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Rotational Magnetic Moment, Magnetic Susceptibilities, and Electron Distribution in the Hydrogen Molecule*

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Measurements have been made on the rotational spectrum of H_2 in the state $I=J=1$ and in strong external magnetic fields. These measurements yield a value for the rotational magnetic moment of H_2 in the rotational state $J=1$ of $\mu_{RH}=(0.88291\pm0.00007)$ nuclear magnetons. From this, the high frequency contribution to the diamagnetic susceptibility is calculated to be $\xi_{HF}=(1.64\pm0.16)\times10^{-31}$ erg gauss^{-2} molecule^{-1}. Since the diamagnetic interaction with the external field increases as the square of the field, the high field measurements yield accurate determinations of the dependence of the diamagnetic susceptibility on the molecules' orientations which is found to be $\xi_{\pm1}-\xi_0=-(3.66\pm0.20)\times10^{-31}$ erg gauss^{-2} molecule^{-1}. ξ_{HF} and $\xi_{\pm1}-\xi_0$ may be combined to give a measure of the quadrupole moment of the electron distribution relative to the internuclear axis with the result that $Q_e^{\text{expt}}=(0.330\pm0.013)\times10^{-16}$ cm^2.

I. INTRODUCTION

MOST of the measurements on the radiofrequency magnetic resonance spectra of H_2 and D_2 up to date[1-3] have been made on the nuclear spectra ($\Delta m_I=\pm1, \Delta m_J=0$). The present experiment is on the rotational spectrum which consists of transitions for which $\Delta m_I=0$, $\Delta m_J=\pm1$. These measurements are somewhat more difficult to make because the signal intensity is lower and also some of the interesting quantities are measurable only with strong external magnetic fields. These measurements are of particular interest because they make possible the evaluation of some of the parameters occurring in the Hamiltonian[4] which are independent of the nuclear orientation and, therefore, cannot be evaluated from measurements on the nuclear spectrum. The measurements on the rotational spectrum at strong external magnetic fields yield values for the rotational magnetic moment, the high frequency contribution to the magnetic susceptibility, and the dependence of the diamagnetic susceptibility on the molecules' orientation. It has been shown[5] that the magnetic susceptibilities determined from measurements on the rotational spectrum may be combined to give a measure of the quadrupole moment of the electron distribution in the molecule. The only previous measurements on the rotational spectrum of H_2 were those of Frisch and Stern[6] which yielded crude values of the rotational magnetic moment, and measurements of Ramsey[7] which improved the numerical value for the rotational magnetic moment but were not accurate enough nor at a high enough external magnetic field to give good measurements of the diamagnetic terms occurring in the Hamiltonian.

TABLE I. Allowed rotational transition frequencies in the strong field limit for H_2 and D_2.

Transition	m_I	m_J	$m_{I'}$	$m_{J'}$	Frequency
AB/KL	∓1	$-1/0$	∓1	$0/+1$	$(1-\sigma_{J1})b\pm(-c+3d/2)$ $\mp f-C_2'+(\sigma_{i0}-\sigma_{i1})a$
BC/GK	∓1	$0/-1$	∓1	$+1/0$	$(1-\sigma_{J1})b\mp(c+3d/2)$ $\pm f-(C_2-C_2')\pm C_3+(\sigma_{i1}-\sigma_{i0})a$
DE/EF	0	$-1/0$	0	$0/+1$	$(1-\sigma_{J1})b\mp3d\mp f-C_2'\mp2C_3$

II. APPARATUS AND PROCEDURE

The apparatus used in these experiments is that described previously[2,3] with improvements. The magnet current regulators[8] were improved to enable them to regulate with the required accuracy at strong magnetic fields. Current variations due to noise or battery voltage drift which could not be corrected by the electronic regulators were taken care of by trigger circuits and motor driven rheostats. Special precautions were taken with the motor driven rheostats to avoid contact troubles. Silver buttons with pressure exerted by a bronze spring provided the movable contact on the constantan strips and proved satisfactory. One difficulty with the current regulator was that, due to the slowness of the galvanometer compared to the magnetic circuit, any fast variations were not corrected.

The procedure in recording and treating the data was similar to that discussed elsewhere.[3] Due to the low intensity of the resonances of the rotational spectra, it was often very difficult to distinguish the resonances from the noise.

There are a number of apparatus effects for which special precautions are required. The central magnetic field is not completely homogeneous but varies considerably along its length. This gives rise to a compli-

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[1] Kellogg, Rabi, Ramsey, and Zacharias, Phys. Rev. **56**, 728 (1939); **57**, 677 (1940).

