3.7×10^{-6} than it is in H_2. The net diamagnetic correction is then 30.5×10^{-6}. Accordingly

$$g_s/g_{p(\text{free})}=658.2087\pm0.0006,$$

where the uncertainties in the diamagnetic corrections have not been included in estimating the precision of the result.

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A Determination of $g_J(^2S_{\frac{1}{2}})$ of Potassium in Terms of the Proton Gyromagnetic Ratio*

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The ratio of the g value of potassium K^{39} in the ground state to the gyromagnetic ratio of the proton in mineral oil has been measured in an experiment combining atomic beam and nuclear resonance techniques. It is found to be $g_J(\text{K})/g_p=658.2274\pm0.0023$. This result, when combined with the measurement of $g_J(\text{H})/g_p$ by Koenig, Prodell, and Kusch, yields

$$g_J(\text{K})/g_J(\text{H})=1.000016\pm0.0004 \text{ percent.}$$

This is to be compared with the recent direct measurement by Pohlman, Bederson, and Eisinger: $g_J(\text{K})/g_J(\text{H})=1.00016\pm0.006$ percent. The present experiment is of superior precision and is in agreement with independent experimental evidence. The present result is found to agree with the theoretically predicted value.

INTRODUCTION

A RECENT direct measurement by Pohlman, Bederson, and Eisinger[1] (PBE) of the ratio of the g_J values of hydrogen and potassium in the ground, $^2S_{\frac{1}{2}}$, states has yielded

$$g_J(\text{K}^{39})/g_J(\text{H}^1)=1.00016\pm0.006 \text{ percent.}$$

In view of the fact that Kusch and Taub[2] found the g_J values of Li, Na, and K to be identical to within one part in 40,000, this large reported difference between the g_J values of K and H is rather surprising. In addition, it is possible to deduce values of $g_J(\text{K})/g_J(\text{H})$ which are in disagreement with the PBE result both from independent experimental evidence and from theoretical considerations. The experimentally determined quantities to be considered are

$$g_J(\text{H})/g_p=658.2171\pm0.0001 \text{ percent,}^3$$
$$g_p/g_J(\text{Na})=15.1927\times10^{-4}\pm0.005 \text{ percent,}^4$$
$$g_J(\text{Na})/g_J(\text{K})=1.00000\pm0.002 \text{ percent.}^2$$

Within the stated precision of the results, the different internal diamagnetic corrections to be applied to the g value of the proton as measured in mineral oil and NaOH can be ignored. From these results it is found that

$$g_J(\text{K})/g_J(\text{H})=0.99999\pm0.007 \text{ percent.}$$

While the disagreement between this result and that of PBE cannot be considered as conclusive evidence of the invalidity of either result, the discrepancy is notable.

Calculations by Phillips[5] on the perturbation of the $^2S_{\frac{1}{2}}$ ground state of K by excited states of the inner electronic core yield the estimate $1\leqslant g_J(\text{K})/g_J(\text{H})\leqslant1.00001$. Relativistic and diamagnetic corrections, to be discussed in Sec. C, increase both limits by 6×10^{-6}. While there is some uncertainty in the perturbation calculations, the disagreement of this limit with the result of PBE is marked.

In view of the discrepancy of the PBE result and that obtained from independent experimental evidence and in view of the fact that no effect as large as that found by PBE can be accounted for on the basis of existing theory, it was decided to redetermine the ratio $g_J(\text{K})/g_J(\text{H})$. In the previous paper Koenig, Prodell, and Kusch[3] (KPK) have described a precision measurement of $g_J(\text{H})/g_p$. The present paper discusses a determination of $g_J(\text{K})/g_p$. The combination of the two results yields a value of $g_J(\text{K})/g_J(\text{H})=1.000016\pm0.0004$ percent, which is of a higher precision than the result of PBE and in marked disgreement with it.

A. PROCEDURE

In the experiment of KPK the ratio $g_J(\text{H})/g_p$ was obtained by measuring a field dependent hyperfine

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1 Pohlman, Bederson, and Eisinger, Phys. Rev. 83, 475 (1951).
2 P. Kusch and H. Taub, Phys. Rev. 75, 1477 (1949).
3 Koenig, Prodell, and Kusch, Phys. Rev. 88, 191 (1952).
4 H. Taub and P. Kusch, Phys. Rev. 75, 1481 (1949).
5 M. Phillips, to be published. We are grateful to Professor Phillips for several discussions relevant to this research.

TABLE I. Tabulation of data. The average of all runs is 658.2264 ±0.0023 average dev. The maximum spread in the data is 0.0098.

