Letters to the Editor

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The Spontaneous Magnetization of a Two-Dimensional Rectangular Ising Model

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THE present letter gives the spontaneous magnetization of a two-dimensional rectangular Ising Model. This is obtained by the method used in a paper by Yang[1] which treats the same problem except for a square lattice. The only difference in setting up the problem for a rectangular lattice is that instead of having the same H for both the vertical and the horizontal interactions, we have H_1 and H_2 (which are Kaufman's H and H'[2]) for the vertical and the horizontal interactions, respectively; i.e.,

$$V_1=\exp\{H_1^*\sum_1^n C_r\},\quad V_2=\exp\{H_2\sum_1^n s_r s_{r+1}\}.$$

This leads to the condition $(x_1+1)(x_2+1)<2$ for temperatures below the critical temperature, where $x_1=\exp(-2H_1)$ and $x_2=\exp(-2H_2)$. The spontaneous magnetization I is expressed as an off-diagonal matrix element as before and the limiting process used there for reducing the calculation of I to an eigenvalue problem remains the same. In other words, all the results in Sec. I and II[3] of Yang's paper are unchanged.

In Sec. III, where an infinite crystal is considered, all the analysis remains the same except that H and H^* are replaced by H_2 and H_1^* in Eq. (60), and Eqs. (62) for A and B are replaced by

$$A=\coth H_2\coth H_1^*=(1+x_2)/x_1(1-x_2),$$
$$B=\tanh H_2\coth H_1^*=(1-x_2)/x_1(1+x_2).$$

In consequence, the modulus k in the elliptic integral is given by, instead of Eq. (78),

$$k=2k_{-1}^{\frac{1}{2}}/(1+k_{-1})=[2x_1/(1-x_1^2)][2x_2/(1-x_2^2)]=1/\sinh 2H_1\sinh 2H_2.$$

In Sec. IV the elliptic transformation (81) is replaced by

$$z=-(\mathrm{cn}u-i[1+k_1]^{\frac{1}{2}}\,\mathrm{sn}u)(\mathrm{dn}u-i[k_1+k_1k_2]^{\frac{1}{2}}\,\mathrm{sn}u)/(1+k_1\,\mathrm{sn}^2u),$$

with

$$\frac{1}{z}\frac{dz}{du}=-i\frac{1-k^2}{(1+k_1)^{\frac{1}{2}}}\frac{1}{\mathrm{dn}u-[k_1(1+k_2)/(1+k_1)]^{\frac{1}{2}}\,\mathrm{cn}u},$$

where $k_1=\sinh^{-2}2H_1$ and $k_2=\sinh^{-2}2H_2$. The essential properties of the variable z as a function of u remain the same as in the square lattice. There are still two singularities per unit cell $(4K\times 4iK')$, although their positions are changed.

In terms of the variable u, one reduces the integral equation (69) again into the integral equation (84). The change in the positions of the singularities of z does not affect one important property of the kernel $J(u',u)$ relevant in our calculation, namely, that $J(u',u)$ has only one simple pole inside a unit cell $(4K\times 4iK')$. This can be shown by examining the zeros of the factors **I** and **III** in $J(u',u)$. The remainder of Sec. V is unchanged except for Eq. (95) which is now

$$\prod_2^\infty\frac{l_\alpha}{4}=\frac{4}{\pi^2}[K(k)]^2(1-k^2)^{\frac{1}{4}}=\frac{4}{\pi^2}K^2\left[1-\left(\frac{2x_1}{1-x_1^2}\frac{2x_2}{1-x_2^2}\right)^2\right]^{\frac{1}{4}}.$$

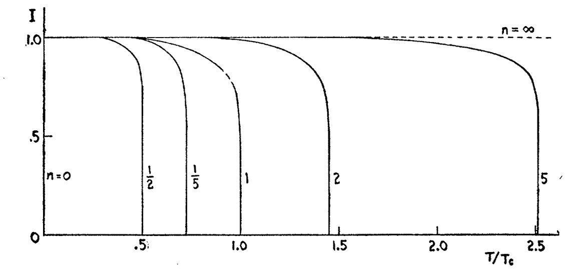

FIG. 1. The spontaneous magnetization of a two-dimensional rectangular Ising model.

The spontaneous magnetization I of a rectangular lattice is therefore

$$I=k'^{\frac{1}{4}}=(1-k^2)^{1/8}=\left[1-\left(\frac{2x_1}{1-x_1^2}\frac{2x_2}{1-x_2^2}\right)^2\right]^{1/8},$$

which is the same result as obtained by Kaugman and Onsager[4] for the long range order of the rectangular lattice using a different method. It reduces to the expression (96) for a square lattice.