[2] Kolsky, Phipps, Ramsey, and Silsbee, Phys. Rev. **79**, 883 (1950); **80**, 483 (1950); **81**, 1061 (1951); Phys. Rev. **87**, 395 (1952).

[3] Harrick, Barnes, Bray, and Ramsey (to be published).

[4] N. F. Ramsey, Phys. Rev. **85**, 60 (1952). It will be noted that Table VIII of this reference disagrees with Table I of the present paper. This is due to a minor error in the earlier table which is corrected in the present Table I.

[5] N F. Ramsey, Phys. Rev. **78**, 221 (1950). Equation (9) of the present paper corrects some errors of sign in the original paper.

[6] O. Frisch and O. Stern, Z. Physik **85**, 4 (1933).

[7] N. F. Ramsey, Phys. Rev. **58**, 226 (1940).

[8] N. J. Harrick, unpublished Ph.D. thesis, Harvard University, 1952.

cated pattern when Rabi's method[1] of a single oscillating field is used. This pattern is sometimes as much as 25 kc wide and can be altered by shimming the magnetic field. A second apparatus effect is observed even when Ramsey's method of separated oscillating fields[9] is used. Although the resonance observed with this method is not appreciably broadened beyond the theoretical value by the non-uniformities of the field, the intensity of the resonance is markedly reduced. This reduction is particularly serious in strong magnetic fields where it is probably due in part to the slowness of the current regulator for the magnetic field and in part to the non-uniformity of the magnetic field over the height of the beam and over the two oscillatory field regions. The variation of the fixed magnetic field in the region of the oscillatory field is a particularly important cause for concern because it might give rise to a dependence of the resonance frequency upon the oscillatory current. However, observations of the resonance frequency with different currents were made, and no appreciable shift was observed. If such a shift were found it presumably could be eliminated by a shortening of the length of the oscillatory field regions and by an increase in the homogeneity of the fixed fields. The low intensities of the resonances, particularly at high fields and for rotational transitions, made the present observations quite difficult and contributed appreciably to the final error.

TABLE II. Data on the rotational spectrum of H_2 at strong magnetic fields. The values of m_I and m_J, respectively, for the different transitions are as follows: EF $(0, 0\leftrightarrow\pm1)$, DE $(0, -1\leftrightarrow0)$, KL $(+1, 0\leftrightarrow+1)$, AB $(-1, -1\leftrightarrow0)$, GK $(1, 0\leftrightarrow+1)$, and BC $(-1, -1\leftrightarrow0)$.

No. of observations	Transition	Position of resonance (cps)
1	ν_D	2,268,903±10
	EF	2,508,945±30
	DE	2,163,820±20
2	ν_D	2,267,247±6
	EF	2,507,255±20
	DE	2,162,117±20
5	ν_D	3,287,223±10
	EF	3,557,275±20
	DE	3,212,781±20
2	ν_D	3,287,197±20
	EF	3,557,202±40
	DE	3,212,701±40
2	ν_D	2,431,730±10
	EF	2,676,529±40
	DE	2,331,484±40
2	ν_D	2,420,760±10
	EF	2,665,265±40
	DE	2,320,217±40
2	ν_D	2,407,886±10
	KL	2,506,576±60
	AB	2,452,469±40
3	ν_D	3,184,690±10
	EF	3,451,738±30
	DE	3,107,145±30
2	ν_D	3,184,690±10
	KL	3,306,310±120
	AB	3,252,680±40
3	ν_D	4,403,595±10
	EF	4,706,392±60
	DE	4,362,917±60
3	ν_D	4,373,575±20
	KL	4,530,035±100
	AB	4,477,593±300
2	ν_D	4,353,350±20
	KL	4,509,285±100
	AB	4,457,180±200
2	ν_D	4,318,906±20
	GK	4,646,835±60
	BC	4,243,533±60
1	ν_D	4,318,855±20
	EF	4,619,159±80
	DE	4,275,600±80

TABLE III. Constants required in calculations on rotational spectrum.

$$a/H=(4257.84\pm0.10)\text{ cps gauss}^{-1}$$
$$6d=346{,}026\pm144\text{ cps}$$
$$2c-3d=54{,}800\pm80\text{ cps}$$
$$2c+3d=400{,}829\pm80\text{ cps}$$
$$C_2'=(209{,}430\pm300)/H\text{ cps}$$
$$C_2=(15{,}380{,}000\pm5{,}000)/H\text{ cps}$$
$$C_3=-(455.1\pm0.2)\times10^6/H^2\text{ cps}$$