Date	Average $g_J(\mathrm{K})/g_p$	Average deviation
Sept. 25, 1951	658.2222	0.0007
Oct. 5	658.2239	0.0009
Oct. 23	658.2282	0.0009
Oct. 24	658.2238	0.0008
Oct. 25	658.2274	0.0006
Oct. 26	658.2223	0.0005
Oct. 30	658.2253	0.0006
Nov. 21	658.2212	0.0006
Nov. 26	658.2251	0.0007
Dec. 11	658.2248	0.0005
Dec. 12	658.2260	0.0010
Dec. 13	658.2294	0.0011
Dec. 17	658.2297	0.0005
Dec. 20	658.2305	0.0003
Jan. 15, 1952	658.2266	0.0009
Jan. 16	658.2268	0.0006
Jan. 19	658.2280	0.0003
Jan. 21	658.2284	0.0009
Jan. 24	658.2249	0.0012
Jan. 28	658.2310	0.0003
Feb. 19	658.2286	0.0013

structure transition in hydrogen in the same magnetic field as that in which the resonance frequency of protons in mineral oil was measured. In the present determination of $g_J(\mathrm{K})/g_p$ a transition was measured in potassium, rather than in hydrogen, with essentially the same apparatus and technique employed in the previous experiment. A correction for the bulk diamagnetism of the cylindrical sample of oil which was used is not critical in the present determination since it is only necessary to compare the ratio of $g_J(\mathrm{K})/g_p$ and $g_J(\mathrm{H})/g_p$ for the same sample of oil to obtain $g_J(\mathrm{K})/g_J(\mathrm{H})$.

The apparatus as described in KPK was modified for this experiment by replacing the hydrogen discharge source with a nickel oven source for potassium and substituting a tungsten hot wire detector for the Pirani gauge. Two stops were attached to the cage surrounding the rf resonant hairpin to provide the same beam height in the transition region as that provided by the aperture of the Pirani gauge in the hydrogen experiment.

The atomic transition

$$(F=2,\ m_F=-1)\leftrightarrow(F=2,\ m_F=-2)$$

was observed. The magnetic field was about 10 percent less than in the previous experiment in order that the potassium line would have the same frequency as that observed in the hydrogen experiment. This made it possible to utilize without change the Klystron oscillator, frequency measuring equipment, and resonant hairpin employed in the hydrogen experiment. The data were taken as described by KPK for the later runs on hydrogen.

To reduce the data of the experiment certain auxiliary information is required:

$$\Delta\nu(\mathrm{K}^{39})=461.723\pm0.010\ \mathrm{Mc}^2,$$

and $B=g_I(\mathrm{K})/g_p=0.0466\pm0.0002$.[6] The spin of K^{39} is $\frac{3}{2}$. The uncertainty in the value of $\Delta\nu(\mathrm{K}^{39})$ introduces an uncertainty in the determination of $g_J(\mathrm{K})/g_p$ of one part in 650,000, while that in B introduces an uncertainty of less than one part in 10^7 in the result. It can readily be shown that the ratio $g_J(\mathrm{K})/g_p$ can be calculated in closed form from the frequency f of the atomic line and the resonance frequency P of the proton:

$$-g_J(\mathrm{K})/g_p=[4f(f+BP+\Delta\nu)+3BP\Delta\nu]/[P(4f+4BP+\Delta\nu)],$$

where g_p is to be interpreted as the nuclear g value of hydrogen measured in a cylindrical sample of mineral oil.

B. ANALYSIS OF DATA

The reduced data of the experiment are listed in Table I. Generally, seven sets of observations of the proton resonance frequency were made, alternating with six sets of observations of the resonance frequency of the atomic transition. The thirteen sets of observations gave twelve values of the ratio $g_J(\mathrm{K})/g_p$, where the magnetic field drift was assumed linear over the time required to make three sets of observations. The mean of each day's determinations together with the average deviation is given in Table I.

An inspection of Table I shows that the spread in the values of $g_J(\mathrm{K})/g_p$ obtained from day to day greatly exceeds the uncertainty in the result for a particular day. A comparison of Table I with the data in Table II of KPK also indicates a greater maximum spread in the results for potassium than for hydrogen. Throughout the present investigation, a continuous search was made for a systematic malfunctioning of the apparatus of the sort which might lead to approximately fixed errors on a particular day but which might vary from day to day. No evidence of such malfunctioning was found, and the apparatus did not appear to be altered in its performance from the hydrogen experiment. The proton resonance and uhf assembly was removed several times during the course of the runs and checked for possible misalignment of the interchange mechanism. Great care was exercised throughout the entire experiment to keep the assembly free of magnetic contaminants. In order to preclude the possibility that the attachment of the beam height stops to the shielding cage gave rise to an asymmetry in the uhf current distribution in the hairpin, a new hairpin was constructed of the same design but with shielding adequate to prevent the stops from perturbing the uhf distribution. The 1952 runs were taken with this new device and indicate no significant change in the dispersion of data. Finally, it is to be noted that on several occasions, runs were taken sequentially on the same day with widely differing amplitudes of the uhf field. The results of the separate runs agreed to within the

[6] H. L. Poss, unpublished Brookhaven National Laboratory report No. 26 (T-10) (1949).

uncertainty of each individual run. The wide range in the results from day to day, therefore, appears to be characteristic of some feature of the apparatus which varies from day to day and was not dependent on easily controlled experimental parameters.

