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The Anomalous Magnetic Moment of the Electron*

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The ratio of the electron spin g value and the proton g value has been measured with high precision. It is found that $g_s/g_p=658.2288\pm0.0006$, where g_p is the g value of the proton measured in a spherical sample of mineral oil. This result, when combined with the measurement by Gardner and Purcell of the ratio of the electron orbital g value and the proton g value, yields for the experimental value of the magnetic moment of the electron

$$\mu_s=(1.001146\pm0.000012)\mu_0.$$

The result is in excellent agreement with the theoretical value recently calculated by Karplus and Kroll:

$$\mu_s=(1.0011454)\mu_0.$$

INTRODUCTION

SEVERAL experiments[1-3] have given direct experimental evidence of the existence of an anomalous electron spin magnetic moment, and have permitted the determination of the magnitude of the anomaly to a moderate precision. New methods developed by Schwinger[4] in quantum electrodynamics have made possible the calculation, to second order, of a radiative correction to the spin magnetic moment of the electron which increased its value from μ_0, given by the Dirac theory, to $\mu_0(1+\alpha/2\pi)=\mu_0(1.00116)$. The experimental results, within the accuracy of measurement and within the validity of the various theoretical approximations which enter into the interpretation of the data, are in excellent agreement with the theoretical prediction.

More recently, Karplus and Kroll[5] have calculated the radiative correction to the electron moment to fourth order and have found for the spin moment μ_s,

$$\mu_s=\mu_0(1+\alpha/2\pi-2.973\alpha^2/\pi^2)=\mu_0(1.0011454).$$

A critical experimental test of the validity of the result subjects the procedures of quantum electrodynamics to test through one of the two directly measurable experimental quantities discussed by that theory. It is the purpose of the present paper to describe an experiment through which μ_s is measured to high precision.

The ratio $g_J(^2S_{\frac{1}{2}}\mathrm{H})/g_p$ is measured in the present experiment, where g_p is the proton g value. The electron spin g value may readily be obtained from g_J. The present result, combined with the recent measurement of $2g_l/g_p$ by Gardner and Purcell,[6] then determines the quantity $g_s/2g_l=\mu_s/\mu_0$. The present experiment avoids the difficulties in interpretation of data which have occurred in previous experiments and is, in addition, designed to obtain a result of very high accuracy. It is instructive to examine the previous data which have led to a determination of μ_s/μ_0.

PREVIOUS RESULTS

The various experimental results which are used in the calculation of g_s/g_l are given in Table I. A symbol to represent each result in subsequent discussion is also given. The results B, C, D, and H have previously been used[3] to calculate g_s/g_l on the basis of the assumption that the atomic states, $^2S_{\frac{1}{2}}\mathrm{Na}$, $^2P_{\frac{1}{2}}\mathrm{Ga}$, $^2P_{\frac{1}{2}}\mathrm{In}$, $^2P_{\frac{3}{2}}\mathrm{Ga}$, $^2P_{\frac{3}{2}}\mathrm{In}$, are pure states, and that Russell Saunders coupling adequately describes the states. The result A, however, permits a calculation of g_s/g_l in which it is necessary to assume the purity of the $^2S_{\frac{1}{2}}$ state only.

It is possible from the data of Table I to obtain three values for $g_J(^2S_{\frac{1}{2}}\mathrm{Na})/g_l$.

$$2D/FA=2(1.00114\pm0.00010), \quad \text{(a)}$$

$$2/GEA=2(1.00109\pm0.00003), \quad \text{(b)}$$

$$2IK/A=2(1.00114\pm0.00006), \quad \text{(c)}$$

$$\text{Average: } 2(1.00112\pm0.00003). \quad \text{(d)}$$

TABLE I. Data used in the calculation of the spin magnetic moment of the electron.

Symbol	Quantity	Result
A	$2g_l/g_p$	-657.475 ± 0.008[a]
B	$g_J(^2P_{\frac{3}{2}}\mathrm{Ga})/g_J(^2P_{\frac{1}{2}}\mathrm{Ga})$	$2(1.00172\pm0.00006)$[b]
C	$g_J(^2S_{\frac{1}{2}}\mathrm{Na})/g_J(^2P_{\frac{1}{2}}\mathrm{Ga})$	$3(1.00242\pm0.00006)$[b]
D	$g_J(^2S_{\frac{1}{2}}\mathrm{Na})/g_J(^2P_{\frac{1}{2}}\mathrm{In})$	$3(1.00243\pm0.00010)$[b]
E	$g_p/g_J(^2S_{\frac{1}{2}}\mathrm{Cs})$	$-15.1911\times10^{-4}\pm0.0003\times10^{-4}$[c]
F	$g_p/g_J(^2P_{\frac{1}{2}}\mathrm{In})$	$-45.6877\times10^{-4}\pm0.0007\times10^{-4}$[c]
G	$g_J(^2S_{\frac{1}{2}}\mathrm{Cs})/g_J(^2S_{\frac{1}{2}}\mathrm{Na})$	1.000134 ± 0.000007[d]
H	$g_J(^2P_{\frac{3}{2}}\mathrm{In})/g_J(^2P_{\frac{1}{2}}\mathrm{In})$	$2(1.00200\pm0.00006)$[e]
I	$g_J(^2S_{\frac{1}{2}}\mathrm{Na})/g_I(\mathrm{Na})$	-24.8839 ± 0.15[f]
J	$g_J(^2S_{\frac{1}{2}}\mathrm{Ca^{II}})/g_J(^2P_{\frac{3}{2}}\mathrm{Ca^{II}})$	1.5013 ± 0.0003[g]
K	$g_I(\mathrm{Na})/g_p$	0.2645182 ± 0.0000007[h]

[a] See reference 6.
[b] See reference 3.
[c] H. Taub and P. Kusch, Phys. Rev. **75**, 1481 (1949).
[d] See reference 7.
[e] A. K. Mann and P. Kusch, Phys. Rev. **77**, 435 (1950).
[f] R. Logan and P. Kusch, Phys. Rev. **81**, 280 (1951).
[g] Rand McNally, Jr., Phys. Rev. **73**, 1130 (1948).
[h] G. Lindstrom, Arkiv Fysik **4**, No. 1 (1951).