III. H_2 ROTATIONAL SPECTRUM AT STRONG MAGNETIC FIELDS

The theory of the hydrogen molecule in a magnetic field has recently been published.[4] In the previous notation,[4] the rotational spectrum consists of the six transitions AB, KL, BC, GK, DE, and EF for which $\Delta m_I=0$, $\Delta m_J=\pm1$. The expressions for the rotational transition frequencies are given in Table I.[4]

The measurements on the rotational spectrum are especially interesting in strong fields because one of the quantities of interest is the diamagnetic term, f, which increases as the square of the magnetic field and is just observable at about 3000 gauss. Measurements were made at various fields ranging from 3000 to 7000 gauss. The upper field limit was set by the disappearance of the interference pattern as mentioned earlier.

The experimental data obtained on the H_2 rotational spectrum are given in Table II. Most of the measurements were made on lines DE and EF. Since $m_I=0$ for these molecules, there are no deflections due to the nuclear magnetic moment; hence a larger signal is obtained for this pair than for AB and KL or BC and GK. Since, at best, the signal on the rotational lines was small, it was easier to make measurements on DE and EF. The transition frequency, ν_D, of D_2 in the state $I=2$, $J=0$ was used to calibrate the magnetic field.[3] Regular and frequent runs were made on this

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[9] N. F. Ramsey, Phys. Rev. **78**, 695 (1950); N. F. Ramsey and H. B. Silsbee, Phys. Rev. **84**, 506 (1951).

TABLE IV. Summary of calculations leading to the rotational magnetic moment of H_2.

No. of observations	Transitions	$(1-\sigma_{J1})b/H$ (cps gauss^{-1})
21	*DE* and *EF*	673.040±0.036
7	*AB* and *KL*	673.053±0.040
2	*BC* and *GK*	673.044±0.040

resonance to give an estimate of any magnetic field drift.

The numerical values of quantities required to carry out the calculations on the present experimental results are given in Table III.[3, 8]

IV. ANALYSIS OF RESULTS

A. Rotational Magnetic Moment

Since the electronic state of the hydrogen molecule is $^1\Sigma$, the nonrotating molecule has no net electronic magnetic dipole moment. There is, however, a magnetic moment induced by the rotation of the molecule. This rotational magnetic moment arises from the circulating protons and electrons and is of a magnitude comparable to the nuclear magnetic moment. Molecular beams are thus deflected due to the presence of a rotational magnetic moment as well as a nuclear magnetic moment.

The calculation for the rotational magnetic moment is carried out in the following manner. Consider, for example, the pair of resonance frequencies *DE* and *EF* of Table I. $(1-\sigma_{J1})b/H$ may be obtained, by iteration, from

$$(1-\sigma_{J1})b/H=(DE+EF)/2H+\left(c-\frac{3d}{2}\right)^2\Big/(a-b)H. \quad (1)$$

In a similar manner, calculations may be carried out on the other pairs of transition frequencies. The results of the measurements on the various pairs of transition frequencies are given in Table IV.

The best value of $(1-\sigma_{J1})b/H$ is

$$(1-\sigma_{J1})b/H=673.043\pm0.036 \text{ cps gauss}^{-1}.$$

Using this value of $(1-\sigma_{J1})b/H$, the rotational magnetic moment, μ_{RH}, of H_2 may be calculated, in the usual notation, from

$$\mu_{\mathrm{RH}}/J=hb/H. \quad (2)$$

When the above experimental value is used in the preceding equation along with the DuMond and Cohen[10] values of the fundamental constants, the result is

$$\mu_{\mathrm{RH}}=0.88291\pm0.00007 \text{ nuclear magneton}, \quad (3)$$

if the small unknown correction σ_{J1} is neglected.