Near the critical temperature the spontaneous magnetization varies proportionally to $(T_c-T)^{1/8}$. It is perhaps noteworthy that the exponent $\frac{1}{8}$ does not change with varying ratios of the vertical and horizontal interactions. One is tempted to conclude that the exponent is dependent only on the dimensionality of the lattice and not on the number of nearest neighbors.

In Fig. 1, I is plotted against T/T_c for various n's, where n is the ratio J_2/J_1 and T_c is the critical temperature for a square lattice with $J=J_1$.

The writer wishes to express his deep gratitude to Dr. C. N. Yang for suggesting this problem and for his guidance throughout this work.

[1] C. N. Yang, Phys. Rev. **85**, 808 (1952).
[2] B. Kaufman, Phys. Rev. **76**, 1232 (1949).
[3] All the section and equation numbers refer to Yang's paper.
[4] L. Onsager, Nuovo cimento **VI**, Suppl. p. 261 (1949).

The Nuclides Ba^{127}, Ba^{128}, and Cs^{128}

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IN the course of study of radioactive nuclides in the region of barium which decay through the electron capture process, a previously unreported isotope of barium was encountered. In addition, further studies were made on the decay characteristics of the isobaric chain

$$Ba^{128}\xrightarrow{\text{2.4 days}}Cs^{128}\xrightarrow{\text{3.1 min}}Xe^{128},$$

with which 3.0-Mev positrons are reported to be associated.[1,2]

A barium fraction was removed from a cesium nitrate target which had been bombarded with 190-Mev deuterons. The short-lived Cs^{128} daughter was separated from the parent barium by the addition of chilled absolute alcohol to a previously fumed and chilled perchloric acid solution of the barium to which cesium carrier had been added. The resultant precipitate of cesium perchlorate was collected in a sintered glass filter and washed with alcohol. The time interval from separation to counting could be made less than a minute. In three independent determinations, half-life values of 3.80, 3.75, and 3.80 minutes were obtained. No other activity was detectable when decay was followed through five half-lives. The cesium daughter accounted for at least 90 percent of the activity present in the barium fraction, as determined on an end-window argon-alcohol-filled counter. This figure does not include the loss of cesium sustained during its chemical separation. Thus, essentially all the activity in the 2.4-day decay is due to Cs^{128}. In addition, the curve for growth of the 3.8-minute cesium into a rapidly purified sample of barium followed that to be expected for the growth of a positron emitter into a parent which contained too small a quantity of particulate radiation to be detected with the end-window counter employed in the experiment. From these two facts it appears that Ba^{128} must decay principally by electron capture, since the equilibrium mixture of isobars contained the positrons reported by Fink and Templeton.[1] K-x-radiation was also found to be associated with the 3.8-minute cesium daughter as determined on a single-channel scintillation pulse analyzer. If these x-rays be due to electron capture rather than conversion of gamma-radiation, it would appear that 25 percent of the Cs^{128} decayed through electron capture. A barium sample was investigated for gamma-radiation on the pulse analyzer, but interference because of Compton recoils from the annihilation radiation rendered results inconclusive.

A barium isotope of 12-minute half-life was found whose radiations were not directly characterized. However, positron emission is probable since electromagnetic radiation seemed to comprise no more than five percent of the total activity detectable on an end-window counter. By four rapid chemical separations made at ten-minute intervals a cesium activity was obtained from the barium whose half-life and radiation characteristics agree with those reported for Cs^{127}.[3] Furthermore, the yield of this nuclide diminished roughly by a factor of two in each of the four successive separations. The 12-minute barium activity is thus Ba^{127}.

[1] R. W. Fink and D. H. Templeton, J. Am. Chem. Soc. **72**, 2818 (1950).
[2] R. W. Fink and E. O. Wiig, J. Am. Chem. Soc. **73**, 2365 (1951).
[3] Fink, Reynolds, and Templeton, Phys. Rev. **77**, 614 (1950).

The Cottrell-Bilby Theory of Yielding of Iron

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ALTHOUGH the mechanism of yielding of iron by Cottrell and Bilby[1] (C-B theory) is the best one that has been proposed, there seem to be ambiguities concerning the method of deriving the equation of temperature and strain rate dependence. It may be much more reasonable to derive it from the following equation:

$$dm/dt=m^2, \tag{1}$$

in which m denotes the transition probability. Equation (1) was previously developed by the author,[2] considering the yield phenomenon as a Markoff process. In the C-B theory, m refers to the probability that a dislocation is released from the carbon atmosphere by an external stress, and it is given by

$$m=B\exp[-U(\sigma/\sigma_0)/kT]. \tag{2}$$

Hence, substituting the value of m from Eq. (2) into Eq. (1), we find

$$\log(v/BkT)+\log(-\partial U/\partial\sigma)=-U/kT, \tag{3}$$

in which $v=d\sigma/dt=$ stress velocity. The change of T involved in $\log(1/T)$ may be neglected compared with that involved in U/kT. Only when the term $\log(-\partial U/\partial\sigma)$ is negligible, does Eq. (3) become

$$BkT=v\exp(U/kT), \tag{4}$$

which is the same formula as in the C-B theory.[1]