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[1] J. E. Nafe and E. B. Nelson, Phys. Rev. **73**, 718 (1948).

[2] G. Breit, Phys. Rev. **72**, 984 (1947).

[3] P. Kusch and H. M. Foley, Phys. Rev. **74**, 250 (1948).

[4] J. Schwinger, Phys. Rev. **73**, 416 (1947).

[5] R. Karplus and N. Kroll, Phys. Rev. **77**, 536 (1950).

[6] J. H. Gardner and E. M. Purcell, Phys. Rev. **76**, 1262 (1949); J. H. Gardner, Phys. Rev. **83**, 996 (1951).

Inasmuch as an atom in a pure S state has no orbital electronic momentum, $g_J(^2S_{\frac{1}{2}})$ must be associated entirely with the electron spin. It has been shown[7] that the g_J values of the alkalis K, Na, Li^7, and Li^6 are equal within one part in 4×10^4, and it seems reasonable to extrapolate to sequence to include hydrogen. (The following paper by Franken and Koenig justifies the extrapolation.) The ground state of hydrogen can be shown to be a pure S state to about 1 part in 10^6. The g_J value of Na in the $^2S_{\frac{1}{2}}$ state is then, within the present approximation, equal to g_s', the spin g value of the electron bound in the ground state of hydrogen. A relativistic correction of two parts in 10^5, essentially due to the increase of mass with the velocity of the bound electron, gives $g_s=2(1.00114\pm0.00003)g_l$. In view of the limited precision of the experimental data, the excellent agreement with the theoretical prediction must be considered as fortuitous.

The result is limited in precision both by the accuracy of the experimental data and by the limited validity of the assumptions which enter into the calculations. Because of the differential diamagnetic effects of the electrons in various molecules, the magnetic moment of either the proton or the Na nucleus measured in two different molecules will presumably differ by some small, unknown amount. In the results (a) and (b), this effect is neglected. The validity of the result is, of course, also limited by the assumption that $g_J(^2S_{\frac{1}{2}}\mathrm{Na}) = g_J(^2S_{\frac{1}{2}}\mathrm{H})$. Finally, it should be noted that in the result (a), use is made of a $^2P_{\frac{1}{2}}$ state as an intermediary in the calculations. The data for D were taken at low magnetic field and those for F at high field. Because of a partial decoupling of the L and S vectors, it is not certain that $g_J(^2P_{\frac{1}{2}})$ is independent of field, to the precision attached to the result (a). The experiment to be discussed in this paper avoids all of these difficulties.

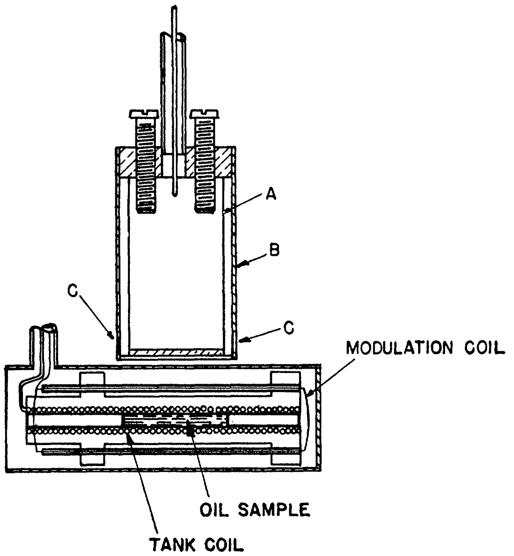

FIG. 1. Cross section of uhf unit and proton resonance probe.

From the data in Table I, it is possible to obtain information on the g values of $^2P_{\frac{1}{2}}$ states. Let

$$N=g_J(^2S_{\frac{1}{2}}\mathrm{Na})/g_l$$

as calculated above. Then

$$\begin{aligned}N/D&=g_J(^2P_{\frac{1}{2}}\mathrm{In})/g_l=\tfrac{2}{3}(1-0.00131\pm0.00010),\\ 2/AF&=g_J(^2P_{\frac{1}{2}}\mathrm{In})/g_l=\tfrac{2}{3}(1-0.00128\pm0.00003),\\ N/C&=g_J(^2P_{\frac{1}{2}}\mathrm{Ga})/g_l=\tfrac{2}{3}(1-0.00130\pm0.00008).\end{aligned}$$

Assuming pure Russell Saunders coupling, $g_J(^2P_{\frac{1}{2}}) = 4g_l/3-g_s'/3$ and $g_s'/g_l=4-3g_J/g_l$. Then

$$\begin{aligned}g_s'(^2P_{\frac{1}{2}}\mathrm{In})/g_l&=2(1.00129\pm0.00003),\\ g_s'(^2P_{\frac{1}{2}}\mathrm{Ga})/g_l&=2(1.00130\pm0.00008).\end{aligned}$$

These results are significantly higher than for an electron in an S state. The difference may be due in part to a lack of purity of these states at zero field as well as the possibility of a mixing of near lying levels by the magnetic field.

From the remaining data on $^2P_{\frac{3}{2}}$ states, under the assumption of Russell-Saunders coupling where $g_J(^2P_{\frac{3}{2}}) = 2g_l/3+g_s'/3$, it is found that

$$\begin{aligned}g_s'(^2P_{\frac{3}{2}}\mathrm{In})/g_l&=2(1.00141\pm0.00013),\\ g_s'(^2P_{\frac{3}{2}}\mathrm{Ga})/g_l&=2(1.00084\pm0.00010),\\ g_s'(^2P_{\frac{3}{2}}\mathrm{Ca^{II}})/g_l&=2(1.0005\pm0.0002).\end{aligned}$$

The significance of these results is not entirely clear, though the values do increase with atomic weight. It is evident that there is an experimental difference in the value of g_s' as measured in S and P states. In view of the greater theoretical simplicity of S states as compared to P states, the result obtained from the S states is probably better.