The contribution of the electrons to the rotational magnetic moment is

$$\begin{aligned}\mu_{\mathrm{Re}}/J&=\mu_{\mathrm{RH}}/J-1\\&=-0.11709\pm0.00007 \text{ nuclear magneton.}\end{aligned}$$

B. High Frequency Terms of the Magnetic Susceptibility

The rotational magnetic moment has been related to the high frequency terms[11] of the H_2 diamagnetic susceptibility by Wick[12, 13] under the assumption that the effect of zero-point vibration in the molecule could be neglected. Ramsey[14] has recently discussed the effects of zero-point vibration on this and has shown that if the high frequency terms are assumed to vary with the internuclear spacing, R, as R^l these terms for H_2 in the zeroth vibrational and first rotational state are

$$\begin{aligned}{}_0^{\mathrm{H}}\chi_1^{\mathrm{HF}}=(e^2/12mc^2)R_e^2L\,{}_0^{\mathrm{H}}\langle(R/R_e)^l\rangle_1\,{}_0^{\mathrm{H}}\langle(R/R_e)^{l-2}\rangle_1^{-1}\\ \times\{1-{}_0^{\mathrm{H}}\langle\mu_R/J\mu_{NM}\rangle_1\}. \quad (4)\end{aligned}$$

TABLE V. Values for the dependence of the diamagnetic susceptibility on the molecules' orientation and of Q_e at various magnetic fields

EF−*DE* (cps)	*H* (gauss)	C_3 (cps)	$2f$ (cps)	$(\xi_{\pm1}-\xi_0)\times10^{31}$ (erg gauss^{-2})	$Q_e\times10^{16}$ (cm^2)
345,125±40	3471.37±0.06	−38	− 749±150	−4.12±0.80	0.362±0.054
345,138±30	3468.84±0.06	−38	− 736±150	−4.05±0.80	0.358±0.054
344,498±30	5029.37±0.10	−17	−1456±150	−3.81±0.39	0.340±0.026
344,500±50	5029.34±0.10	−17	−1458±150	−3.82±0.39	0.341±0.026
345,045±50	3720.49±0.06	−33	− 849±150	−4.06±0.80	0.358±0.054
345,048±50	3703.71±0.06	−33	− 846±150	−4.08±0.80	0.360±0.054
344,594±40	4872.50±0.10	−19	−1356±150	−3.78±0.39	0.348±0.026
344,593±40	4872.50±0.10	−19	−1357±150	−3.78±0.39	0.348±0.026
343,478±80	6737.40±0.10	−10	−2508±150	−3.66±0.21	0.330±0.014
343,559±100	6607.75±0.10	−10	−2427±160	−3.68±0.21	0.327±0.014
KL−*AB* (cps)					
54,107±60	3684.01±0.06		− 693±100	−3.38±0.49	0.310±0.033
53,630±120	4872.50±0.10		−1170±150	−3.28±0.40	0.303±0.027
52,391±260	6691.47±0.10		−2409±260	−3.56±0.30	0.323±0.020
GK−*BC* (cps)					
403,301±90	6607.83±0.10	−10	−2452±130	−3.72±0.20	0.339±0.014

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[10] J. W. M. DuMond and E. R. Cohen, Phys. Rev. **82**, 556 (1951).

[11] J. H. Van Vleck, *The Theory of Electric and Magnetic Susceptibilities* (Clarendon Press, Oxford, 1932).

[12] G. C. Wick, Z. Physik **85**, 25 (1933).

[13] G. C. Wick, Nuovo cimento **10**, 118 (1933).

[14] N. F. Ramsey, Phys. Rev. **87**, 1077 (1952).

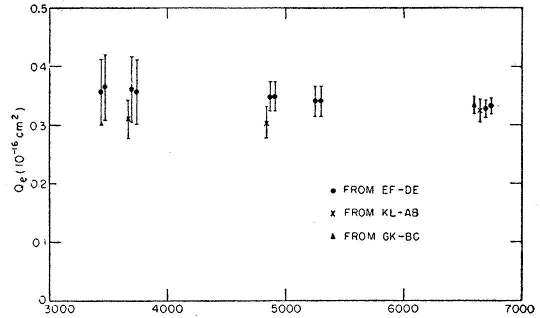

FIG. 1. The quadrupole moment of the electron distribution measured for various transitions and magnetic fields.

The value to be used for l is not determined in the present experiment. However, if one uses a dimensional argument analogous to that used by Newell[15] in a related problem, the value for l in the present case would be 2±2. With this value,[10]

$$ {}_0^H\chi_1^{\mathrm{HF}} = (0.0990\pm0.0010) \times10^{-6} \text{ erg gauss}^{-2}\text{ mole}^{-1}. \tag{5}$$

Per molecule, the high frequency terms of the susceptibility equal

$$ {}_0^H\xi_1^{\mathrm{HF}} = (1.64\pm0.16) \times10^{-31} \text{ erg gauss}^{-2}\text{ molecule}^{-1}. \tag{6}$$

The error in the last two quantities is due almost exclusively to the uncertainty in the value of l to be used for the zero-point vibration correction.

