigenvalue of  $U_D$ . In the same way it follows that the singlet state belongs to the  $+1$  eigenvalue. Comparing these results with those of the last part of Sec. III, we see that

- (1) the  $^1S$  states cannot decay into three photons;
- (2) the  $^3S$  states cannot decay into two photons, a result already known from space-symmetry considerations.

TABLE I. Values of  $J$ ,  $P$ , and  $U$  for states of positronium.

State	$J$	$P$	$U$
$^1S$	0	odd	+1
$^3S$	1	odd	-1
$^1P$	1	even	-1
$^3P$	0, 1, 2	even	+1

Clearly these results can be generalized to states of higher  $\mathbf{l}$ , with the result that the eigenvalues of  $U_D$  involve an extra factor  $(-1)^l$ . A classification of the  $S$  and  $P$  states of positronium is given in Table I, according to the eigenvalues of  $\mathbf{J}$ ,  $P$ , and  $U_D$ . It may be noted that the states which are odd under charge conjugation are just those which would not exist if the two particles were identical.

By using the property of invariance under charge conjugation it is possible to deduce another result for positronium—that the  $m = +1$  and  $m = -1$  levels of the  $^3S$  states are not separated by a magnetic field. Since the interaction of external electromagnetic fields with the charged particles is formally the same as for the radiation field, we require such fields also to change sign on charge conjugation. We then consider the behavior of states of positronium under the two successive transformations of charge conjugation and of rotation through  $180^\circ$  about an axis perpendicular to the axis of quantization, which is parallel to the magnetic field. Under the first transformation the magnetic field is reversed, while the states are unaltered (apart from a change of sign, which does not affect the energy eigenvalues of the states). Under the rotation the magnetic field is re-reversed, while the  $m = +1$  and  $m = -1$  states are interchanged. Since the only effect of the two transformations, which each leave the Hamiltonian invariant, is the interchange of the two states, the energies of the two must be exactly equal.

A third result that can be deduced, of less practical value, is that in electron-positron scattering there is no mixing of singlet and triplet states, since for the same values of  $\mathbf{J}$  and  $P$  they belong to different eigenvalues of  $U_D$ .

These results are absolute selection rules, independent of any perturbation expansion.

#### V. CHARGED BOSON FIELDS

For the sake of completeness we give the explicit form of  $U$  for a charged scalar field  $\phi$ . The transformation laws for such a field under charge conjugation are

$$\begin{aligned} U\phi(x)U^{-1} &= \eta\phi^*(x), \\ U\phi^*(x)U^{-1} &= \eta^{-1}\phi(x), \end{aligned}$$

where  $\eta$  is an arbitrary factor of modulus unity, which can be set equal to unity without loss of generality. In a way similar to that used in Sec. II we find the following transformation laws for  $a_s^*$ ,  $b_s^*$ ,  $a_s$ ,  $b_s$ , the free particle creation and annihilation operators:

$$\begin{aligned} Ua_sU^{-1} &= b_s, \\ Ua_s^*U^{-1} &= b_s^*. \end{aligned}$$

As in Sec. II we define new operators  $\alpha_s^*$ ,  $\beta_s^*$ ,  $\alpha_s$ ,  $\beta_s$  by

$$\alpha_s = (a_s + b_s)/\sqrt{2}, \quad \beta_s = (a_s - b_s)/\sqrt{2},$$

and the Hermitian conjugate equations. These new operators satisfy

$$\begin{aligned} U\alpha_s U^{-1} &= \alpha_s, \\ U\beta_s U^{-1} &= -\beta_s, \end{aligned} \quad (13)$$

and the usual commutation relations among themselves. The operator satisfying Eq. (13) is  $(-1)^{\sum_s \beta_s^* \beta_s}$ , so  $U$  may be represented as

$$U = (-1)^{\sum_s \beta_s^* \beta_s}.$$

## VI. EXTENSION TO NEUTRAL FIELDS

Although a neutral field is not generally assumed to interact directly with the electromagnetic field, one can attempt to determine the behavior of a neutral field under charge conjugation from its interaction with charged fields. For any single interaction term an appropriate assumption about the transformation law of the neutral field will make this term invariant under charge conjugation; for a neutral boson field  $\varphi(x)$  this assumption will be either