THEORY OF THE EXPERIMENT

The atomic beams method in combination with nuclear resonance techniques was used to measure the ratio of the electronic g value of hydrogen in the $^2S_{\frac{1}{2}}$ state and the proton g value.

The relevant portion of the term values of the hfs levels of hydrogen in the $^2S_{\frac{1}{2}}$ state is given by

$$W_{F,\,m_F}=m_Fg_p\mu_0H\pm\tfrac{1}{2}h\nu_{\mathrm{H}}(1+2m_Fx+x^2)^{\frac{1}{2}},\qquad(1)$$

where $x=g_J(1-g_p/g_J)\mu_0H/h\nu_{\mathrm{H}}$, and the $\pm$ sign is to be associated with $F=1$, 0, respectively. The frequency of the line $(1, 0\leftrightarrow1, -1)$ was measured rather than that of the other observable line $(1, 1\leftrightarrow0, 0)$, because its frequency is lower and easier to produce and measure, and because the value of g_J/g_p calculated from the data is less dependent on the frequency of transition for the chosen line than for the other observable line.

From Eq. (1) the transition frequency f of the line $(1, 0\leftrightarrow1, -1)$ is

$$f=\tfrac{1}{2}[(1+x^2)^{\frac{1}{2}}-(1-x)]\nu_{\mathrm{H}}+g_p\mu_0H/h.\qquad(2)$$

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[7] P. Kusch and H. Taub, Phys. Rev. 75, 1477 (1949).

The frequency of the proton resonance is

$$P = -g_p \mu_0 H/h. \tag{2a}$$

If f and P are measured in the same magnetic field, Eqs. (2) and (3) give g_J/g_p in closed form

$$g_J/g_p = -[2f(f+P+\nu_{\mathrm{H}})+P\nu_{\mathrm{H}}]/P[\nu_{\mathrm{H}}+2(f+P)]. \tag{3}$$

g_p in (2) is, of course, the g value of the proton measured in atomic hydrogen. g_p in (2a) is the g value of the proton measured in the environment of the particular sample of proton containing material used in a nuclear resonance experiment. The substitution of (2a) in (2) may, however, be made without any meaningful loss of accuracy since the relative diamagnetic correction does not affect P within the limit to which the frequency f is measured in the present experiment. g_J/g_p is then expressed in terms of three measurable frequencies. P is the Larmor precession frequency of a proton in a magnetic field H and may be measured by use of nuclear resonance absorption techniques. ν_{H} is known from the preceeding paper, and f is the atomic transition frequency measured in this experiment.

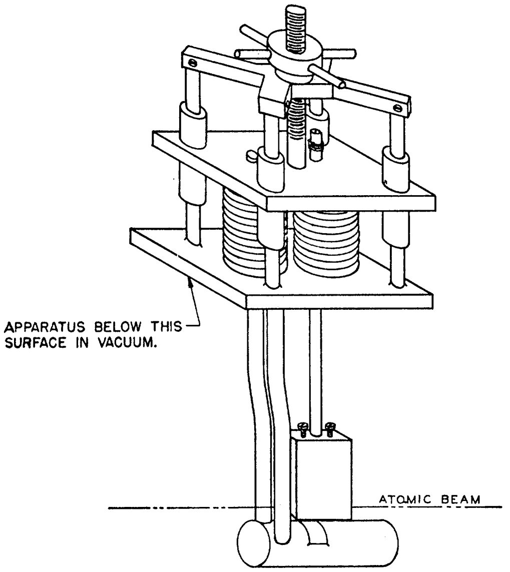

FIG. 2. Probe mount. The mechanism for raising and lowering hairpin and proton sample is indicated.

APPARATUS

A. The Magnet

The major equipment for the present experiment was the atomic beams apparatus described in the previous paper. The large external electromagnet, not used by Prodell and Kusch, was added to produce a uniform magnetic field in the region in which the atomic transitions occurred. The magnet was so designed that an accuracy of one part in 10^6 in the measurement of g_s/g_p appeared to be attainable. The main requirement was an extremely homogeneous field over the transition region, so that neither the atomic line nor the proton resonance line would be excessively broadened by the nonuniformities of the magnetic field.

The magnet, of Armco magnet iron, is a low voltage, water-cooled electromagnet with 30-cm hand lapped pole faces initially parallel within 0.003 cm and with a 5.7-cm gap. The magnetic field was about 1500 gauss. To achieve a high degree of homogeneity, it was necessary to shim the magnet extensively. Geometric factors, such as fringing of the field, and different path lengths of flux for different parts of the pole faces contributed to the variation of the field over the desired region. It was possible to compensate for these effects by large shims near the edges of the magnet faces, and by an adjustment of the assembly bolts so that the gap became slightly wedged shaped. The most important factor contributing to the inhomogeneities of the field were, however, local irregularities in the iron, which to some extent varied from day to day.

The first successful attempts at shimming were by the use of 0.003–0.008 cm steel shims with an area of about 12 cm². By trial and error it was possible to improve the character of the field considerably, but not sufficiently to eliminate field broadening of the observed lines. The main difficulty was that effects of the various shims were not independent. Continuous experimentation during the course of this research led to improved shimming procedures, and for the last few runs it was possible to obtain the theoretical line widths. Small nickel shims, about 5 mm square, held in place with Scotch Electrical Tape were used. The small size of the shims greatly reduced their mutual interaction, as did the reduced permeability of the nickel. At best, it was possible to have the field vary less than one part in 10^5 over the transition region. However, the field would retain this character for only about two weeks of use.