C. Dependence of Magnetic Susceptibility upon Molecular Orientation

The last two terms in the Hamiltonian for H_2 and D_2 [4] are included to account for the diamagnetic interaction of the molecule with the external magnetic field. Because these quantities depend only on the rotational quantum number, they can be measured only by observing the rotational transitions and not the nuclear transitions. A value of f may be obtained by taking differences among any pair of rotational transitions in Table I. Consider, for example, the pair of transitions DE and EF:

$$EF-DE=6d+2f+4c_3. \tag{7}$$

Now, the difference of diamagnetic susceptibility for a molecule with $m_J=\pm1$ and $m_J=0$ is given[4] by

$$\xi_{\pm1}-\xi_0=2hf/H^2. \tag{8}$$

The quantity f is just observable at about 3000 gauss and increases as the square of the magnetic field. For this reason, the dependence of the susceptibility upon orientation is best determined by experiments in strong magnetic fields. The results of different measurements of the dependence of the diamagnetic susceptibility upon the molecular orientation are given in Table V. The best average value of the 6600-gauss measurements is

$$\xi_{\pm1}-\xi_0=(-3.66\pm0.20) \times10^{-31} \text{ erg gauss}^{-2}\text{ molecule}^{-1}. \tag{9}$$

D. Quadrupole Moment of Electron Distribution

Ramsey[5] has shown that $\xi_{\pm1}-\xi_0$ and ξ^{HF} can be combined to give a measure of the quadrupole moment of the electron distribution of a molecule. Relative to the internuclear axis, the quadrupole moment is given by the following expression:[5]

$$\begin{aligned} Q_e&=-\textstyle\sum_j\langle 3\gamma_j^2-r_j^2\rangle \\ &=-(20mc^2/e^2)[(\xi_{\pm1}-\xi_0)-\tfrac{3}{5}\xi^{\mathrm{HF}}]. \end{aligned} \tag{10}$$

The results of the different measurements of Q_e are given in Table V. Figure 1 is a plot of Q_e for various magnetic fields as obtained from these measurements. From the resonances DE and EF alone, there appears to be a slight trend for Q_e to decrease with increasing magnetic field, but little significance should probably be attached to this apparent trend because it is less than the estimated experimental error. Since all pairs of transitions give the best agreement in high fields and since the experimental error is least there, the best value of Q_e is the average obtained from high field measurements and is

$$Q_e=(0.330\pm0.013)\times10^{-16}\text{ cm}^2. \tag{11}$$

This result agrees, within the experimental error, with the value quoted previously.[16] The previous value was the highest field result available at that time and was taken entirely from measurements on DE and EF. The accuracy of the present value is improved over the previously quoted value.

V. DISCUSSION

This calculated value of Q_e may be compared to a calculated value[8] from the theoretical expression of James and Coolidge.[17] Q_e is related to the quantity $N(R)$ calculated by James and Coolidge by

$$Q_e=R^2-[2N(R)/e]a_0^2, \tag{12}$$

where a_0 is the Bohr radius. If James and Coolidge's[17]

TABLE VI. Interaction constants of H_2 obtained from the rotational spectrum of H_2 at strong magnetic fields.

Quantity	Present value
$(1-\sigma_{J1})b/H$	0.673043±0.000036 kcps gauss^{-1}
μ_{RH}/J	0.88291±0.00007 nuclear magnetons
μ_{Re}/J	−0.11709±0.00007 nuclear magnetons
${}_0^H\chi_1^{HF}$	$(0.0990\pm0.0010)\times10^{-6}$ erg gauss^{-2} mole^{-1}
${}_0^H\xi_1^{HF}$	$(1.64\pm0.16)\times10^{-31}$ erg gauss^{-2} molecule^{-1}
$\xi_{\pm1}-\xi_0$	$-(3.66\pm0.20)\times10^{-31}$ erg gauss^{-2} molecule^{-1}
${}_0^H\langle Q_e\rangle_1^{\mathrm{expt}}$	$(0.330\pm0.013)\times10^{-16}$ cm^2
${}_0^H\langle Q_e\rangle_1^{\mathrm{theor}}$	$(0.344\pm0.010)\times10^{-16}$ cm^2

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[15] G. F. Newell, Phys. Rev. **80**, 476 (1950).

[16] Harrick, Barnes, and Bray, Phys. Rev. **85**, 716 (1952).

