It might turn out that the first excited state of Sr^{88} has spin 2, even parity in accordance with the Goldhaber-Sunyar rule.[7] This would remove the last[8] of the four even-even nuclei with spin one in the first excited state which were quoted by Goldhaber and Sunyar[7] as exceptions to their rule.

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† Now with the Bartol Research Foundation, Swarthmore, Pennsylvania.
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A Variational Method for Radiationless Transitions

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RADIATIONLESS transitions are usually formulated by means of a time independent perturbation theory. Since the initial state is unstable, the corresponding wave function may be written as a sum of terms, several of which are not closed. The system formed is, therefore, analogous to a "collision complex" whose break-up may be described by a time independent theory.

Consider, in particular, the auto-ionization of an excited atomic system. Suppose $\Psi_i(1\cdots n)$ is a properly antisymmetrical closed wave function, normalized to unity, and $\Psi_f(1\cdots n)$ is a wave function describing the final (unbound) state for which the transition probability is required. Then, if only these two terms are included, the total wave function Ψ corresponding to an energy E may be written as the linear combination

$$\Psi=c\Psi_i+d\Psi_f,$$

where the correct Ψ satisfies the appropriate Schrödinger equation

$$(H-E)\Psi=0;$$

c, d are constants and $\Psi_f\sim\chi(2\cdots n)\phi(1)\,\sigma(1\cdots n)$, where $\chi(2\cdots n)$ is the core wave function $\sigma(1\cdots n)$ the spin wave function for the whole system, and $\phi(1)$ represents an outgoing spherical wave

$$\phi(1)=r_1^{-1}\exp[i(kr_1-\alpha\log 2kr_1)]P_l(\cos\theta).$$

Ψ_f may be normalized so that

$$\int\Psi_f^*(k')\Psi_f(k)d\tau=\frac{4\pi^2}{(2l+1)}\delta(k-k'),$$

so that the constants c, d are related to P, the probability of auto-ionization per atom per unit time, by the equation

$$P=\frac{\hbar}{2m}\frac{8\pi}{(2l+1)}k\left|\frac{d}{c}\right|^2\left(1+\left|\frac{d}{c}\right|^2\right)^{-1}.$$

The variational methods developed by Hulthén[1] and Kohn[2] for collision problems may be extended for the computation of the ratio d/c and hence P. Using the notation of Kato,[3] we define

$$L\equiv H-E,$$

and Ψ_t, a trial wave function depending on p constants a_m, the final state wave function having the same boundary conditions as Ψ_f with the exception that d_t replaces d.

It follows that, if $\omega\equiv\Psi_t-\Psi$,

$$I(a_m, d_t)\equiv\sum_{\text{spin}}\int\Psi_t^*L[\Psi_t]d\tau$$
$$=-\frac{\hbar^2}{2m}(8\pi ik)d^*(d-d_t)+\sum_{\text{spin}}\int\omega^*L[\omega]d\tau.$$

This is the finite form of the usual variational equation, the integral on the right-hand side being of the order ω^2.

Following Hulthén, we may require $d_t-d\to 0$ and then the best function $\Psi_t(a_m, d_t)$, of the trial form chosen, and the best value of $|d/c|$ may be found from the equations

$$I(a_m, d_t)=0,\qquad m=1, 2, \cdots p,$$
$$\partial I/\partial a_m=0,$$

which (as d_t, a_m need not be real) are $(2p+2)$ in number.

Kohn's method may also be applied, leading to the equations

$$\partial I/\partial a_m=0,\quad m=1, 2, \cdots p,$$
$$\partial I/\partial d_t=\frac{\hbar^2}{2m}\frac{8\pi ik}{(2l+1)}d^*,$$
$$d^*d=d_t^*d_t-\frac{2m}{\hbar^2}\frac{(2l+1)}{8\pi ik}I.$$

A comparison of these variational methods and the usual perturbation treatment is being carried out for a number of auto-ionizations undergone by helium.

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The Minimum of Electrical Resistance at Low Temperatures

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THE initial discovery[1,2] of a resistance minimum at low temperatures in gold has been followed since the war by increasing interest in the analogous phenomenon in other metals.[3–7] An extended investigation has been in progress over the past two years on alloys of Cu, Ag, and Au, and some aspects have already been reported.[6] More recently particular attention has been paid to a very characteristic minimum in Cu occurring generally at about 10.5°K.

The following conclusions have now been definitely established through experiments on a rather wide range of alloys specially prepared from spectrographically pure metals. As solute, elements were chosen which tend to occur as traces in very pure samples of the parent.

(1) Pure copper and gold as parent metals do not exhibit the characteristic minimum.

(2) A variety of different solute elements when alloyed with the parent metal are capable of producing the minimum.

(3) The initial introduction of an effective "impurity" causes characteristic general scattering and a proportionately increasing resistive minimum together with a corresponding rise in the temperature of the minimum. The second effect, however, "saturates" very rapidly as the appropriate concentration (in the order of $<10^{-2}$ atomic percent to 10^{-1} percent dependent on solute and solvent) is approached. Thereafter the only further effect is to provide additional random scattering while both the magnitude and location of the minimum stay constant.

(4) In particular, it has been established in the case of copper-rich alloys that: (a) oxygen, silver, and nickel as solutes do not produce the minimum; (b) tin (which enters into homogeneous solid solution) and (c) carbon, lead, and bismuth (which at best have very small solid solubility) all produce the minimum.

(5) Also in the case of copper-rich alloys, very dilute ternary solutions of carbon and tin can almost annul the minimum over a short range of solute concentration. At the same time, anomalous behavior is observed in the over-all residual resistance.[8] (See following letter.)