B. The uhf Unit

In order to induce the desired transitions in the atomic beam, it is necessary to apply a varying magnetic field perpendicular to the direction of the static field at a frequency of 3655 Mc. The open "hairpin" construction used by Prodell and Kusch was not suitable for the present experiment. Since the transition frequency of the line observed in this experiment is critically dependent on the field, the extent of the rf field must be more closely confined to a known limited region in the present experiment. In addition, the atomic beam apparatus readily propagated energy with a frequency of 3655 Mc, and considerable interference was produced in the Pirani gauge detector by use of an open hairpin.

A resonant, carefully shielded hairpin, shown in Fig. 1, was used. A rectangular box (A), open at both ends and approximately one-half wavelength high, was fed by a loop connected to the rf source through a coaxial line. The cavity had a low Q and could be tuned by means of two screws at the top of the unit. The cavity was enclosed in a copper box (B), with small openings

(*C*) for the passage of the beam. The shield confines the radiation within the desired limits, and the shorted turn formed by the box produces a current sheet and therefore an approximately uniform magnetic field distribution along the beam direction. The hairpin was mounted on a horizontal brass tube, about 9 cm long and 2.86 cm in outside diameter, which contained the sample and the associated coils necessary for the detection of the proton resonance. The entire assembly, Fig. 2, could be moved vertically in vacuum through bellows, so that either the hairpin could be placed in position for observing the atomic transitions, or the water (oil) sample could be raised into the region of the field otherwise occupied by the beam.

C. The Proton Resonance Apparatus

In all cases the sample in which the proton resonance was observed was cylindrical to give the best coincidence in space of the beam and the proton sample. The proton resonance was detected by a circuit[8] which is sensitive only to absorption of energy by the precessing protons. The sample is placed in the tank coil of an rf oscillator which is coupled to an rf amplifier, a detector, and audio amplifiers, so that any change in the level of the oscillator at an audio rate is present at the output of the audio stage. The average dc level at the detector is fed back through a one-second time constant RC circuit to the oscillator to prevent slow wandering of the oscillator level. In actual operation it was found desirable to reduce the filament voltage of all tubes in the circuit to 4 volts. The result was an increased signal to noise ratio as well as increased stability of the feedback circuit.

The magnetic field at the sample was modulated by a pair of rectangular coils. These coils, the oscillator tank coil, and the sample were assembled as a unit to be inserted into the brass tube under the hairpin. In the earlier runs, the tank coil was wound on a glass tube containing the water sample, while the modulating coils were wound on Lucite spacers mounted on the glass tube as well. This unit could be slipped into the brass tube, and electrical contact made by clips. Glass plugs cemented into the glass tube determined the sample length.

For the later runs, a new Lucite holder was designed so that a thin walled glass tube, 4.7-mm i.d., containing the oil sample could be removed without removing the coils. This gave a more flexible working arrangement as well as better electrical stability, since the Lucite holder and coils were permanently mounted in the brass cylinder.

When setting the magnetic field for a run, 60-cycle modulation of about 2 gauss peak to peak was used to display the entire resonance on an oscilloscope. A sinusoidal sweep of 120 cycles derived from the filtered output of a full wave 6H6 rectifier made it possible to adjust the oscillator frequency to the proton resonance frequency corresponding to the unmodulated magnetic field by setting the two resonances occurring on the oscilloscope screen per modulation cycle to coincidence. With large modulation, a 60-cycle signal was induced in the tank circuit from the modulating coils. The interference was conveniently eliminated by injecting a 60-cycle correction voltage of adjustable phase and amplitude through two one-millihenry chokes directly across the parasitic suppressor in the grid lead of the oscillator.

In the actual measurement of the line frequency, a modulation amplitude less than 0.01-gauss rms at a frequency of 99 cycles was used. The frequency was chosen because it is high enough to eliminate flicker effect noise, low enough for the relaxation time of the sample, and not related to 60 cycles in any simple manner so that 60-cycle pick-up could not introduce any errors. The modulation voltage was supplied by a Hewlett Packard 201B audio oscillator feeding the coils through a 6-volt filament transformer.

The output of the audio amplifier, a 99-cycle signal proportional to the slope of the resonance line, in as much as the line in all cases had a width greater than the modulation amplitude, was fed into a narrow band amplifier. The output was displayed on the oscilloscope and the maximum of the resonance curve was determined by setting for null output. By this procedure the resonance could be reset to within 15 cycles out of 6.5 Mc consistently. Repetition of measurements gave a significant improvement in the accuracy of the measurements.

FREQUENCY MEASUREMENT

The main data of the experiment are the frequencies f and P. A 2K42 klystron mounted in a water-cooled oil bath and powered by well-regulated A and B supplies was used to generate f, a frequency of approximately 3655 Mc. The frequency of the klystron was varied by manual adjustment of the repeller voltage. The output was sufficiently free of modulation and constant during the time required to make a setting and measurement of the transition frequency to permit measurement of the frequency to 1 kc. The frequency was measured by beating a part of the klystron output with 3680 Mc from a crystal harmonic generator fed by 240 Mc plus added 80-Mc sidebands from the frequency standard. The 25-Mc beat was heard on a receiver, which was essentially calibrated at each observation by the second harmonic of a General Radio 620A Heterodyne Frequency Meter. The procedure yielded a least count of 2 kc in f.

Measurement of P, a frequency of the order of 6.5 Mc, to the same precision was somewhat more difficult. Initially, 5 Mc from the frequency standard was mixed with 1.5 Mc from a General Radio 805 signal generator. The 6.5-Mc beat was allowed to leak into the oscillator associated with the resonance detection circuit. This

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[8] R. V. Pound and W. D. Knight, Rev. Sci. Instr. **21**, 219 (1950).

frequency, beating with that produced by the proton resonance oscillator, produced an audiofrequency which appeared on the oscilloscope superimposed on the proton signal. By adjusting the signal generator, zero beat could readily be attained. The tenth harmonic of the signal generator output was measured on a GR Heterodyne Frequency Meter. The least count of this method was 100 cycles at 6.5 Mc. It was necessary, then, to estimate to 0.1 division on the frequency meter to achieve an accuracy of 1 part in 6.5×10^5.