[17] H. M. James and A. S. Coolidge, Astrophys. J. **87**, 447 (1938).

expression for $N(R)$ is used in this equation, one finds

$$\begin{aligned}{}_0^{\mathrm{H}}\langle Q_e\rangle_1^{\mathrm{theor}} = a_0^2[{}_0^{\mathrm{H}}\langle (R/R_e)^2\rangle_1 (R_e/a_0)^2 - 0.884 \\ -0.614\ {}_0^{\mathrm{H}}\langle (R-R_e)/R_e\rangle_1 (R_e/a_0) \\ +0.410\ {}_0^{\mathrm{H}}\langle (R-R_e)^2/R_e^2\rangle_1 (R_e/a_0)^2]. \quad (13)\end{aligned}$$

The various averages can be obtained from Ramsey's[14] paper on the effects of zero-point vibration and when these are used, the above equation gives

$${}_0^{\mathrm{H}}\langle Q_e\rangle_1^{\mathrm{theor}} = (0.344 \pm 0.010) \times 10^{-16}\ \mathrm{cm}^2. \quad (14)$$

The agreement between the experimental value of Eq. (11) and the theoretical value of Eq. (14) is within the overlap of the errors.

The results of the present measurements are all summarized in Table VI. These are in good agreement with values previously quoted.[4] There are large improvements in accuracy with the present results. $(1-\sigma_{J1})b/H$ is improved by a factor of 100 over previous measurements.[7] The best existing values for γ_p limits the accuracy of the rotational magnetic moment as calculated from the present results, but, there still is an improvement of about 100 over previous measurements. The dependence of the diamagnetic susceptibility on the orientation of the molecules' rotational angular momentum, $\xi_{\pm 1} - \xi_0$, is improved from a 65 percent uncertainty to only 5 percent; and the quadrupole moment of the electron distribution, Q_e, from 40 percent to 4 percent uncertainty.

The experimental value of Q_e compares favorably to the theoretically calculated value. The usefulness of the experimental value of Q_e lies in a check of the wave functions used in the theoretical calculation of Q_e and the wave function used to calculate the gradient of the electric field at the nucleus resulting from the electrons.

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Multiple Scattering of Neutrons. III. Scattering by Spin-Dependent Forces and Polarization Phenomena

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The treatment presented in two preceding papers is extended to cover the case of spin-dependent forces. A general theorem is derived which reduces the problem of scattering by spin-dependent forces to the problem of scattering by spin-independent forces, which has previously been solved analytically. It is shown that observations on the neutrons returning from the scatterer can be used in many practical cases to determine the ratio of the two scattering amplitudes. Similarly, for very small probability of depolarization per single collision, the depolarization of the neutrons transmitted through the scatterer offers information about the spin dependence of the scattering amplitudes. The necessary material for the evaluation of the observation is presented in table form. A comparison is made with a paper by Borowitz and Hamermesh in which the same subject has been treated by a different method.

I. INTRODUCTION

IN two preceding papers[1] the multiple scattering of slow neutrons in a plane parallel sheet of infinite extension has been discussed. The physical assumptions underlying the treatment were the following: The scattering probability is spherically symmetrical; the scattering is purely elastic; capture processes are admitted. It was also assumed that no correlation exists for the scattering from the various centers. The present paper, as announced at the end of II, extends the treatment to the case of spin-dependent nuclear forces.[2]

The physical interest of the problem here discussed can perhaps best be explained as follows. It has been shown in an earlier paper[3] that the probability Q for the inversion of the spin of the incident neutron during a scattering process is given by

$$Q = \frac{2i(i+1)}{3(2i+1)} \frac{(a_1-a_0)^2}{ia_0^2+(i+1)a_1^2}. \quad (1)$$

Here i denotes the spin of the nucleus in units of $h/2\pi$; a_0 and a_1 stand for the scattering amplitude of the system nucleus plus neutron if its total angular momentum is $i-\frac{1}{2}$ and $i+\frac{1}{2}$, respectively.

One can see by a simple analysis of (1) that Q has a maximum value of $\frac{2}{3}$; this maximum value is taken on for the case of

$$a_0 + 3a_1 = 0. \quad (2)$$

The relation (2) is approximately satisfied, for example, in the case of H. One can see in general that sizeable values of Q will only occur if the two amplitudes have opposite sign.

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[1] Halpern, Luneburg, and Clark, Phys. Rev. **53**, 173 (1938) (I); O. Halpern and R. K. Luneburg, Phys. Rev. **76**, 1811 (1949) (II).

[2] O. Halpern, Phys. Rev. **75**, 1633 (1949).

[3] O. Halpern and M. H. Johnson, Phys. Rev. **55**, 898 (1939).