

The de Haas-van Alphen Effect in Enriched Tin Isotopes

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The de Haas-van Alphen effect has been investigated in tin isotopes in the liquid helium temperature range, and no conclusive evidence has been found indicating a dependence of the effect upon atomic mass. However, some new features of the effect have been investigated which are interpreted as being caused by at least two independent groups of electrons. One group of electrons, having effective masses $m_2=1.1m_0$ (where m_0 is the free electron mass) parallel to the tetragonal axis and $m_1=0.16m_0$ perpendicular to the tetragonal axis, and having a chemical potential of $E_0=0.25$ electron volt, gives rise to short period oscillations in the magnetic susceptibility as the magnetic field is varied. A second group of electrons having a chemical potential $E_0=0.20$ electron volts and for which the effective masses are not so clearly defined gives rise to longer period oscillations such that the observed de Haas-van Alphen oscillations exhibit beats between the long and short period terms.

INTRODUCTION

THE de Haas-van Alphen effect in tin was first observed by Shoenberg¹ who has since completed a more comprehensive investigation² of the effect in tin as well as in several other elements. The effect has also been observed in tin by Verkin, Lazarev, and Rudenko³ and Croft, Love, and Nix.⁴ Shoenberg's studies indicated that the periodicity of the oscillations of the anisotropy of the magnetic susceptibility, when plotted against the reciprocal of the magnetic field, was a reasonably reproducible feature in different single crystals of tin, while the amplitude of these oscillations was not in general a very reproducible feature. Furthermore, since the periodicity of the oscillations is determined by rather clearly defined electronic parameters, it was felt that a comparison of the periods would reveal an isotope shift if it existed. Qualitatively, the argument may be stated as follows. The de Haas-van Alphen effect is caused by a relatively few electrons, from 10^{-3} to 10^{-6} per atom, having highly anisotropic effective masses, which therefore occupy some of the positions in phase space for which the curvature of the Fermi surface is very great. Thus, if the electronic structure is altered at all by a variation of the mass of

the lattice points, the de Haas-van Alphen electrons might be expected to reflect this change.

In any event, the theory as developed by Peierls,⁵ Blackman,⁶ and Landau⁷ assumes that the de Haas-van Alphen electrons are contained in one or more regions in phase space having ellipsoidal constant energy surfaces which have the symmetry of the particular lattice under consideration. As Shoenberg has shown, one such ellipsoid is inadequate in fully explaining the de Haas-van Alphen effect in tin. The present paper proposes one ellipsoid to account for the short period oscillations and at least one more less simple surface to account for the long period oscillations.

Inasmuch as the theory of the de Haas-van Alphen effect has been covered in detail elsewhere⁵⁻⁷ only a brief statement of the theoretical results will be given here. Modifications of this theory and methods for determining the pertinent electronic parameters from the experimental data will be discussed in a later section.

For not too high fields nor too low temperatures, Landau's final expression for the difference of magnetic susceptibility ($\Delta\chi$) per unit mass in two directions at right angles to each other is given by

$$\Delta\chi = \sum \frac{A\Delta m}{\rho} \left\{ \frac{\pi^2}{6} \left(\frac{k}{E_0} \right)^{\frac{1}{2}} - \frac{1}{T^{\frac{1}{2}}} \left(\frac{2\pi^2 kT}{\beta H} \right)^{\frac{1}{2}} \right. \\ \left. \times \exp \left(-\frac{2\pi^2 kT}{\beta H} \right) \sin \left(\frac{2\pi E_0}{\beta H} - \frac{\pi}{4} \right) \right\}, \quad (1)$$

where A is a constant given by

$$A = \frac{e^2 E_0}{\pi^4 c^2 \hbar (2k)^{\frac{1}{2}} m^{\frac{1}{2}}}, \quad (2)$$

ρ is the density, β is an effective double Bohr magneton

TABLE I. Sources, purities, and masses of the tin specimens.

Av. atomic mass	Source	Purity (%)	Mass (grams)
116.17	AEC	99.9	0.1373
118.70 Nat. Sn	JM*530 Lab. No. 4600	99.997	0.1549
121.63	AEC	99.7	0.1240
123.77	AEC	99.7	0.1207

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* DuPont Predoctoral Fellow in Physics. This work is part of a thesis submitted in partial fulfillment of the requirements for the Ph.D. degree at Yale University.

† Assisted by the ONR.

¹ D. Shoenberg, *Nature* **164**, 225 (1949).

² D. Shoenberg, *Phil. Trans. Roy. Soc.* **245** (No. 891) 1 (1952).