For the later runs, a 10-kc blocking oscillator driven by the 50-kc output of the frequency standard was built. Harmonics of 10-kc in the 6.5-Mc region were received and identified on a National HRO receiver with an internal crystal calibrator. The frequency to be measured was also fed into the receiver. One of the difference frequencies, an audio note less than 5 kc, was measured on a GR Audio Interpolation Oscillator, with a least count of 1 cycle.

In the measurement of both f and P, the procedure was to track the unknown frequency with the frequency measuring device at all times while a setting on a line was made. In this manner effects due to oscillator drift could be minimized. Frequency measurements, then, were not factors which limited the precision of the experiment, especially in view of the statistically significant body of data which was obtained.

EXPERIMENTAL PROCEDURE

The critical point of the experiment was the necessity of placing the sample in the same portion of the field as that occupied by the beam. An exact superposition was impossible, inasmuch as the beam was 2.8 mm high and approximately 0.05 mm wide compared with a proton sample of circular cross section approximately 4.7 mm in diameter. It was attempted to place the sample so that its axis of symmetry coincided with the axis of the beam. This was not always accomplished exactly, and after a run or set of runs, the entire probe mount (Fig. 2) was removed from the apparatus and carefully measured to determine any vertical error in the placement of the sample. At the end of every run the vertical variation of the field was measured. It was at most 1 part in 1.5×10^5 per mm, so that over the height of the sample there could exist field variations as great as 1 part in 3.2×10^4.

To correct for errors due to incorrect positioning of the sample in the face of a gradient of the field, two procedures may be used. The simpler of the two effectively treats the beam and sample as geometric lines defined by their axes of symmetry; if these do not coincide, the measured values of f and P are corrected by an amount determined by the field differential over the distance of separation. The second correction procedure takes account of the fact that the field does not vary linearly but has a second derivative. Therefore, the average value of the field over the beam or sample will not be the same as the field at the axis of the beam or sample. In as much as the beam and sample are of different heights and the varying thickness of the sample weights different parts of the field differently, the second derivative of the field will cause an effective or virtual relative displacement of the beam and sample. A correction can be applied in a straightforward manner, if it is assumed that there is a quadratic variation of the magnetic field in the vertical direction, and no variation of the field in the direction of the field. The validity of this assumption will be discussed in succeeding sections.

TABLE II. g_J/g_p.

Run	I	Ia	II	III	IV
March 17, 1951	658.2123±0.0007		658.2151	658.2176	658.2169
March 20, 1951	.2172±0.0006		.2200	.2169	.2162
March 26, 1951	.2180±0.0005		.2208	.2173	.2172
June 28, 1951	.2131±0.0004		.2141	.2154	.2153
July 12, 1951	.2142±0.0002		.2151	.2181	.2196
July 26, 1951	.2167±0.0002		.2177	.2171	.2163
August 3, 1951*A*	.2170±0.0003		.2180	.2180	.2175
August 3, 1951*B*	.2165±0.0003		.2175	.2175	.2170
Average:			.2173	.2172	.2170
Probable error:			0.0006	0.0002	0.0003

The C field current was monitored by a type K2 L & N potentiometer connected across an L & N 0.001-ohm shunt in series with the magnet windings. In the first two runs no attempt was made to stabilize the current, obtained from two submarine storage batteries. For the remaining runs, however, the current was held constant within 1 part in 2×10^5 by continuous manual adjustment of a mercury rheostat. Drift in the A and B field current as well as geometrical distortions of the C magnet resulting from temperature changes contributed to a variation in time of the magnetic field in the transition region. This variation served the useful purpose of causing each setting on a line to occur at a different position of the dial of the frequency measuring devices, and in this way minimized prejudice of the observer in adjusting and reading the frequency.

The procedure adopted in taking data was to make about 13 sets, alternating between the two lines, of three or four determinations of the frequency of the center of the line. At each observation the time and, in those runs where it was not held constant, the current in the C field were recorded. The data were reduced by interpolating in time or current between the average of the first and third sets of readings to obtain a frequency P to be paired with the average of the second set of readings f. The interpolation process was repeated for all groups of three adjacent sets of readings. Each pair of frequencies was substituted into Eq. (3), and the average of the results of the substitution was taken as the value of g_J/g_p for the run. This value of g_J/g_p depends then on about 21 measurements of P and 18 measurements of f.

EXPERIMENTAL RESULTS

In Table II the values obtained for g_J/g_p in the various runs are tabulated. Column I gives the result of each run without any corrections, that is, the result of Eq. (3) after direct substitution of the measured and interpolated frequencies. The estimated probable error, which gives a measure of the internal consistency of the run, is tabulated in column Ia. Column II lists the results corrected to measurement of the proton frequency in a spherical sample of mineral oil. Several diamagnetic effects to be discussed later are involved. In column III the results of the various runs corrected for variations of the magnetic field over the sample and beam by the first method (see preceeding section on experimental procedure) are tabulated, whereas in column IV the results, after correction for the quadratic variation of the field over the finite samples, are tabulated. It is seen that the spread of the average result in the last three columns is 1 part in 2×10^6. To appreciate this excellent agreement, it is necessary to consider the substantial, accidental or deliberately invoked, variations in the experimental conditions from one run to the next.

In the runs of March 17, 20, and 26, the proton resonance was observed in a 0.13 molar solution of $CuSO_4$ and the proton resonance frequency was measured by the first of the two techniques described in the section on frequency measurements. The sign of the gradient was deliberately changed between the runs of March 17 and March 20 by the simple expedient of altering the angle between the pole faces. To reduce the field inhomogeneity introduced by the change in the shape of the gap, readjustment of shims was required. The vertical gradient of the field in the runs of March 20 and March 26 was approximately the same. However, in the runs of March 17 and 20 the C field was allowed to drift, while in the run of March 26, the current in the C magnet was maintained at a constant value. The total field drift during the course of the two earlier runs was about 1 part in 10^4, while in the later runs, it was only about 1 part in 5×10^4. The atomic line width was about 130 kc on March 17, and about 145 kc on the other two March runs. A theoretical line width of 60 kc is characteristic of a 4-cm hairpin.

