In order to calculate the permeability for the rf $H_\perp$, we decompose the $H_\perp$ into two circularly polarized components whose permeabilities are shown by Polder[3] to be, (in rationalized units)

$$\begin{aligned}\mu_+/\mu_0&=1+\gamma M/(\omega_0-\omega),\\ \mu_-/\mu_0&=1+\gamma M/(\omega_0+\omega),\end{aligned}\tag{1}$$

where $M$ is the saturation magnetization of the sample and $\omega$ is the operating frequency. Knowing $\mu_+$ and $\mu_-$, it can be shown that the effective $\mu_\perp$ is given by[3]

$$\mu_\perp=\frac{2\mu_+\mu_-}{\mu_++\mu_-}=\left[1+\frac{\gamma M\omega_0+\gamma^2M^2}{(\omega_0{}^2-\omega^2)+\gamma M\omega_0}\right]\mu_0.\tag{2}$$

It is interesting to note that $\mu_\perp$ becomes infinite, not when $\omega=\omega_0=\gamma H_0$ as for an infinite medium magnetized in the direction of propagation, but when $\omega=\omega_0(1+\gamma M/\omega_0)^{1/2}$. Thus, the maximum magnetic displacement in the medium will occur at different frequencies depending upon the direction of the dc magnetic field relative to the field vectors of the wave.[3] This is due to the fact that for a uniform plane wave there is no rf $B$ in the direction of propagation as can be seen from Maxwell's curl equation. The electrons therefore experience a restraint from turning their spin axes in a longitudinal direction, since there will be an rf $H$ along the $z$ axis opposing their alignment in this direction. This restraint is equivalent to a demagnetizing force which raises the resonant frequency. No such restraint to electron precession exists when the $H_0$ is in the direction of propagation.

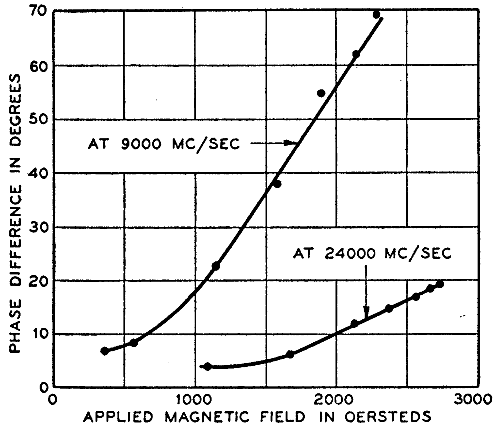

FIG. 1. Phase difference as a function of applied magnetic field, at 24,000 Mc/sec and 9000 Mc/sec.

If $\omega$ is much higher than the resonance frequency, so that $\omega_0{}^2$ and $M\gamma\omega_0$ can be neglected compared to $\omega^2$, the above equation can be simplified to

$$\mu_\perp=[1-(\gamma M\omega_0+\gamma^2M^2)/\omega^2]\mu_0.\tag{3}$$

The relative phase retardation of the wave with its magnetic vector parallel to $H_0$, with respect to the wave whose magnetic vector is perpendicular to $H_0$, is therefore given by

$$\theta_\gamma=2\pi l\left(\frac{1}{\lambda_{\parallel}}-\frac{1}{\lambda_\perp}\right)=\frac{l(\epsilon\mu_0)^{1/2}}{2}\left[\frac{\gamma^2M^2}{\omega}+\frac{\gamma^2MH_0}{\omega}\right].\tag{4}$$

This is to be compared to the Faraday rotation angle given by Hogan:[1]

$$\theta_F=\tfrac{1}{2}l(\epsilon\mu_0)^{1/2}[\gamma M].\tag{5}$$

It is thus seen that far above resonance, the double refraction effect will be much smaller than the Faraday rotation and will be inversely proportional to frequency. It is also to be noted that the relative phase retardation is a linear function of the applied steady magnetic field after the material is saturated, unlike the Voigt and Cotton-Mouton effects which are quadratic functions of the magnetic field, since these latter deal with nonsaturable materials.

In the experimental setup to check the above relations, the ferrite sample is placed in a dominant mode circular wave guide with reflections eliminated by proper tapering and with cross polarizing absorbers used to prevent cross-polarized reflections from the rectangular guide ends. The transverse magnetic field is applied at 45° to the rf $E$ vector. The phase difference is measured by noting the ellipticity of the output with linearly polarized input. The results for measurements made at 24,000 Mc/sec and at 9000 Mc/sec are shown in Fig. 1. The measurements at these two frequencies were made with the same type of ferrite but with somewhat different geometrical shapes. From the results one notes that the phase difference is a linear function of the magnetic field over the high field range and that the effect is much smaller at the higher frequencies as predicted by the above analysis.