³ Verkin, Lazarev, and Rudenko, *Doklady Akad. Nauk. SSSR* **69**, 773 (1949). *J. Exp. Theor. Phys. USSR* **20**, 995 (1950).

⁴ Croft, Love, and Nix, *Phys. Rev.* **86**, 650 (1952).

⁵ R. Peierls, *Z. Physik* **81**, 186 (1933).

⁶ M. Blackman, *Proc. Roy. Soc. (London)* **A166**, 1 (1938).

⁷ L. D. Landau, see appendix to D. Shoenberg, *Proc. Roy. Soc. (London)* **A170**, 341 (1939).

given by

$$\beta = e\hbar/m''c, \quad (3)$$

and m' , m'' , and Δm are functions of the relevant effective masses which depend upon the geometry of the experiment and the symmetry of the crystal lattice. E_0 is the energy at the ellipsoidal surface of constant energy in phase space which is given in general by

$$E_0 = (1/2m_0)(\alpha_{11}p_x^2 + \alpha_{22}p_y^2 + \alpha_{33}p_z^2 + 2\alpha_{12}p_xp_y + 2\alpha_{23}p_y p_z + 2\alpha_{31}p_z p_x). \quad (4)$$

Here the α_{ij} are the ratios of the true electronic mass, m_0 , to the relevant effective masses. As will be illustrated later, these determine the definitions of m' , m'' , and Δm . Usually, the ellipsoidal constant energy surface is so oriented with respect to the crystal lattice that $\alpha_{12} = \alpha_{23} = \alpha_{31} = 0$.

THE EXPERIMENTAL WORK

I. The Torsion Balance Method

If a magnetically anisotropic single crystal is placed in a homogeneous magnetic field, the magnetization will not in general be in the same direction as the applied field. Hence, the crystal will experience a couple (C) per unit mass given by

$$C = \mathbf{I} \times \mathbf{H}, \quad (5)$$

where \mathbf{I} is the magnetization per unit mass. For the case of a tin single crystal placed in a horizontal magnetic field with a binary axis and the tetragonal axis horizontal, this becomes

$$C = \Delta\chi \cdot H^2 \sin\phi \cos\phi, \quad (6)$$

where ϕ is the angle between H and the tetragonal axis. $\Delta\chi$ is here the difference between the susceptibilities along the tetragonal axis and a binary axis, i.e., the quantity in which we are interested, as given by Eq. (1).

In this work, a torsion balance similar to that described by Sydoriak⁸ was used to determine $\Delta\chi$, the chief difference being that provision was made for transferring liquid helium to the apparatus directly, thus minimizing the probability of disturbing the alignment. Temperatures were determined by observation of the vapor pressure of the helium bath with mercury and oil manometers. The Mond vapor pressure tables⁹ were used, and temperatures were held to better than 0.01°K.

The crystal being investigated was mounted on the lower end of a long quartz rod approximately 1 mm in diameter, the upper end of which was rigidly affixed to a small brass frame carrying a front silvered mirror and a damping ring which dipped into Octoil-S. This frame was in turn fastened to the lower end of a flattened nichrome wire torsion member. Light from an illuminated vertical slit placed at the focus of a one meter focal length lens (which also served as a balance

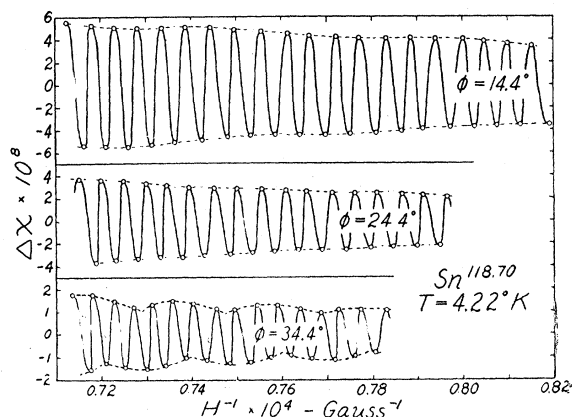


FIG. 1. $\Delta\chi$ in emu versus H^{-1} in gauss⁻¹ for natural tin, Sn^{118.7}, for various angles between the tetragonal axis and the magnetic field. At $\phi = 34.4^\circ$ modulations caused by a short period term become important.

case window) was reflected from the mirror and brought to focus so that the diffraction pattern image of the slit could be observed with a traveling microscope. Torsion fibers were calibrated by observation of the periods of oscillation when systems of known moment of inertia were attached.