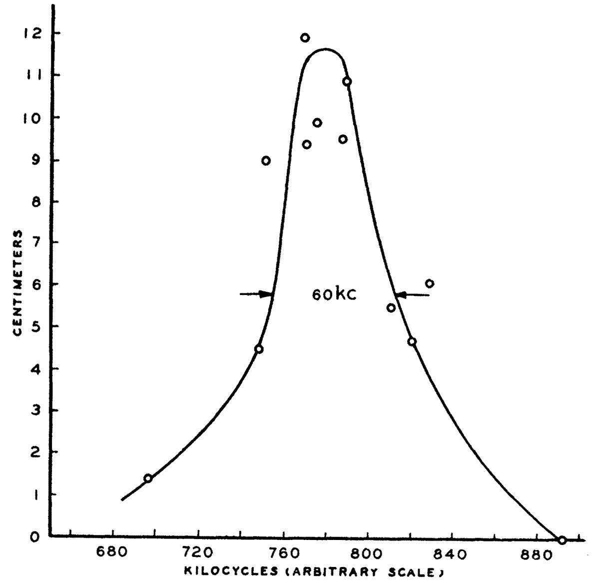

FIG. 3. A plot of the resonance line of hydrogen observed on July 26, 1951. The ordinate is the observed change in beam intensity and the abscissa is the deviation of the applied frequency from a fixed frequency. The scatter of points arises principally from mechanical unsteadiness of the Pirani gauge.

After the series of March runs, a careful measurement of the position of the beam in the transition region as well as a measurement of the position of the water sample under the observational conditions was made. A small relative shift was found; corrections for this shift by each of the two procedures possible in the face of a magnetic field gradient lead to the corrected values in columns III and IV of Table II. In spite of the fact that the corrections are as great as one part in 2×10^5, the maximum discrepancy between the results of the three runs is only about 1 part in 6×10^5. The agreement, in view of the varying field inhomogeneities and different observational procedures, is impressive.

Three runs taken in April, not included in Table II, were made with a new sample holder, tank coil, and modulating coils. The water sample was deliberately made considerably longer than the hairpin. The values of g_J/g_p after reduction to a spherical sample of mineral oil were 658.2208, 658.2141, and 658.2046, with a mean of 658.2132. The large spread of values presumably arises from a differential mean field in the hairpin and the longer water sample. It is significant that the errors of different signs appear in the three runs. Moreover, it should be noted that subsequent examination of the sample holder showed contamination with magnetic particles, and it is not inconceivable that the iron particles gave rise to the total discrepancy. The interpretation of the April runs is, therefore, doubtful, and the data were discarded. No magnetic contamination of relevant portions of the apparatus was detected in any other run.

The runs of June, July, and August were made with the Lucite holder. Squibb's mineral oil was used as the sample material in all these runs. Extraordinary care was taken to prevent magnetic contamination. A new uhf unit was also installed. In this series of runs the current in the C field was held constant, and the proton resonance frequency was measured with an improved procedure, previously described.

The runs of June 28 and July 12 were both taken with a rather poor field and the atomic line widths were 150 and 170 kc, respectively. The shims were adjusted before each run. The vertical gradient was about three times as great in the second of these runs as in the first; the rather poor homogeneity was accompanied by a low intensity of the atomic line and observation was difficult. In spite of these difficulties, the corrected result is in remarkable agreement with those obtained in the March runs.

The experience during the runs of June 28 and July 12 indicated a progressive deterioration of the magnetic field. A major readjustment and reshimming appeared to be indicated. The magnet yoke was completely disassembled, and all shims removed from the pole faces. The component parts of the yoke were cleaned and lightly greased, and the magnet reassembled. The magnet was then shimmed with small nickel shims, and a portion of the gap was found with a field variation less than 1 part in 10^5 over the transition region.

Mechanisms were provided to insure that the axis of the oil sample coincided with the beam axis in a horizontal plane. Finally, a considerable rearrangement of the uhf unit was made; in particular, the joints were all resoldered, and the tuning screws were adjusted to bring the hairpin closer to resonance at the transition frequency.

The run of July 26, listed in Table II, was taken under the best conditions of any of the runs. The atomic line width was reduced to the theoretical value of 60 kc by careful adjustment of the rf amplitude. The line is shown in Fig. 3. Unfortunately, a mechanical unsteadiness of the beam prevented the observation of a smoother curve. It was apparent from the approximately theoretical magnitude of the "flop" observed that all previous runs had been taken with less than optimum rf amplitude, presumably because of poor tuning of the hairpin.

Though the low rf amplitude in the previous runs made observation difficult because of low intensity of the line, errors arising from an asymmetrical broadening of the line by excessive rf amplitude in the face of field inhomogeneties and other effects are avoided.

An earlier run taken on July 26 with a grossly excessive rf amplitude gave a result of 658.2284 for g_J/g_p. It is possible that the discrepancy with other data arises from effects associated with the large rf amplitude; it is our belief, however, in view of the consistency of all other valid data, that the result is a consequence of a systematic blunder in observation, and the data were rejected.

The two runs of August 3 were taken to determine the effect of rf amplitude on the experimental result. In the first run, the rf amplitude was adjusted so that the flop was only half of its maximum value. The second run was taken at the optimum rf amplitude, and both the line width and intensity had the theoretical value. A third run was taken with an excessive rf amplitude, which broadened the line to about $2\frac{1}{2}$ times its theoretical width and reduced its intensity by about 20 percent. The result for g_J/g_p was 658.2192. This result is in the direction of the previously discarded run, although it is difficult to see how an effect as large as that observed on July 26 could be attributed to any reasonable power broadening. The third result of August 3 is discarded because of the excessive power level in the hairpin and also because of instabilities which appeared in the C field throughout the run.