It seems reasonable to attribute the large day to day variation in data to a varying homogeneity of the magnetic field. In view of the fact that detailed information on the actual field distributions could not be obtained, it is not possible to make any exact statements as to the effect of field inhomogeneities on the line shape. Resonance curves were obtained on several occasions and found to exhibit considerable assymmetry, although the field did not appear to be excessively inhomogeneous by the standards of the hydrogen experiment. It is to be noted, however, that the natural width of the potassium line (12 kc) is considerably less than the natural width of the hydrogen line (60 kc), owing to the smaller average velocity of the potassium atoms. The role of the field inhomogeneity in determining the position of the maximum of the line is evidently related to the line width, but no further conclusions can be made at this time.

The average of the seven 1952 runs is 658.2277 and that of the last six runs in 1951 is 658.2276. However, the average of the first eight runs in 1951 is 658.2243. In view of this grouping of individual runs, it is possible that the field distribution may not have varied from day to day in a random manner. In the absence of evidence concerning the actual field distributions leading to an assignment of statistical weights to the individual runs, the best value of $g_J(\mathrm{K})/g_p$ is the mean value of all the runs. In view of the uncertainty in the randomness of the field distributions, it seems doubtful that the statistical probable error (± 0.0005) is a realistic measure of the precision of the result. We have arbitrarily chosen to use as a precision measure the average deviation (± 0.0023). Then,

$$g_J(\mathrm{K})/g_p = 658.2264 \pm 0.0023,$$

where g_p is the gyromagnetic ratio of the proton as measured in a cylindrical sample of mineral oil. After correction to a measurement of the proton resonance frequency in a spherical sample of mineral oil,

$$g_J(\mathrm{K})/g_p = 658.2274 \pm 0.0023.$$

C. INTERPRETATION OF RESULT

The result of this experiment when combined with that of KPK yields

$$g_J(\mathrm{K})/g_J(\mathrm{H}) = 1.000016 \pm 0.0004 \text{ percent.}$$

This is to be compared with the recent result of PBE:

$$g_J(\mathrm{K})/g_J(\mathrm{H}) = 1.00016 \pm 0.006 \text{ percent,}$$

which indicates a discrepancy in the g values ten times as great as that found in the present experiment. Two considerations indicate that the present result is more reliable:

(1) The width of the potassium line observed by PBE was one part in 4000 of their resonance frequency; in the present case a typical width was one part in 65,000 of the resonance frequency.

(2) The maximum spread of data in the PBE experiment was 260 parts in a million compared with 15 parts in a million in the present determination.

The present result can be interpreted in terms of three effects: (*a*) the relativistic behavior of the electron, (*b*) the diamagnetism of the electronic core, and (*c*) the perturbation of the potassium ground state arising from exchange interactions with excited states of the electronic core.

The effect of the relativistic behavior of the electron on its observed spin moment has been discussed by Margenau.[7] If g_s' is the spin g value of the bound valence electron and g_s that of a free electron, then $g_s' = 1 + \Delta$, where $\Delta \equiv (-2T/3mc^2)$ and T is the average kinetic energy of the valence electron. Δ can be calculated exactly for hydrogen and, as shown by KPK, is equal to -18×10^{-6}. The calculation of the average kinetic energy in many electron atoms requires an approximate treatment. Lamb[8] has derived the approximation $T = 2W - W_0$, where W is the ionization potential of the atom considered and W_0 is the ionization potential of a hydrogen atom in the same electronic state. Foley[9] has compared the Lamb approximation with a numerical calculation utilizing Hartree-Foch functions in the case of sodium and has found an agreement to within four percent. The Lamb approximation yields a Δ for potassium of -10×10^{-6}. The authors place an uncertainty in this calculation of $\pm 2 \times 10^{-6}$.

The correction to the observed g value in many electron atoms due to the diamagnetic polarization of the electronic core has been shown by Lamb[8] to be $g_s'/g_s = 1 + \delta$, where $\delta \equiv -(T-W)/3mc^2$. The Lamb approximation for $T-W$ yields, for potassium $\delta = -2 \times 10^{-6}$.

The difference in the relativistic correction for hydrogen and the relativistic and diamagnetic corrections for potassium produces, then, a shift of $(+6 \pm 2) \times 10^{-6}$ in the g value of potassium relative to that of hydrogen. Since the observed shift is $(+16 \pm 4) \times 10^{-6}$, there is a discrepancy of $(+10 \pm 5) \times 10^{-6}$. Calculations by Phillips[5] on the perturbation of the potassium ground state by excited states of the electronic core indicate a discrepancy equal to or less than $+10^{-5}$. The result obtained in this experiment is in agreement, therefore, with the theoretical estimate.

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[7] H. Margenau, Phys. Rev. **57**, 383 (1940).
[8] W. E. Lamb, Jr., Phys. Rev. **60**, 817 (1941).
[9] H. M. Foley (private communication).