Because of the close spacing of the susceptibility oscillations in tin, it was essential that very small angular displacements be utilized in the measurements. For example, a displacement of a few degrees would be sufficient in some cases to move the crystal from a position of maximum susceptibility to one of minimum susceptibility. Hence, the angular displacements utilized never exceeded 0.2° and were in most cases less than 0.05°. These were reproducible to $\pm 0.0003^\circ$, since the slit image observed with the traveling microscope was a well-defined diffraction pattern.

II. The Magnet and Its Calibration

Magnetic fields up to 14 kilogauss were supplied by a water-cooled magnet capable of rotation about a vertical axis and having 6-in. diameter flat pole pieces with a gap of 4 cm. Again because of the close spacing of the susceptibility peaks in tin, it was essential that the field be homogeneous to a high degree over the volume of the sample and also time invariant. Exploration of the gap with a search coil showed the field to be homogeneous (to about two parts in 10⁴) within 5 mm in any direction from the center of the gap. This proved to be quite satisfactory since the tin crystals were never larger than 3 mm in any one dimension. A degenerative feedback voltage regulator used in conjunction with the motor-generator set which supplied power to the magnet assured that variations of the field could be limited to one or two gauss during the course of each measurement. The magnetic field was calibrated with a proton resonance fluxmeter to an accuracy of approximately 0.03 percent with proper precautions being taken against hysteresis.

⁸ S. G. Sydoriak and J. E. Robinson, Phys. Rev. **75**, 118 (1949).

⁹ H. Van Dijk and D. Shoenberg, Nature **164**, 151 (1949).

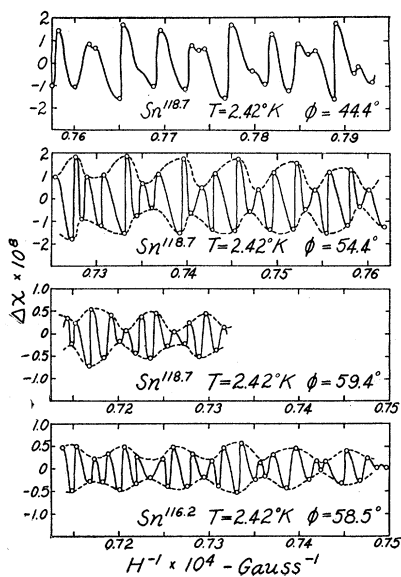


FIG. 2. $\Delta\chi$ in emu versus H^{-1} in gauss $^{-1}$ for natural tin, $\text{Sn}^{118.7}$, and $\text{Sn}^{116.2}$ for various angles between the tetragonal axis and the magnetic field. The long and short period terms are both important at these angles and temperatures. The last two curves illustrate the reproducibility for different isotopes.

III. The Specimens

After several unsuccessful methods for growing small tin single crystals had been tried, the following procedure was finally adopted. A small (approximately 150 mg) bead of tin was placed in a 9-mm Pyrex tube sealed off at one end which was then evacuated. Next the tin was melted with a Bunsen burner. Upon removal of the Bunsen burner, the portion of the Pyrex tube directly beneath the molten tin was touched to

the surface of a beaker of hot water, thus assuring that the crystallization proceeded from the point of contact of the tin with the tube. This procedure produced single crystals with roughly 25 percent efficiency. The resulting crystals were etched with concentrated hydrochloric acid until good reflections were obtained. They were then oriented using an optical method described by Chalmers.¹⁰ While a given crystal, with a binary axis vertical, was still fastened to the goniometer head with Duco cement, the torsion balance was manipulated so that the quartz rod hung plumb above the crystal. The rod was then fastened to the crystal with Duco cement, and in this way the crystal retained the desired orientation during the setting of the cement. The bond between the crystal and the goniometer was then severed by judicious application of acetone with a small brush. Because the anisotropy of tin single crystals is very small at room temperature, the position of the tetragonal axis in the horizontal plane was determined by observation of the position of the magnet for which the amplitude of the de Haas-van Alphen effect just vanished at liquid helium temperatures, a procedure due to Shoenberg.

Pertinent information regarding the tin specimens¹¹ is recorded in Table I. The $\text{Sn}^{116.17}$ and $\text{Sn}^{121.63}$ were supplied in the form of SnO_2 and were reduced in a hydrogen furnace. It should also be pointed out that the stated purities are probably not very accurate, for the samples were crystallized and etched several times in general before suitable crystals were obtained.