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## Internal Conversion of the $Sr^{88}$ Gamma-Rays*

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WHEN investigating the decay of 105-day $Y^{88}$, Peacock and Jones[1] determined the internal conversion coefficients of the 0.9- and 1.85-Mev $Sr^{88}$ gamma-rays as $2.7\times10^{-4}$ and $1.3\times10^{-4}$, respectively. Using the theoretical data then available, these authors characterize the 1.85-Mev transition as electric dipole, the 0.9-Mev transition as magnetic dipole. Ling and Falkoff[2] based their analysis of the angular correlation data[3] on this assignment and showed that the spin combination 2–1–0 with a mixture of $E2$ and $M1$ for the 900-kev transition fits the experimental results.

Recently Bunker, Langer, and Moffat[4] found that the disintegration of $Rb^{88}$ leads to the same levels in $Sr^{88}$. They assigned even parity to all the levels in $Sr^{88}$, discarding the conversion experiments.

In view of this disagreement we decided to reinvestigate the internal conversion of the $Sr^{88}$ gamma-rays. This seemed the more important, as at least one experiment had indicated an electric dipole transition which is a type of transition rarely found in radioactive decays.

Using a $Y^{88}$ source obtained from the MIT cyclotron group,[5] we compared with a lens spectrometer of 2.5 percent resolution the internal conversion electron peaks with the photoelectron peaks from a gold converter of known efficiency. In Table I the experimental conversion coefficients are compared with the theoretical values of Rose *et al.*[6] The agreement between the two experiments is satisfactory. The conversion coefficients characterize the 900-kev transitions as $E1$, the 1.85-Mev transition as $M1$ or $E2$.

TABLE I. Internal conversion coefficients of the $Sr^{88}$ gamma-rays. The experimental values are total conversion coefficients, the theoretical values $K$-conversion coefficients.

| | Experiment $\alpha$ | | Theory $\alpha K$ | | | |
|---|---|---|---|---|---|---|
| | Peacock and Jones | This paper | $E1$ | $E2$ | $M1$ | $M2$ |
| $10^5\times\alpha_K$ (0.9 Mev) | 27 | $34\pm7$ | 28 | 68 | 66 | 165 |
| $10^5\times\alpha_K$ (1.85 Mev) | 13 | $17\pm4$ | 7.8 | 15 | 15.5 | 29 |

For the explanation of the angular correlation experiments one is now faced with the necessity of having to mix $E1$ and $M2$. If such a mixture occurs, it provides us with information concerning the reduction of the $E1$ matrix element. However, before this question can be decided, better angular correlation data are necessary.

It might turn out that the first excited state of $Sr^{88}$ has spin 2, even parity in accordance with the Goldhaber-Sunyar rule.[7] This would remove the last[8] of the four even-even nuclei with spin one in the first excited state which were quoted by Goldhaber and Sunyar[7] as exceptions to their rule.

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† Now with the Bartol Research Foundation, Swarthmore, Pennsylvania.

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## A Variational Method for Radiationless Transitions

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RADIATIONLESS transitions are usually formulated by means of a time independent perturbation theory. Since the initial state is unstable, the corresponding wave function may be written as a sum of terms, several of which are not closed. The system formed is, therefore, analogous to a "collision complex" whose break-up may be described by a time independent theory.

Consider, in particular, the auto-ionization of an excited atomic system. Suppose $\Psi_i(1\cdots n)$ is a properly antisymmetrical closed wave function, normalized to unity, and $\Psi_f(1\cdots n)$ is a wave function describing the final (unbound) state for which the transition probability is required. Then, if only these two terms are included, the total wave function $\Psi$ corresponding to an energy $E$ may be written as the linear combination

$$\Psi=c\Psi_i+d\Psi_f,$$

where the correct $\Psi$ satisfies the appropriate Schrödinger equation

$$(H-E)\Psi=0;$$

$c$, $d$ are constants and $\Psi_f\sim\chi(2\cdots n)\phi(1)\,\sigma(1\cdots n)$, where $\chi(2\cdots n)$ is the core wave function $\sigma(1\cdots n)$ the spin wave function for the whole system, and $\phi(1)$ represents an outgoing spherical wave

