Letters to the Editor

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The Spontaneous Magnetization of a Two-Dimensional Rectangular Ising Model

C. H. Chang
University of Washington, Seattle, Washington
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THE present letter gives the spontaneous magnetization of a two-dimensional rectangular Ising Model. This is obtained by the method used in a paper by Yang[1] which treats the same problem except for a square lattice. The only difference in setting up the problem for a rectangular lattice is that instead of having the same H for both the vertical and the horizontal interactions, we have H_1 and H_2 (which are Kaufman's H and H'[2]) for the vertical and the horizontal interactions, respectively; i.e.,

$$\mathrm{V}_1=\exp\{H_1^*\sum_1^n \mathrm{C}_r\},\quad \mathrm{V}_2=\exp\{H_2\sum_1^n \mathrm{s}_r\mathrm{s}_{r+1}\}.$$

This leads to the condition $(x_1+1)(x_2+1)<2$ for temperatures below the critical temperature, where $x_1=\exp(-2H_1)$ and $x_2=\exp(-2H_2)$. The spontaneous magnetization I is expressed as an off-diagonal matrix element as before and the limiting process used there for reducing the calculation of I to an eigenvalue problem remains the same. In other words, all the results in Sec. I and II[3] of Yang's paper are unchanged.

In Sec. III, where an infinite crystal is considered, all the analysis remains the same except that H and H^* are replaced by H_2 and H_1^* in Eq. (60), and Eqs. (62) for A and B are replaced by

$$A=\coth H_2\coth H_1^*=(1+x_2)/x_1(1-x_2),$$
$$B=\tanh H_2\coth H_1^*=(1-x_2)/x_1(1+x_2).$$

In consequence, the modulus k in the elliptic integral is given by, instead of Eq. (78),

$$k=2k_{-1}^{\frac{1}{2}}/(1+k_{-1})=[2x_1/(1-x_1^2)][2x_2/(1-x_2^2)]$$
$$=1/\sinh 2H_1\sinh 2H_2.$$

In Sec. IV the elliptic transformation (81) is replaced by

$$z=-(\mathrm{cn}u-i[1+k_1]^{\frac{1}{2}}\,\mathrm{sn}u)(\mathrm{dn}u-i[k_1+k_1k_2]^{\frac{1}{2}}\,\mathrm{sn}u)/(1+k_1\,\mathrm{sn}^2u),$$

with

$$\frac{1}{z}\frac{dz}{du}=-i\frac{1-k^2}{(1+k_1)^{\frac{1}{2}}}\frac{1}{\mathrm{dn}u-[k_1(1+k_2)/(1+k_1)]^{\frac{1}{2}}\,\mathrm{cn}u},$$

where $k_1=\sinh^{-2}2H_1$ and $k_2=\sinh^{-2}2H_2$. The essential properties of the variable z as a function of u remain the same as in the square lattice. There are still two singularities per unit cell $(4K\times 4iK')$, although their positions are changed.

In terms of the variable u, one reduces the integral equation (69) again into the integral equation (84). The change in the positions of the singularities of z does not affect one important property of the kernel $J(u',u)$ relevant in our calculation, namely, that $J(u',u)$ has only one simple pole inside a unit cell $(4K\times 4iK')$. This can be shown by examining the zeros of the factors **I** and **III** in $J(u',u)$. The remainder of Sec. V is unchanged except for Eq. (95) which is now

$$\prod_2^\infty\frac{l_\alpha}{4}=\frac{4}{\pi^2}[K(k)]^2(1-k^2)^{\frac{1}{2}}=\frac{4}{\pi^2}K^2\left[1-\left(\frac{2x_1}{1-x_1^2}\,\frac{2x_2}{1-x_2^2}\right)^2\right]^{\frac{1}{2}}.$$

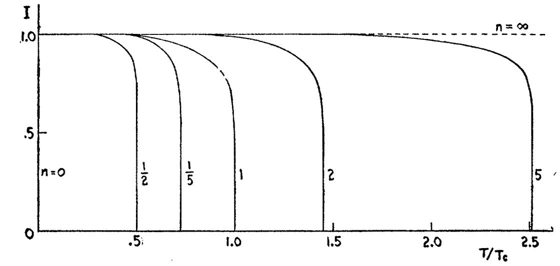

FIG. 1. The spontaneous magnetization of a two-dimensional rectangular Ising model.

The spontaneous magnetization I of a rectangular lattice is therefore

$$I=k'^{\frac{1}{4}}=(1-k^2)^{1/8}=\left[1-\left(\frac{2x_1}{1-x_1^2}\,\frac{2x_2}{1-x_2^2}\right)^2\right]^{1/8},$$

which is the same result as obtained by Kaugman and Onsager[4] for the long range order of the rectangular lattice using a different method. It reduces to the expression (96) for a square lattice.

Near the critical temperature the spontaneous magnetization varies proportionally to $(T_c-T)^{1/8}$. It is perhaps noteworthy that the exponent $\frac{1}{8}$ does not change with varying ratios of the vertical and horizontal interactions. One is tempted to conclude that the exponent is dependent only on the dimensionality of the lattice and not on the number of nearest neighbors.

In Fig. 1, I is plotted against T/T_c for various n's, where n is the ratio J_2/J_1 and T_c is the critical temperature for a square lattice with $J=J_1$.

The writer wishes to express his deep gratitude to Dr. C. N. Yang for suggesting this problem and for his guidance throughout this work.

[1] C. N. Yang, Phys. Rev. **85**, 808 (1952).
[2] B. Kaufman, Phys. Rev. **76**, 1232 (1949).
[3] All the section and equation numbers refer to Yang's paper.
[4] L. Onsager, Nuovo cimento **VI**, Suppl. p. 261 (1949).

The Nuclides Ba^{127}, Ba^{128}, and Cs^{128}

M. Lindner and R. N. Osborne
California Research and Development Company, Livermore, California
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IN the course of study of radioactive nuclides in the region of barium which decay through the electron capture process, a previously unreported isotope of barium was encountered. In addition, further studies were made on the decay characteristics of the isobaric chain

$$Ba^{128}\xrightarrow{\text{2.4 days}}Cs^{128}\xrightarrow{\text{3.1 min}}Xe^{128},$$

with which 3.0-Mev positrons are reported to be associated.[1,2]

A barium fraction was removed from a cesium nitrate target which had been bombarded with 190-Mev deuterons. The short-lived Cs^{128} daughter was separated from the parent barium by the addition of chilled absolute alcohol to a previously fumed and chilled perchloric acid solution of the barium to which cesium carrier had been added. The resultant precipitate of cesium perchlorate was collected in a sintered glass filter and washed with alcohol. The time interval from separation to counting could be made less than a minute. In three independent determinations, half-life values of 3.80, 3.75, and 3.80 minutes were obtained. No other activity was detectable when decay was followed through five half-lives. The cesium daughter accounted for at least 90 percent of the activity present in the barium fraction, as determined on an end-window argon-alcohol-filled counter. This figure does not include the loss of cesium sustained during its chemical separation. Thus, essentially all the activity in the 2.4-day decay is due to Cs^{128}. In addition, the curve for growth of the 3.8-minute