EXPERIMENTAL RESULTS

The general features of the de Haas-van Alphen effect in tin are illustrated in Figs. 1, 2, and 3, where

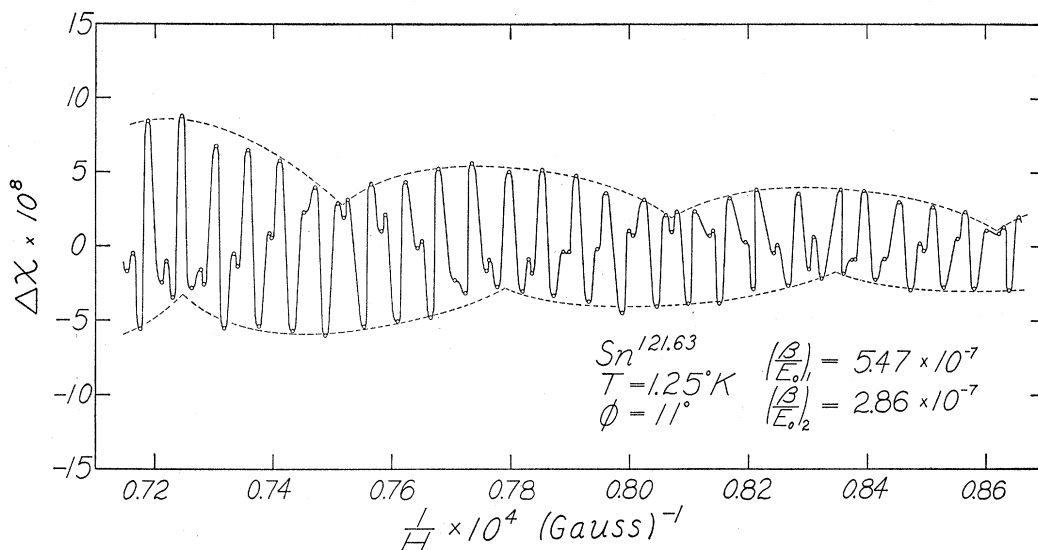


FIG. 3. $\Delta\chi$ in emu versus H^{-1} in gauss $^{-1}$ for $\text{Sn}^{121.63}$ at 1.25°K and $\phi = 11^\circ$. The presence of two periodic terms accounts for the cusp-shaped envelope.

¹⁰ B. Chalmers, Proc. Phys. Soc. (London) **47**, 733 (1935).

¹¹ The enriched isotopes were produced by Carbide and Carbon Chemicals Division, Oak Ridge National Laboratory, Y-12 Area, Oak Ridge, Tennessee and obtained on allocation from the Isotopes Division of the AEC.

$\Delta\chi$ in emu has been plotted against H^{-1} . We note that at high temperatures and for small angles between the magnetic field and the tetragonal axis, only one term in Eq. (1) is important, i.e., a long period term. For lower temperatures and larger values of ϕ however, a shorter period term also becomes important, and modulations characteristic of the summation of two sinusoidal variations are observed. For $\phi=34.4^\circ$ and $T=4.22^\circ\text{K}$ the cusp-shaped modulation first observed by Shoenberg, and for which he proposed no explanation, begins to appear. The explanation of this unusual type of modulation becomes clear however, in view of Fig. 3, where the temperature was sufficiently low and the magnetic field regulation good enough to permit resolution of both frequencies even for $\phi=11^\circ$. In order to verify that such a curve could be synthesized from two sinusoidal terms, two audiofrequency oscillators were connected in parallel to the vertical plates of a cathode-ray oscilloscope, and with a little manipulation of frequencies and amplitudes such a curve was indeed produced.

At 2.42°K and $\phi=54.4^\circ$ the short period term is most important and the more common types of beats are observed. The final two curves of Fig. 2 for two different isotopes and only slightly different angles illustrate the reproducibility of the measurements from one isotope to another.

If the values of H^{-1} for which maxima in $\Delta\chi$ occur are plotted against successive integers as in Fig. 4, a straight line is obtained verifying the strict periodicity of $\Delta\chi$ in H^{-1} provided, of course, that only one term is important. The slope of such a curve is just β/E_0 , the period of the oscillations. For curves of anisotropy *versus* H^{-1} displaying beats, both periods could often be determined unambiguously by standard means. Following Shoenberg, the square of these periods has