Corresponding to each four series of U–σ/σ_0 curves in the C-B paper,[1] the curves of $U/\alpha(2AW\rho)^{\frac{1}{2}}$ *versus* σ/σ_0 were drawn as solid lines (as shown in Fig. 1), which approximate the curves in the C-B paper. In Fig. 1 the α's are suitably chosen constants, which are different for each four series, respectively; and $\alpha(2AW\rho)^{\frac{1}{2}}$ has such a unit that the ordinates become dimensionless. It is found that these curves are approximately equivalent to the curve of $-\log(\sigma/\sigma_0)$, as shown by the dashed line in Fig. 1. That is,

$$U\doteqdot-(1/n)\log(\sigma/\sigma_0), \tag{5}$$

where $1/n=\alpha(2A\bar{W}\rho)^{\frac{1}{2}}$. Substituting $U=-(1/n)\log(\sigma/\sigma_0)$ in Eq. (3) gives

$$\log(v/nBkT\sigma_0)=(1/nkT+1)\log(\sigma/\sigma_0). \tag{6}$$

Since $1/nkT$ was found to be much larger than unity in the cases corresponding to these four series of U–σ/σ_0 curves, Eq. (6) becomes

$$nkT\log(v/nBkT\sigma_0)=\log(\sigma/\sigma_0). \tag{7}$$

Thus, the logarithm of yield stress should be proportional to the absolute temperature. The experimental results[3] are in good agreement with Eq. (7), as shown in Fig. 2.

It is interesting that from Eq. (7) the dependence of yield stress on strain rate $\dot{\epsilon}$ is given by the power law:

$$\sigma\propto\dot{\epsilon}^{nkT}. \tag{8}$$

If one uses the parameters corresponding to the No. 2 curve in the C-B paper,[1] then the exponent at room temperature is

$$nkT=0.022,$$

which is in good agreement with the value 0.020 obtained in Deutler's experiments[4] upon mild steel.

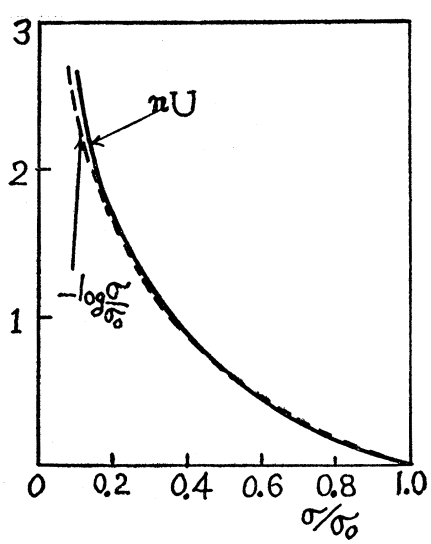

FIG. 1. The activation energy as a function of σ/σ_0 (from data of Cottrell and Bilby).

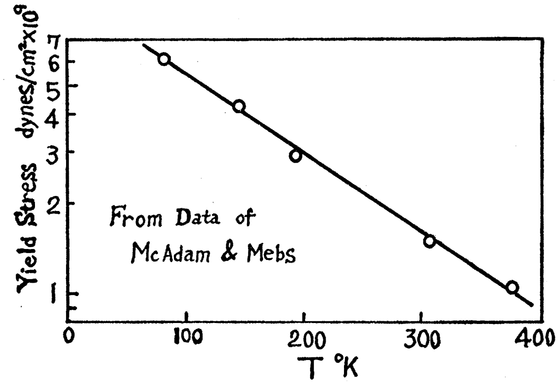

FIG. 2. The dependence of yield stress on temperature (from data of McAdam and Mebs).

[1] A. H. Cottrell and B. A. Bilby, Proc. Phys. Soc. (London) **62A**, 49 (1949).
[2] T. Yokobori, J. Phys. Soc. Japan **7**, 44 (1952).
[3] D. J. McAdam, Jr. and R. W. Mebs, Proc. Am. Soc. Testing Materials **43**, 661 (1943).
[4] C. Zener and J. H. Hollomon, J. Appl. Phys. **15**, 22 (1944).