Data have, therefore, been discarded in arriving at the final result, for well-defined reasons. The agreement of the means of the results in columns II, III, and IV indicates that the effect of the gradient of the magnetic field as well as that of a quadratic variation of field in the vertical direction is random. No direct observation of the gradient and its effect in the two horizontal directions has been made. However, under the widely varying experimental conditions in the several runs, it appears to be reasonable that effects arising from gradients other than the vertical gradient are also random. In addition, the observed width of the lines could be approximately explained by the vertical gradient alone, so that a total variation of the field over the 4-cm length of the sample larger than that over the height of the sample is precluded.

The result of the experiment is taken to be the mean of the results in columns III and IV since the detailed validity of the assumptions underlying the gradient corrections in both these columns is uncertain. We have, then,

$$g_J/g_p \text{ (spherical sample mineral oil)} = 658.2171 \pm 0.0006,$$

where the stated uncertainty is twice the probable error in column IV and three times that in column III to allow for statistically uncompensated errors arising from gradients not here considered, and to include uncertainties in certain of the diamagnetic corrections.

DIAMAGNETIC CORRECTIONS

The measured proton resonance frequency P' is not identically the frequency corresponding to the externally applied field, but rather that corresponding to the external field modified by several small diamagnetic effects. If P' is substituted into Eq. (3) the result, tabulated in column I of Table II, will be the ratio of g_J to the g value of the proton as measured in a sample of particular shape and material, since the small difference between P and P' will have a negligible effect on the numerator and the bracketed denominator term of the equation. To reduce the result in column I to that of column II, which gives the values of g_J/g_p, before gradient corrections, for g_p measured in a spherical sample of mineral oil, several diamagnetic corrections must be considered. These arise from (1) the bulk diamagnetism of the sample material itself which modifies the field in a cylindrical sample but not in a spherical sample, (2) fields due to paramagnetic ions in the sample, and (3) the differential diamagnetic shielding of the proton by the orbital electrons in the molecules containing the proton. For water, all three corrections must be applied, while for oil the only correction which need be applied is that due to (1). For an infinite cylinder the field at the proton is $H_0(1-\frac{2}{3}\pi k)$, where

H_0 is the applied field and k the diamagnetic susceptibility of the water or oil. The sample used in the present experiment was a cylinder with a ratio of length to diameter of 8.5. The data of Dickenson[9] show that such a cylinder may be considered infinite in calculating the diamagnetic correction. To the present precision the same result is obtained if the cylinder is approximated by an ellipsoid. The ratio of the susceptibility of mineral oil and water is[10] 1.01 and $k_{\mathrm{H_2O}}=-0.72\times10^{-6}$. The field at the proton in both materials is, therefore, increased by $H_0 2\pi k/3=1.5\times10^{-6}H_0$. The effect of (1) is to decrease the value of g_J/g_p measured in a cylindrical sample from that measured in a spherical sample. To reduce the observed values to the case of a spherical sample, the value of g_J/g_p is, therefore, to be increased by 0.0010 for both oil and water.

The effect (2) occurs only in the case of the three runs on water. For the 0.13 molar $CuSO_4$ solution used in the water runs, the correction due to the paramagnetism of the Cu^{++} ions is somewhat uncertain. Were the ions derived from $CuCl_2$, the correction would result in a decrease in the measured value of g_J/g_p of 0.7 part in 10^6. Dickenson gives data on the effect of Cu^{++} derived from the chloride, as well as data on the effect of the Ni^{++} ions derived from the sulfate and the chloride. For nickel, the corrections are somewhat different in each case, and it is reasonable to assume that the same may be true for copper. In the absence of further data, however, the correction of -0.7 part in 10^6 will be applied to g_J/g_p. The uncertainty will not affect the final result by more than 1 part in 10^7.

For a fixed magnetic field, the difference in the average magnetic field at the proton in mineral oil and in water due to shielding produced by the different electron configurations in the two molecules is given by Gutowsky and McClure[11] to be 3.4 parts in 10^6. The field at the proton in water is higher than in oil. Then to the uncorrected results for the water runs, a correction of $(1.5+3.4-0.7)\times6.58\times10^{-4}=+0.0028$ must be applied. These corrections, added to the data in the first column of Table II, give the results tabulated in column II. A rough estimate of diamagnetic effects arising from the mechanical structures associated with the apparatus shows a negligible effect.

DISCUSSION

In order to obtain the magnetic moment of the electron, it is necessary to know the relation between g_J of atomic hydrogen and g_s of an unbound electron. A hydrogen atom with a spinless nucleus in zero magnetic field is in a pure S state, and its g value would be entirely due to the electron spin. However, the presence of the nuclear moment as well as the externally applied field mixes a small part of the ${}^2D_{\frac{3}{2}}$, $m_J=\pm\frac{1}{2}$ levels with the two degenerate ground levels. The amount of mixing may be calculated with the formulas derived by Perl and Hughes.[12] A rough estimate of the mixing may, however, be made. The interaction energy of a Bohr magneton with a magnetic field corresponds to 2.8 Mc per gauss, so that the perturbation energy of the atom in the applied field of 1500 gauss is about 4200 Mc. The $S-D$ separation is approximately 0.9×13.6 ev $=3\times10^9$ Mc. The amount of mixing will then be on the order of $4.2/3\times10^6$ or 1.3 parts in 10^6. The effect on the g value will be proportional to the square of the mixing, or 1.7 parts in 10^{12}, a completely negligible effect.

The effect of the magnetic field due to the nuclear moment is smaller still. The field at the first Bohr orbit due to the proton moment is approximately $\mu/a_0^3=90$ gauss, as compared to the 1500 gauss of the external field. These estimates are in agreement with the more accurately calculated results. Within the limits of the experiment, then, the ground state may be considered pure. $g_J({}^2S_{\frac{1}{2}}\mathrm{H})$ is, therefore, equal to g_s', the spin g value of the electron bound in the hydrogen atom.