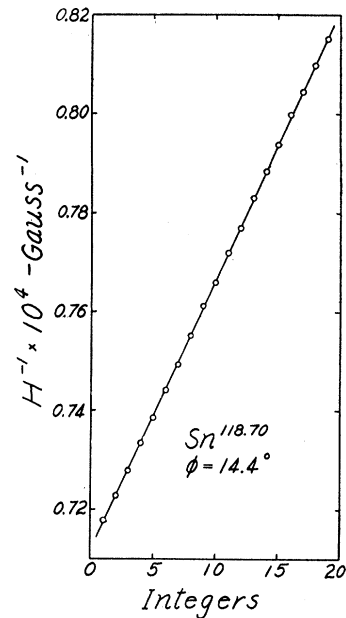


FIG. 4. Typical plot of values of H^{-1} in gauss⁻¹ for which maxima in $\Delta\chi$ occur *versus* integers, illustrating the strict periodicity of the susceptibility oscillations in H^{-1} .

been plotted against $\cos^2\phi$ in Fig. 5. This permits comparison of the different enriched isotopes investigated, and, as will be discussed later, provides a scheme for determining the electronic structure which gives rise to the two periodic terms. The heavy points refer to cases where both periods were obtained from a single curve of $\Delta\chi$ *versus* H^{-1} . Usually 20, but sometimes as many as 80, periods were observed at each angle ϕ , i.e., each point on Fig. 5 represents a curve of $\Delta\chi$ *versus* H^{-1} such as those in Figs. 1, 2, and 3. It is quite evident that if any isotope shift in the period does exist, it is beyond the accuracy of these measurements.

In Fig. 6 the amplitude, a at 14 kilogauss and 4.22°K for the long period oscillations is plotted as a function

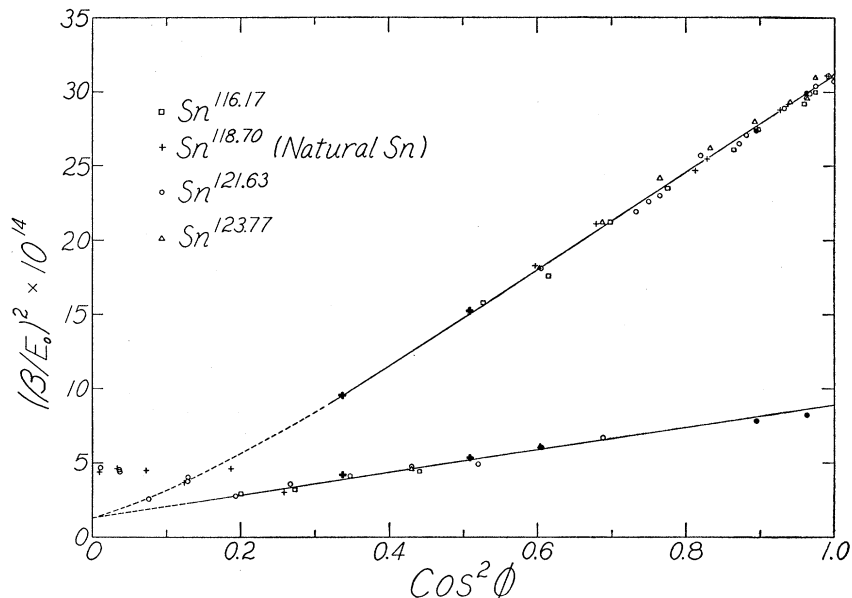


FIG. 5. Comparison of periods for the various isotopes. The linear relation between the period squared, $(\beta/E_0)^2$, and $\cos^2\phi$ provides a scheme for analyzing the electronic structure which causes the de Haas-van Alphen effect.

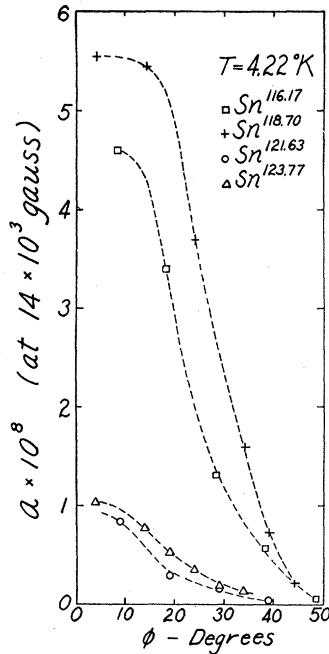


FIG. 6. Comparison at various angles of the long period term amplitudes of the susceptibility oscillations for the various enriched isotopes.

of ϕ for the various isotopes investigated. In order to obtain these amplitudes where strong modulations were present it was necessary to smooth the curves of $\Delta\chi$ versus H^{-1} . The large difference in amplitudes is not surprising in view of the purities listed in Table I and in view of Shoenberg's experiments on the effect of impurities. It might be significant that the two heaviest isotopes did exhibit much smaller amplitudes than did the light isotopes, but on the other hand they were also the most impure, so it is doubtful that any definite conclusions can be drawn from Fig. 6.