$$\phi(1)=r_1^{-1}\exp[i(kr_1-\alpha\log 2kr_1)]P_l\,(\cos\theta).$$

$\Psi_f$ may be normalized so that

$$\int\Psi_f^*(k')\Psi_f(k)d\tau=\frac{4\pi^2}{(2l+1)}\delta(k-k'),$$

so that the constants $c$, $d$ are related to $P$, the probability of auto-ionization per atom per unit time, by the equation

$$P=\frac{\hbar}{2m}\frac{8\pi}{(2l+1)}k\left|\frac{d}{c}\right|^2\left(1+\left|\frac{d}{c}\right|^2\right)^{-1}.$$

The variational methods developed by Hulthén[1] and Kohn[2] for collision problems may be extended for the computation of the ratio $d/c$ and hence $P$. Using the notation of Kato,[3] we define

$$L\equiv H-E,$$

and $\Psi_t$, a trial wave function depending on $p$ constants $a_m$, the final state wave function having the same boundary conditions as $\Psi_f$ with the exception that $d_t$ replaces $d$.

It follows that, if $\omega\equiv\Psi_t-\Psi$,

$$\begin{aligned}I(a_m,\,d_t)&\equiv\sum_{\text{spin}}\int\Psi_t^*L[\Psi_t]d\tau\\&=-\frac{\hbar^2}{2m}(8\pi ik)d^*(d-d_t)+\sum_{\text{spin}}\int\omega^*L[\omega]d\tau.\end{aligned}$$

This is the finite form of the usual variational equation, the integral on the right-hand side being of the order $\omega^2$.

Following Hulthén, we may require $d_t-d\rightarrow 0$ and then the best function $\Psi_t(a_m,\,d_t)$, of the trial form chosen, and the best value of $|d/c|$ may be found from the equations

$$\left.\begin{aligned}I(a_m,\,d_t)&=0,\\ \partial I/\partial a_m&=0,\end{aligned}\right.\qquad m=1,\,2,\,\cdots p,$$

which (as $d_t$, $a_m$ need not be real) are $(2p+2)$ in number.

Kohn's method may also be applied, leading to the equations

$$\partial I/\partial a_m=0,\quad m=1,\,2,\,\cdots p,$$

$$\partial I/\partial d_t=\frac{\hbar^2}{2m}\frac{8\pi ik}{(2l+1)}d^*,$$

$$d^*d=d_t^*d_t-\frac{2m}{\hbar^2}\frac{(2l+1)}{8\pi ik}I.$$

A comparison of these variational methods and the usual perturbation treatment is being carried out for a number of auto-ionizations undergone by helium.

The authors with to thank Professor D. R. Bates for his interest in the problem.

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## The Minimum of Electrical Resistance at Low Temperatures

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THE initial discovery[1,2] of a resistance minimum at low temperatures in gold has been followed since the war by increasing interest in the analogous phenomenon in other metals.[3–7] An extended investigation has been in progress over the past two years on alloys of Cu, Ag, and Au, and some aspects have already been reported.[6] More recently particular attention has been paid to a very characteristic minimum in Cu occurring generally at about 10.5°K.

The following conclusions have now been definitely established through experiments on a rather wide range of alloys specially prepared from spectrographically pure metals. As solute, elements were chosen which tend to occur as traces in very pure samples of the parent.

(1) Pure copper and gold as parent metals do not exhibit the characteristic minimum.

(2) A variety of different solute elements when alloyed with the parent metal are capable of producing the minimum.

(3) The initial introduction of an effective "impurity" causes characteristic general scattering and a proportionately increasing resistive minimum together with a corresponding rise in the temperature of the minimum. The second effect, however, "saturates" very rapidly as the appropriate concentration (in the order of $<10^{-2}$ atomic percent to $10^{-1}$ percent dependent on solute and solvent) is approached. Thereafter the only further effect is to provide additional random scattering while both the magnitude and location of the minimum stay constant.

(4) In particular, it has been established in the case of copper-rich alloys that: (a) oxygen, silver, and nickel as solutes do not produce the minimum; (b) tin (which enters into homogeneous solid solution) and (c) carbon, lead, and bismuth (which at best have very small solid solubility) all produce the minimum.

(5) Also in the case of copper-rich alloys, very dilute ternary solutions of carbon and tin can almost annul the minimum over a short range of solute concentration. At the same time, anomalous behavior is observed in the over-all residual resistance.[8] (See following letter.)