Relativistic effects, essentially due to the increase in mass of the electron bound in the hydrogen atom, serve to decrease g_s' from g_s, the g value of the free electron.[13]

$$g_s'/g_s=[1+2(1-\alpha^2)^{\frac{1}{2}}]/3\cong1-\alpha^2/3=1-17.8\times10^{-6}.$$

Application of this factor to our experimental result yields

$$g_s/g_p=658.2288\pm0.0006.$$

A combination of this result with that of Gardner and Purcell,

$$2g_l/g_p=657.475\pm0.008,$$

where g_p is also measured in a spherical sample of mineral oil, yields

$$g_s/g_l=2(1.001146\pm0.000012).$$

The greater part of the uncertainty arises from the stated uncertainty in the result of Gardner and Purcell. The agreement with the value calculated by Karplus and Kroll is excellent. Because of the large experimental uncertainties, the result cannot, however, be considered as conclusive evidence of the validity of the theoretical calculation.

The ratio $g_s/g_{p(\mathrm{oil})}$ is not of fundamental significance except when combined with other observations in which a measurement of the proton resonance frequency in a spherical sample of mineral oil is made. There is a rather large diamagnetic shielding of the proton in H_2. Ramsey[14] has calculated the magnetic shielding constant in this case to be $\sigma=26.8\times10^{-6}$. In addition, Gutowsky and McClure[11] have shown that the diamagnetic shielding of the proton in mineral oil is greater by

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[9] W. C. Dickenson, Phys. Rev. **81**, 717 (1951).

[10] I. Cooter, Bureau of Standards, private communication.

[11] H. S. Gutowsky and R. E. McClure, Phys. Rev. **81**, 276 (1951).

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[14] N. F. Ramsey, Phys. Rev. **78**, 699 (1950).

3.7×10^{-6} than it is in H_2. The net diamagnetic correction is then 30.5×10^{-6}. Accordingly

$$g_s/g_{p(\text{free})}=658.2087\pm0.0006,$$

where the uncertainties in the diamagnetic corrections have not been included in estimating the precision of the result.

The authors wish to express their appreciation to Mr. Jim Woods and his staff in the machine shop for the construction of the apparatus; to Mr. Richard Blume for his work in the design and construction of a suitable frequency standard; and to Dr. Irwin Cooter of the National Bureau of Standards for his measurement of the diamagnetic susceptibility of our mineral oil sample.

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A Determination of $g_J(^2S_{\frac{1}{2}})$ of Potassium in Terms of the Proton Gyromagnetic Ratio*

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The ratio of the g value of potassium K^{39} in the ground state to the gyromagnetic ratio of the proton in mineral oil has been measured in an experiment combining atomic beam and nuclear resonance techniques. It is found to be $g_J(\text{K})/g_p=658.2274\pm0.0023$. This result, when combined with the measurement of $g_J(\text{H})/g_p$ by Koenig, Prodell, and Kusch, yields

$$g_J(\text{K})/g_J(\text{H})=1.000016\pm0.0004\text{ percent.}$$

This is to be compared with the recent direct measurement by Pohlman, Bederson, and Eisinger: $g_J(\text{K})/g_J(\text{H})=1.00016\pm0.006$ percent. The present experiment is of superior precision and is in agreement with independent experimental evidence. The present result is found to agree with the theoretically predicted value.

INTRODUCTION

A RECENT direct measurement by Pohlman, Bederson, and Eisinger[1] (PBE) of the ratio of the g_J values of hydrogen and potassium in the ground, $^2S_{\frac{1}{2}}$, states has yielded

$$g_J(K^{39})/g_J(H^1)=1.00016\pm0.006\text{ percent.}$$

In view of the fact that Kusch and Taub[2] found the g_J values of Li, Na, and K to be identical to within one part in 40,000, this large reported difference between the g_J values of K and H is rather surprising. In addition, it is possible to deduce values of $g_J(\text{K})/g_J(\text{H})$ which are in disagreement with the PBE result both from independent experimental evidence and from theoretical considerations. The experimentally determined quantities to be considered are

$$g_J(\text{H})/g_p=658.2171\pm0.0001\text{ percent,}^3$$
$$g_p/g_J(\text{Na})=15.1927\times10^{-4}\pm0.005\text{ percent,}^4$$
$$g_J(\text{Na})/g_J(\text{K})=1.00000\pm0.002\text{ percent.}^2$$

Within the stated precision of the results, the different internal diamagnetic corrections to be applied to the g value of the proton as measured in mineral oil and NaOH can be ignored. From these results it is found that

$$g_J(\text{K})/g_J(\text{H})=0.99999\pm0.007\text{ percent.}$$

While the disagreement between this result and that of PBE cannot be considered as conclusive evidence of the invalidity of either result, the discrepancy is notable.

Calculations by Phillips[5] on the perturbation of the $^2S_{\frac{1}{2}}$ ground state of K by excited states of the inner electronic core yield the estimate $1\leqslant g_J(\text{K})/g_J(\text{H})\leqslant1.00001$. Relativistic and diamagnetic corrections, to be discussed in Sec. C, increase both limits by 6×10^{-6}. While there is some uncertainty in the perturbation calculations, the disagreement of this limit with the result of PBE is marked.

In view of the discrepancy of the PBE result and that obtained from independent experimental evidence and in view of the fact that no effect as large as that found by PBE can be accounted for on the basis of existing theory, it was decided to redetermine the ratio $g_J(\text{K})/g_J(\text{H})$. In the previous paper Koenig, Prodell, and Kusch[3] (KPK) have described a precision measurement of $g_J(\text{H})/g_p$. The present paper discusses a determination of $g_J(\text{K})/g_p$. The combination of the two results yields a value of $g_J(\text{K})/g_J(\text{H})=1.000016\pm0.0004$ percent, which is of a higher precision than the result of PBE and in marked disgreement with it.

A. PROCEDURE

In the experiment of KPK the ratio $g_J(\text{H})/g_p$ was obtained by measuring a field dependent hyperfine

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