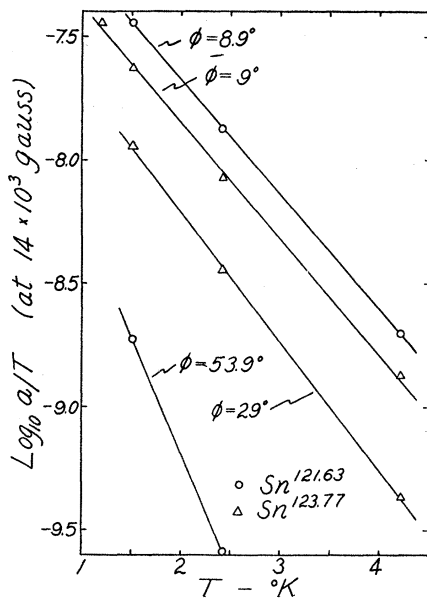


FIG. 7. $\text{Log}_{10}(a/T)$ at 14 kilogauss versus T . Values of β are derived from the slopes of such plots.

It should be mentioned in passing that unsuccessful attempts were made to observe the de Haas-van Alphen effect in Te, As, Pb, and Ge. Pertinent data regarding these crystals is listed in Table II. With the exception of the As crystal, all of these specimens were of high quality. Unfortunately, while As appears most likely to exhibit the de Haas-van Alphen effect, it is difficult to obtain as a single crystal.

CALCULATION OF ELECTRONIC PARAMETERS

Inspection of Eq. (1) indicates that β may be calculated from the temperature dependence of the amplitude, i.e., a plot of $\log(a/T)$ versus T (for fixed H and ϕ) should yield a straight line from the slope of which β may be calculated. Several such plots appear in Fig. 7 for different values of ϕ . The three upper lines refer to the long period oscillations while the lower one refers to the short period oscillations. Again because of the modulations, it was necessary to smooth the curves of $\Delta\chi$ versus H^{-1} in order to obtain a . Once β has been determined, E_0 is readily obtained from the period, β/E_0 . Values derived from Fig. 7 are listed in Table III. These values show good agreement with those of

TABLE II. Sources, purities, and orientations of crystals not exhibiting the de Haas-van Alphen effect.

Crystal	Source	Purity	Orientation
Te	JM 795 Lab. No. 4938	99.99%	Hex. axis horizontal
As	Lab. Stock	Unknown	Trig. axis horizontal
Pb	JM F688 Lab. No. 1162	99.999%	001 axis vertical
Ge	Dr. R. N. Hall, G. E. Res. Lab.	33 ohm cm	001 axis vertical

Shoenberg, who has verified that E_0 for the long period term is independent of ϕ within experimental error.

Close inspection of Fig. 7 shows that the points could be more accurately fitted to a curve which is slightly concave upward. This is to be expected inasmuch as Eq. (1) is a valid approximation only when $E_0 \gg kT \gg \beta H / 2\pi^2$. If the latter half of this inequality is not satisfied, the product of the exponential and the sine in Eq. (1) must be replaced by

$$\sum_{p=1}^{\infty} \frac{\sin(2\pi p E_0 / \beta H - \frac{1}{4}\pi) (-1)^{p+1}}{2p^{\frac{1}{2}} \sinh(2\pi^2 p kT / \beta H)}, \quad (7)$$

which predicts the presence of harmonic terms. For the lowest temperatures and the highest fields in this investigation kT was very nearly equal to $\beta H / 2\pi^2$. However, taking account of this fact would change the calculated values of β only slightly, so this discrepancy was neglected.

The presence of apparently independent long and short period terms in the susceptibility variations suggests that it is necessary to postulate at least two independent groups of electrons to account for the de Haas-van Alphen effect in tin. Since a straight line is obtained when the period squared for the short period oscilla-

tions is plotted against $\cos^2\phi$ (see Fig. 5), the assumption is made that for this group of electrons the constant energy surface in phase space is an ellipsoid of revolution. The electrons then have effective masses m_1 at right angles to the tetragonal axis and m_3 along the tetragonal axis. This assumption is made in spite of the fact that, as Shoenberg has shown, an anisotropy does exist in the plane perpendicular to the tetragonal axis. However, since the constant energy surface for the long period term appears more complicated, as may be judged from Fig. 5, it is most convenient to attribute this anisotropy to this group of electrons or to still other groups which cause the irregular dependence of period upon ϕ at large angles.