$$U\varphi(x)U^{-1} = \varphi(x), \quad (14a)$$

or

$$U\varphi(x)U^{-1} = -\varphi(x). \quad (14b)$$

Once one of these has been chosen there exist a whole set of interaction terms which are forbidden by the condition that the Hamiltonian be invariant under charge conjugation; consequently, selection rules concerning neutral particles may be derived from this invariance principle. It should be noted that it is possible formally to retain invariance under charge conjugation without drawing any conclusions about neutral particles; this can be done by assuming that the charge-conjugate of a neutral field is a different neutral field (the field of the antiparticle). For example, if a neutral particle consisted of a bound state of a proton and a negative  $\mu$ -meson, the charge-conjugate state would involve a negative proton and a positive  $\mu$ -meson. Thus the requirement that a neutral boson field follow either Eqs. (14a) or (14b) is really an additional assumption.

The interactions of a neutral scalar field  $\varphi$  with a single charged field are of the form

$$\begin{aligned} \varphi(x)\phi^*(x)\phi(x), \\ \varphi(x)\bar{\psi}(x)\psi(x), \\ \varphi(x)\Phi_\mu^*(x)\Phi_\mu(x), \end{aligned} \quad (15)$$

( $\phi$ ,  $\psi$ , and  $\Phi_\mu$  are scalar, spinor, and vector fields, respectively) where only terms linear in  $\varphi$  have been included. As long as only a single charged field is involved, vector interactions of the form  $\partial_\mu \varphi j_\mu(x)$ , where  $j_\mu$  is the current of the charged field, can be reduced to contact interactions of the charged field and terms quadratic in  $\varphi$ , neither of which contribute to the decay of a single neutral meson. Each of the forms (15) corre-

sponds to a decay of the neutral meson via a charged pair and each requires the transformation law (14a) for invariance under charge conjugation. Thus for all of these interactions the decay of the neutral scalar meson into three photons is forbidden, as previously noted. If interactions involving two charged fields are considered it is possible to get interaction terms which require (14b). One such interaction is a modified form of the vector interaction,

$$\partial_\mu \varphi(x) j_\mu(x) \bar{\chi}(x) \chi(x).$$

Here  $j_\mu$  can be  $\bar{\psi} \gamma_\mu \psi$  or the gauge-invariant currents of either of the fields  $\phi$  or  $\Phi_\mu$ , and  $\chi$  is another spinor field. This interaction should allow the decay into three photons. Similar considerations hold for a pseudoscalar neutral field; in this case both the pseudovector and the pseudoscalar interactions with a single charged field require Eq. (14a).

For the neutral vector field the simple vector and tensor interactions both require Eq. (14b) (for each component of the vector field). Interaction terms requiring Eq. (14a) can also be constructed; for example,

$$\partial_\mu \varphi \bar{\psi} \gamma_\mu \psi \bar{\chi} \gamma_\nu \chi.$$

A vector meson field obeying Eq. (14a) could not permit decay into two or three photons, but only into four. For the neutral pseudovector field the pseudovector interaction requires Eq. (14a) while the tensor interaction requires Eq. (14b) so that only one of these is allowed by our assumptions, as has been pointed out by Pais and Jost.<sup>3</sup>

In addition to the decays into a number of photons, some other selection rules for the decay of neutral bosons may be established. If the neutral boson field is even under charge conjugation (14a) the following decay scheme is forbidden:

$$B^0 \rightarrow \pi^0 + \pi^0 + \gamma, \quad (16)$$

where  $\pi^0$  is the usual neutral meson. This has been pointed out<sup>11</sup> as a consequence of Furry's theorem for a pseudoscalar  $B^0$ , but is independent of the spatial transformation properties of  $B^0$ . More generally, the decay into any number of neutral mesons (assumed to be even under charge conjugation) and an odd number of photons is impossible. If the neutral boson field is odd under charge conjugation (14b) many decay schemes are forbidden, including

$$\begin{aligned} B^0 &\rightarrow \pi^0 + \pi^0, \\ B^0 &\rightarrow \pi^+ + \pi^- \text{ for scalar } B_0, \\ B^0 &\rightarrow e^+ + e^- \text{ for scalar or pseudoscalar } B^0. \end{aligned}$$

On the other hand, (16) now becomes possible.

We should like to thank Professor R. E. Peierls for several very useful comments. We are deeply indebted to Professor G. C. Wick for many helpful discussions and suggestions.

<sup>11</sup> A. Pais, Phys. Rev. 86, 663 (1952).