Then for the short period electrons we have

$$E_0 = (p_x^2 + p_y^2)/2m_1 + p_z^2/2m_3, \quad (8)$$

and

$$\beta = -\frac{e\hbar}{c} \left(\frac{m_3 \cos^2\phi + m_1 \sin^2\phi}{m_1^2 m_3} \right)^{\frac{1}{2}}. \quad (9)$$

Using Eq. (9) and values of β derived from Fig. 5 on the basis that $E_0 = 40.1 \times 10^{-14}$ erg, the following effective masses are obtained for the electrons causing

TABLE III. Values of the effective double Bohr magneton, the period, and the chemical potential for the de Haas-van Alphen electrons in tin.

Specimen	ϕ	$\beta \times 10^{20}$ (erg gauss ⁻¹)	$\beta/E_0 \times 10^7$ (gauss ⁻¹)	$E_0 \times 10^{14}$ (ergs)	
Sn ^{123.77}	9°	17.9	5.54	32.2	long period
Sn ^{123.77}	29°	16.1	4.93	32.6	
Sn ^{121.63}	8.9°	18.2	5.52	32.9	short period
Sn ^{121.63}	53.9°	8.02	2.00	40.1	

the short period oscillations:

$$m_1 = 0.16m_0, \quad m_3 = 1.1m_0, \quad (E_0 = 0.25 \text{ ev}).$$

The number of electrons per atom involved in the de Haas-van Alphen effect is then given by

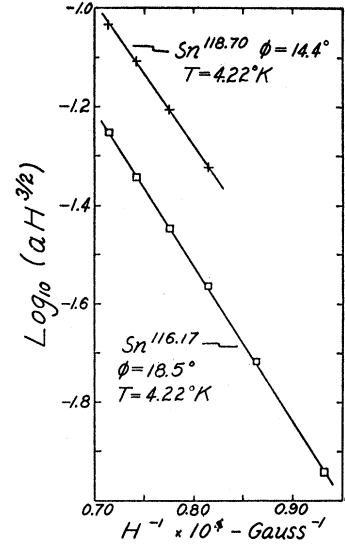
$$n = (8\pi/3h^3)(V/N)m_1 m_3^{\frac{1}{2}} (2E_0)^{\frac{3}{2}}, \quad (10)$$

where V is the gram molecular volume and N is Avogadro's number. Substituting the values of m_1 , m_3 , and E_0 listed above yields $n = 2.6 \times 10^{-3}$ electrons per atom for the electrons causing the short period oscillations.

Since the relation between $(\beta/E_0)^2$ and $\cos^2\phi$ for the long period term, if considered linear, would extrapolate to imaginary periods for large values of ϕ , little can be said with finality regarding the electrons causing the long period oscillations. However, using the average value $E_0 = 32.6 \times 10^{-14}$ erg = 0.20 electron volt from Table III and making the assumption that Eq. (9) is suitable for small values of ϕ , a value of $m_1 = 0.10m_0$ is obtained. No further analysis seems justified here.

According to recent theoretical work by Dingle² collision broadening of the energy levels due to impurities will reduce the amplitude of the oscillations by the

FIG. 8. $\text{Log}_{10}(aH^{\frac{3}{2}})$ versus H^{-1} . Values of the amplitude damping parameter, x , resulting from collision broadening of the energy levels are derived from the slopes of such plots.



same amount as if the temperature appearing in the exponential of Eq. (1) were raised by an amount x . It can readily be shown then that $T+x$ is given by the product of $-\beta/2\pi^2k$ and the slope of the straight line obtained when $\log(aH^{\frac{3}{2}})$ is plotted against H^{-1} . Two illustrative plots for the long period term appear in Fig. 8, and values of x for the various isotopes are compared in Table IV with no indications of an isotopic dependence being present. The strong modulations prohibited similar analysis of the short period term. The dependence of x upon T and ϕ might be of interest, but again the data do not justify any analysis. The correlation between x and purity is only fair, since higher values of x are to be expected for the relatively impure Sn^{123.77} crystal. In general, the values of x listed in Table IV are a bit lower than those quoted by Shoenberg which could conceivably be attributed to better equilibrium between the sample and the helium bath.

CONCLUSIONS

Within the experimental error of this work, there is no evidence of an isotope shift in the de Haas-van Alphen effect in tin. However, it now appears that tin is a poor choice for such an investigation because of

TABLE IV. Values of the amplitude damping parameter, x , for the various tin specimens.

Specimen	$\phi(0)$	T (°K)	x (°K)
Sn ^{116.17}	8.5	4.22	0.9
Sn ^{116.17}	18.5	4.22	0.4
Sn ^{118.70}	14.4	4.22	0.2
Sn ^{118.70}	24.4	4.22	0.6
Sn ^{121.63}	8.9	4.22	2.1
Sn ^{121.63}	8.9	1.51	2.8
Sn ^{123.77}	9.0	4.22	1.1
Sn ^{123.77}	9.0	2.42	0.9

the relatively large effective masses of the electrons involved. Perhaps a crystal, such as zinc, in which the de Haas-van Alphen electrons have much smaller effective masses, which corresponds to higher curvature of the Fermi surface, would be more likely to show an isotopic dependence. The presence of only one periodic term in this case would also simplify comparison. On the other hand, it is highly probable that the presence of impurities would influence more drastically the effective masses in zinc than in tin.

Nevertheless, it can be concluded that there are at least two groups of electrons responsible for the de Haas-van Alphen effect in tin. While a more thorough investigation of the dependence of the period upon orientation for large values of ϕ might further clarify the electronic structure, it appears doubtful that such

an investigation would add much of value to an understanding of the de Haas-van Alphen effect.

It is a pleasure to thank Professor C. T. Lane who suggested and directed this problem. I am also indebted to Professor D. Shoenberg, who sent me, prior to publication, the manuscript of his excellent and comprehensive paper² on the de Haas-van Alphen effect. Many thanks are due the staff of the General Electric Research Laboratory where early stages of this work were completed. In particular, I wish to thank Dr. M. D. Fiske of the Low Temperature Group for his interest, Mr. F. C. Kelley for the reduction of the oxides of the tin isotopes, and Dr. R. N. Hall for the germanium single crystal. The magnet used in this research was most generously loaned by the General Electric Company.

The Abundances of the Elements

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The cosmic abundances of the elements were estimated by Goldschmidt (1937) from a study of terrestrial and meteoritic abundances and a comparison of these with Russell's data on the sun. More recently Brown has prepared a table weighting the proportions of the iron and silicate phases according to an estimated proportion of these phases in the earth. The writer has recently proposed that the chondritic meteorites themselves may represent an average sample, and has shown that this assumption is consistent with the density of the moon, which on the basis of his recent discussion in regard to the origin of the solar system should also be approximately a sample of nonvolatile materials. A table of abundances is prepared assuming that these meteorites do represent such a mean sample. This table does not differ markedly from Goldschmidt's, but is distinctly different from Brown's, both because of a different weighting of the phases in meteorites and because of a different choice of data from the literature. Iron is much less abundant than estimated by Brown and somewhat less abundant than Goldschmidt's estimate. There is some indication for markedly low abundances of selenium and tellurium and bromine and iodine, which may indicate some escape of these elements during the formation of the meteorites. Mercury is low, almost certainly because of its loss as a volatile substance during the formation of the meteorites.

THE abundance of the elements in nature is a subject of interest to astronomers, physicists, and geophysicists. These abundances were discussed in great detail by Goldschmidt, and his work has remained a classic in this field up to the present time. He and his co-workers made very many determinations of the abundances of the elements in the rocks of the earth and in the meteorites, and he critically reviewed the work of other people in a classic paper published in 1937-1938.¹ Russell² first attempted an estimate of the relative abundances of the elements in the stars and sun, and his conclusion that the composition of the stars and of the earth and meteorites is very nearly the same remains a valid conclusion today; in fact

it has not been possible in general to draw a definite conclusion as to whether the stars on the one hand, and the earth and the meteorites on the other have exactly or only approximately the same composition. In the case of certain stars the composition is certainly different from that of other stars, but whether these are or are not unusual stars and therefore the exception to the rule, is difficult to decide.

Goldschmidt weighted the stone, iron, and troilite phases in the proportions of 10:2:1, approximating in this way what he believed to be the composition of the terrestrial planets. He and other workers in the field used these three types of phases, often determining the abundance of the elements in each of the three phases and weighting them in the proportions indicated. In some cases the original determinations were made upon the whole meteorite, and Goldschmidt attempted to separate out the abundances in each one of these

¹ V. M. Goldschmidt, *Geochem. Verteilungsgesetze der Elemente IX*, Skrifter av det Norske Videnskaps-Akad. i Oslo I. Mat.-Naturv. Klasse No. 4, 1937. Goldschmidt gives references to many other researchers, including his co-workers.

² H. N. Russell, *Astrophys. J.* **70**, 11 (1929).